MathBlast3

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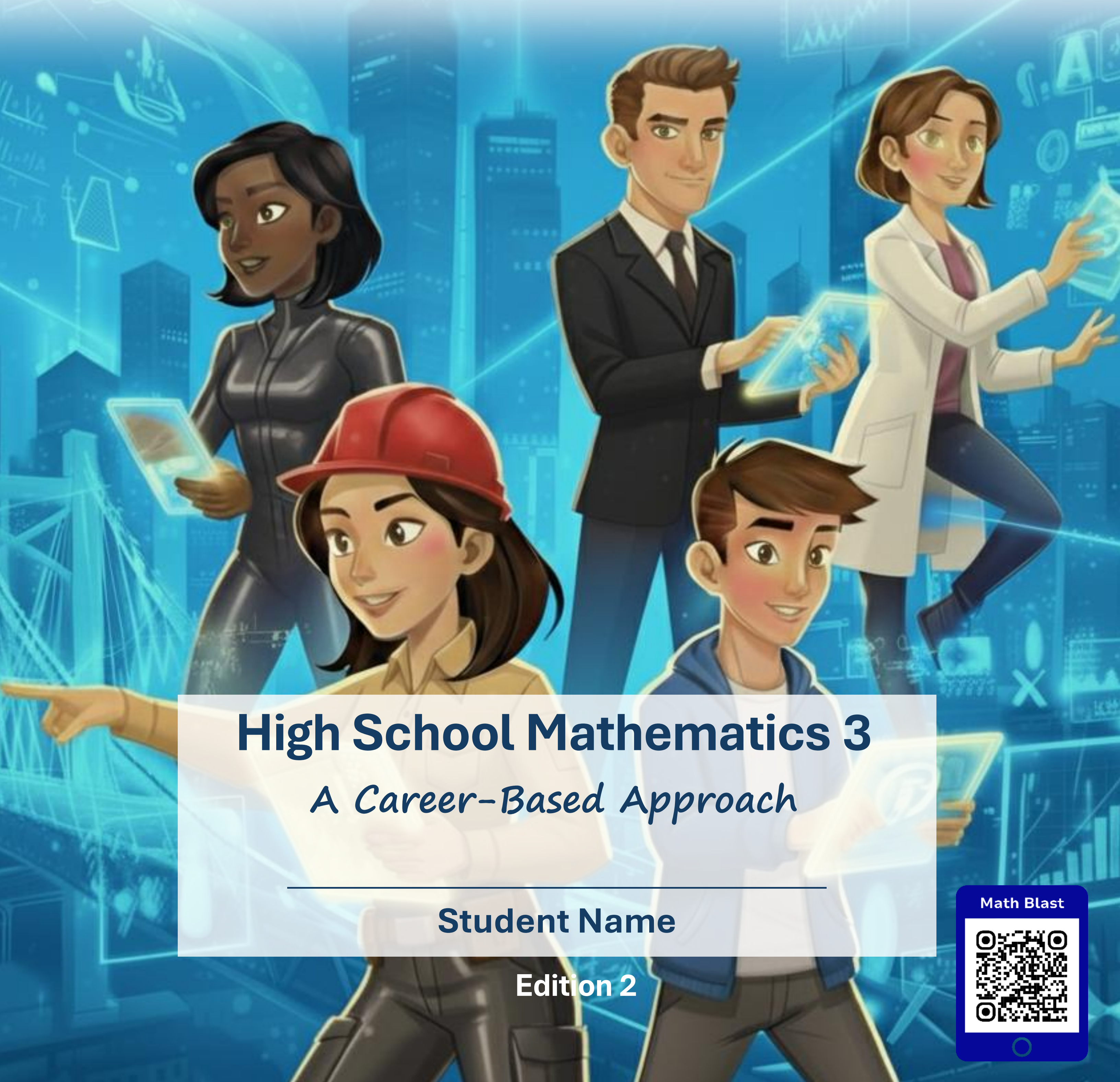

High School Mathematics 3

A Career-Based Approach

Student Name

Edition 2

Math Blast

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A Career-Based Approach

Edition 2
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Cybersecurity

High School Mathematics 3

North Carolina Math 3 Standards

Unit 1

Seeing Structure in Expressions *Interpret the structure of expressions.*	
NC.M3.A-SSE.1 **NC.M3.A-SSE.1a** **NC.M3.A-SSE.1b**	Interpret expressions that represent a quantity in terms of its context. a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.
NC.M3.A-SSE.2	Use the structure of an expression to identify ways to write equivalent expressions.
NC.M3.A-SSE.3	Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.
Creating Equations *Create equations that describe numbers or relationships.*	
NC.M3.A-CED.1	Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.
NC.M3.A-CED.2	Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.
NC.M3.A-CED.3	Create systems of equations and/or inequalities to model situations in context.
Interpreting Functions *Understand the concept of a function and use function notation.*	
NC.M3.F-IF.2	Use function notation to evaluate piecewise defined functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Interpreting Functions *Interpret functions that arise in applications in terms of the context.*	
NC.M3.F-IF.4	Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.
Interpreting Functions *Analyze functions using different representations.*	
NC.M3.F-IF.7	Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range;

	intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.
Building Functions *Build a function that models a relationship between two quantities.*	
NC.M3.F-BF.1	Write a function that describes a relationship between two quantities.
NC.M3.F-BF.1a	a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).
NC.M3.F-BF.1b	b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.
Building Functions *Build new functions from existing functions.*	
NC.M3.F-BF.3	Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k\cdot f(x)$, $f(x)+k$, $f(x+k)$ to include $f(k\cdot x)$ for specific values of k (both positive and negative).
NC.M3.F-BF.4 **NC.M3.F-BF.4a** **NC.M3.F-BF.4b** **NC.M3.F-BF.4c**	Find an inverse function. a. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations. b. Determine if an inverse function exists by analyzing tables, graphs, and equations. c. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, f-1, with a table, graph, or equation and use it to solve problems in terms of a context.

1.1 Key Features of Functions

Warm-Up

Write all you know about the following terms.

Domain	x-Intercept	Slope	Vertex
Range	y-Intercept	Asymptote	End Behavior

Main Topic | **Key Features of Functions**

A zero-day exploit is launched against a network. The number of infected machines **triples every hour** because each newly infected machine immediately tries to infect others.

TASK 1

Security teams must identify a mathematical equation to estimate how quickly the entire network will be compromised and prioritize containment if there are 2187 machines.

Definitions

Function: When the x-values (inputs) do NOT repeat.
Domain: The set of the 1st elements (x-values).
Range: The set of the 2nd elements (y-values).

What is the domain in task 1?

What is the range in task 1?

Main Topic	Applications of Key Features of Functions in Cybersecurity

A Security Information and Event Management (SIEM) system is collecting security logs from all devices. The log server has a total storage capacity of 5,000 GB. The system currently has 100 GB already stored and is receiving new logs at a steady rate of 10 GB per day.

Let S(d) be the amount of available storage (in GB) after d days.

$$\boldsymbol{S(d) = 5000 - (100 + 10d)}$$

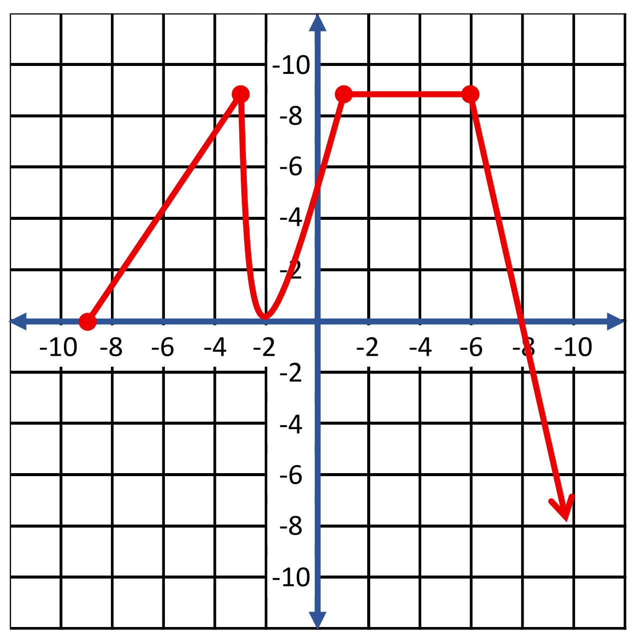
What is the domain (possible days)?

What is the range (possible available storage)?

TASK 2

Is the graph a function?

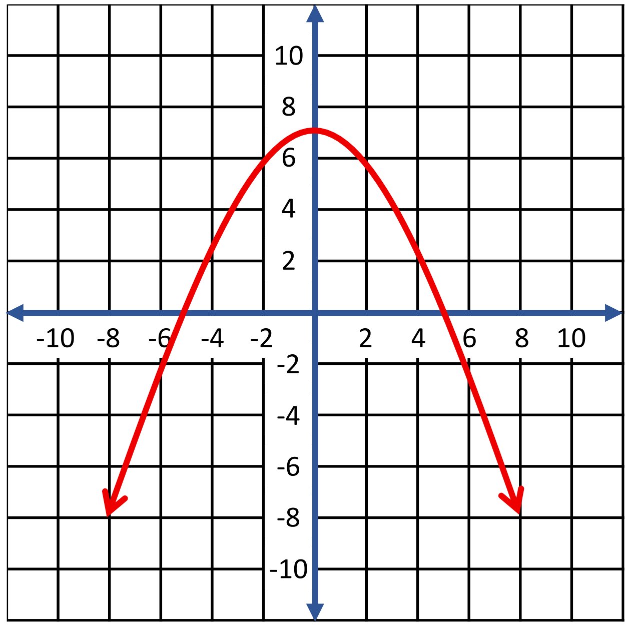
Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



TASK 3

Is the graph a function?

Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



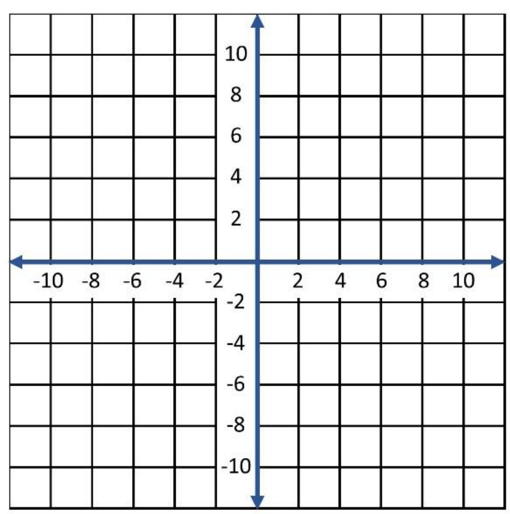
TASK 4

Is the graph a function?

$$y - 2x = 6$$

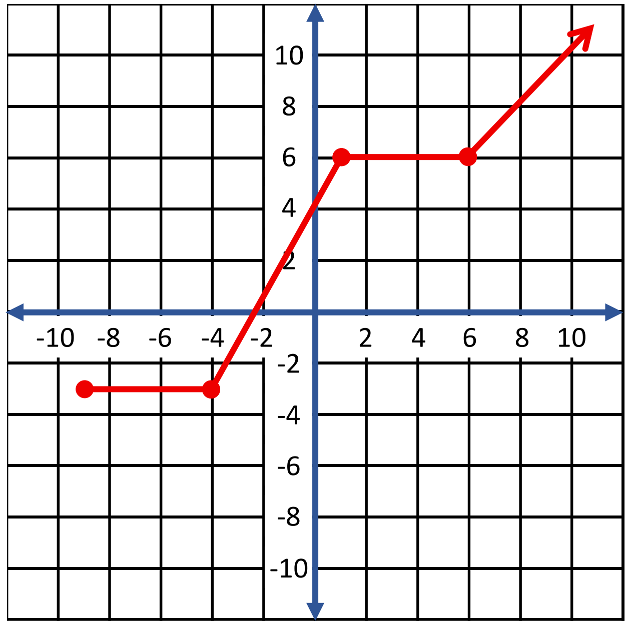
x	y

Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



Find the key features of the functions. Put a check mark in the circle if the graph is a function or not.

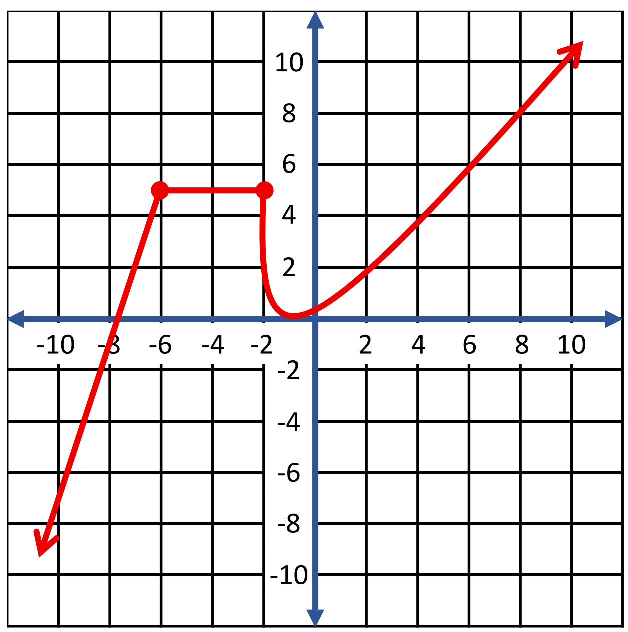
Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



Not a Function

Function

Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	

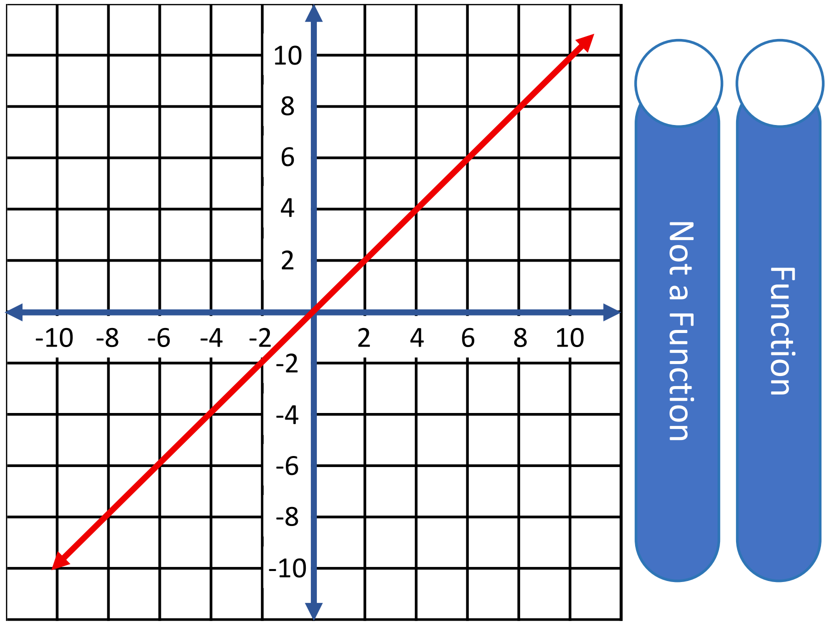


Not a Function

Function

Find the key features of the functions. Put a check mark in the circle if the graph is a function or not.

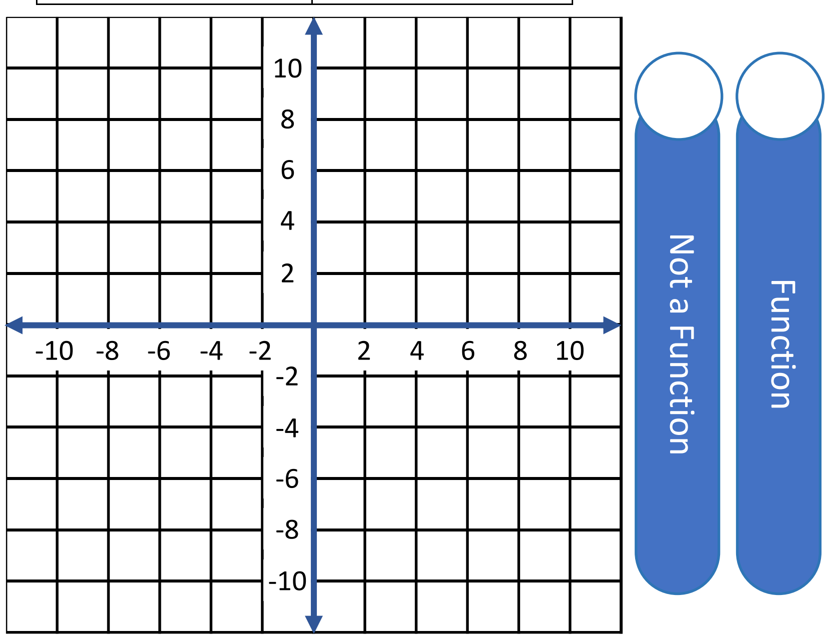
Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



$$y = x^2 + 2$$

x	y

Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	

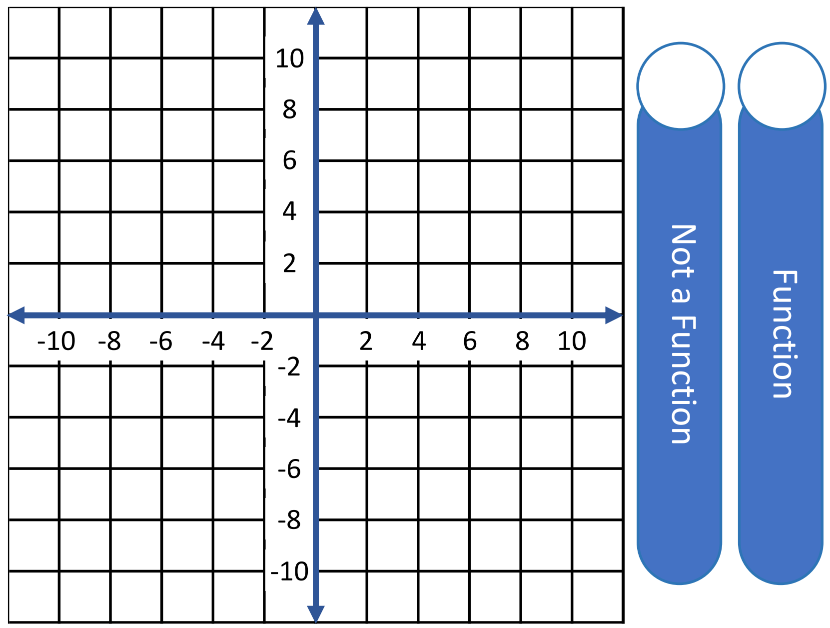


Find the key features of the functions. Put a check mark in the circle if the graph is a function or not.

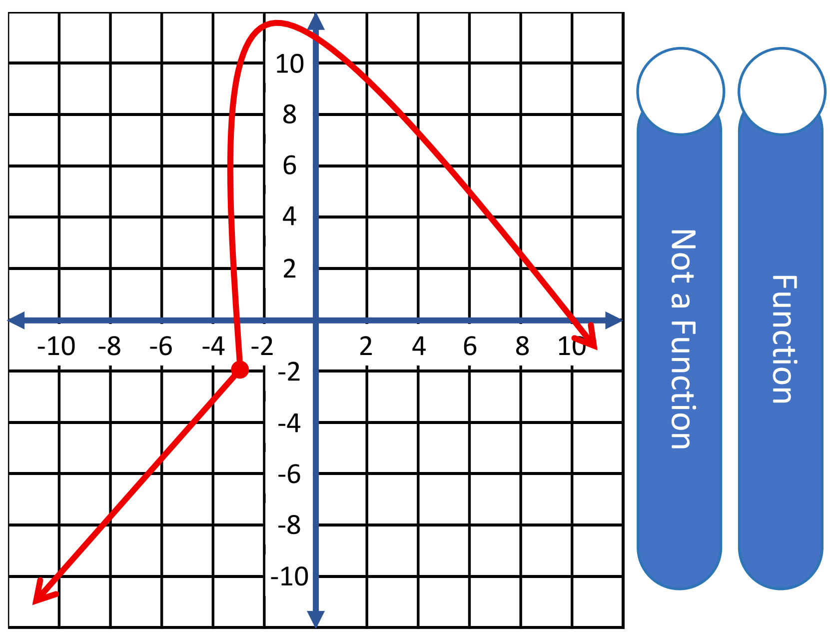
$$y = |x - 1| + 3$$

x	y

Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



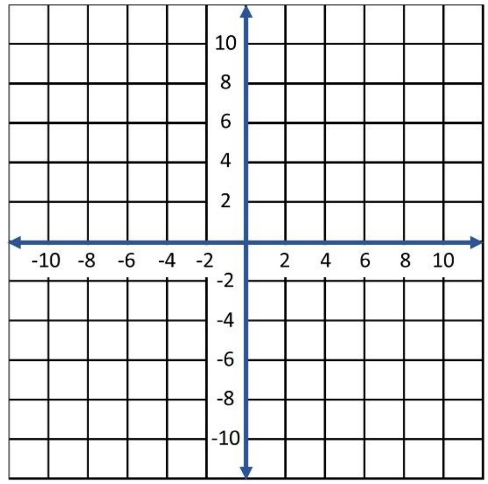
Domain	
Range	
x-intercept	
y-intercept	
Max	
Min	
Interval of Increasing	
Interval of Decreasing	
Left End Behavior	
Right End Behavior	



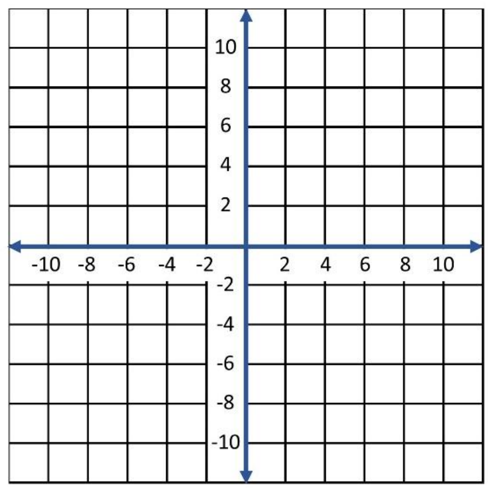
Honors Level

Sketch the graph given its features

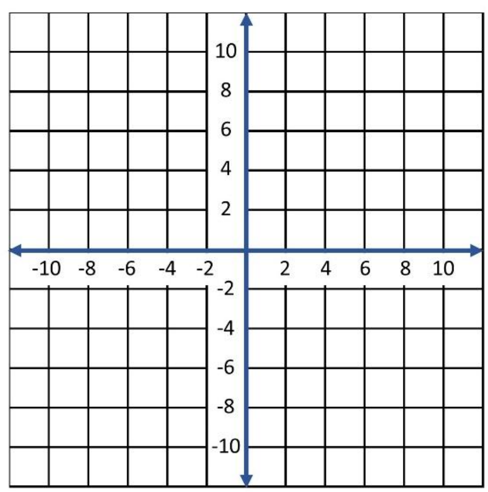
Domain	$(-\infty, \infty)$
Range	$(-\infty, \infty)$
x-intercept	{-4, -1, 3}
Left End Behavior	∞
Right End Behavior	$-\infty$



Domain	$(-\infty, \infty)$
Range	$(-7, \infty)$
x-intercept	{-2, -1, 0, 2}
Left End Behavior	∞
Right End Behavior	∞



Domain	$(-\infty, \infty)$
Range	$(-\infty, \infty)$
x-intercept	{-5, -3, -1, 2, 4}
Left End Behavior	$-\infty$
Right End Behavior	∞



Honors Level

What is the fundamental difference between a relation and a function?

Conceptually, what does the Vertical Line Test verify about a graph?

Explain the difference between the domain and the range of a function.

What does it mean for a function to be increasing over an interval?

Conceptually, what do the end behavior of a function's graph describe?

End-of-Course Prep

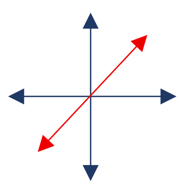
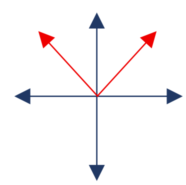
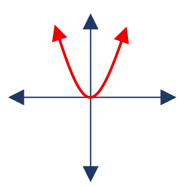
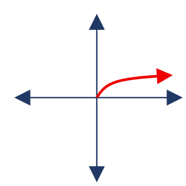
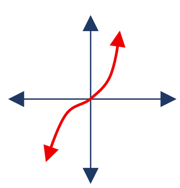
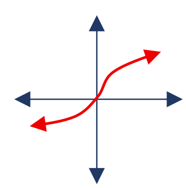
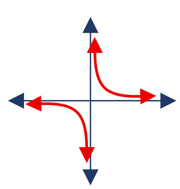
1.2 Parent Functions and Their Transformations

Warm-Up

Write all you notice from $f(x)$ to $g(x)$.

$f(x) = x^2 \rightarrow g(x) = x^2 + 3$ $\qquad$ $f(x) = x^2 \rightarrow g(x) = (x-3)^2 + 1$

Main Topic **Parent Functions**

Parent Function	Graph	Parent Function	Graph
Linear Function $y = x$		Absolute Value Function $y = \lvert x \rvert$	
Quadratic Function $y = x^2$		Square Root Function $y = \sqrt{x}$	
Cubic Function $y = x^3$		Cube Root Function $y = \sqrt[3]{x}$	
Reciprocal Function $y = \frac{1}{x}$		Write any observations on the graphs of the functions.	

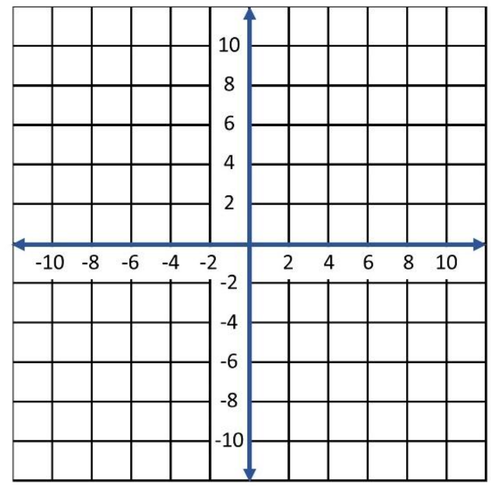
Main Topic | **Quadratic Function Transformations**

TASK 1

$$\boldsymbol{f(x) = (x-1)^2 + 2}$$

- Identify the parent function.

- Sketch the parent function and its transformation.
- Label your graph or use a different color for the transformed graph.
- Find the domain and range.



Create a quadratic function transformation rule.

____

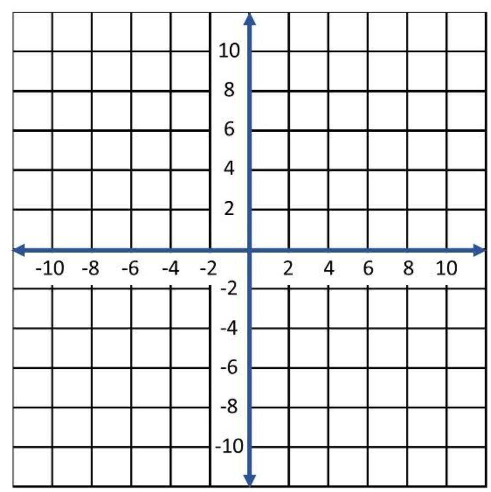
____

TASK 2

$$\boldsymbol{f(x) = -(x+4)^2}$$

- Identify the parent function.

- Sketch the parent function and its transformation.
- Label your graph or use a different color for the transformed graph.
- Find the domain and range.



Create a quadratic function transformation rule.

____

____

$f(x) = (x-2)^2 + 8$ is shifted 3 units to the right and 4 units down.
Write the equation of the transformed function.

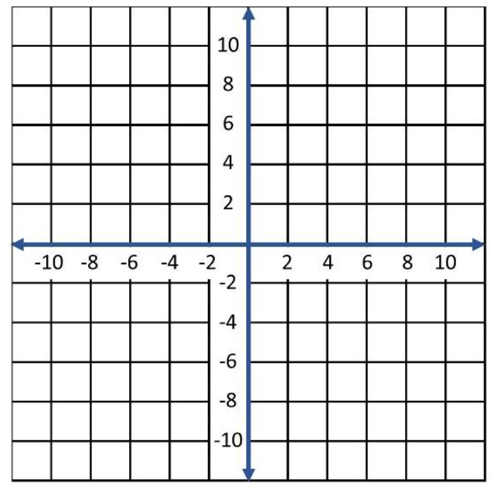
Main Topic Square Root Function Transformations

TASK 3

$$f(x) = \sqrt{x+3}$$

- Identify the parent function.

- Sketch the parent function and its transformation.
- Label your graph or use a different color for the transformed graph.
- Find the domain and range.



Create a square root function transformation rule.

____

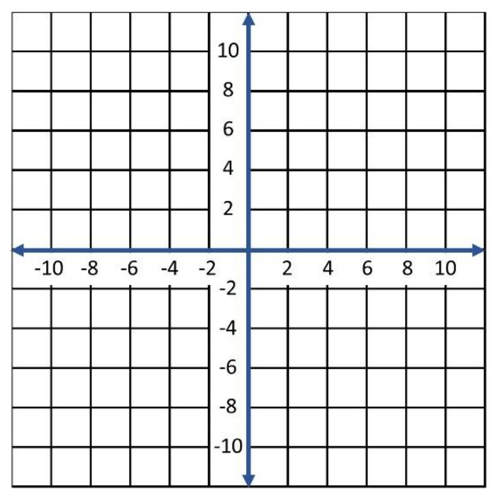
____

TASK 4

$$f(x) = \sqrt{-x+5}$$

- Identify the parent function.

- Sketch the parent function and its transformation.
- Label your graph or use a different color for the transformed graph.
- Find the domain and range.



Create a square root function transformation rule.

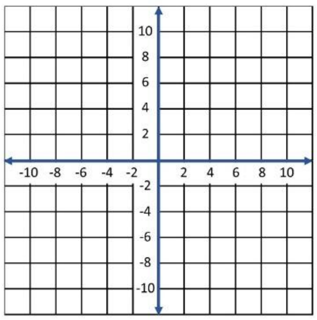
____

____

$f(x) = -\sqrt{x+7} + 3$ is reflected over the y-axis, shifted 2 units to the left, and 4 units up. Write the equation of the transformed function.

Practice

Identify the parent function. Sketch the parent function and its transformation.
Find the domain and range. Label your graph or use a different color for the transformed graph.



$$f(x) = (x + 3)^2 - 1$$

Parent Function:

Domain:

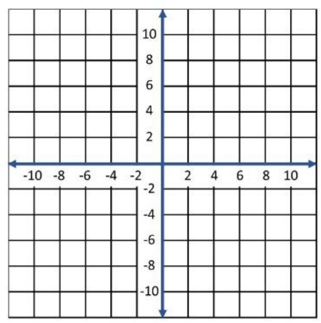
Range:

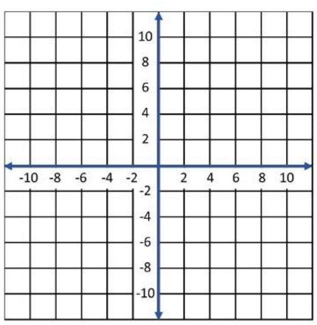
$$f(x) = x^2 + 3$$

Parent Function:

Domain:

Range:





$$f(x) = -\sqrt{x + 2}$$

Parent Function:

Domain:

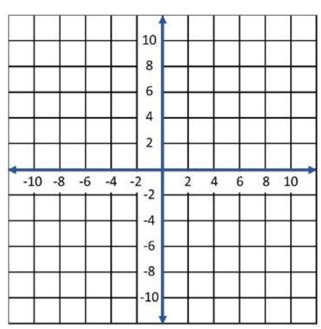
Range:

$$f(x) = |x + 3| - 2$$

Parent Function:

Domain:

Range:



Honors Level

What are the transformation rules when functions are shifted vertically?

What are the transformation rules when functions are reflected over the x-axis?

What are the transformation rules when functions are reflected over the y-axis?

What are the transformation rules when functions are compressed horizontally?

What are the transformation rules when functions are compressed vertically?

End-of-Course Prep

1.3 Absolute Value Equations

Warm-Up

Write the equation of $\boldsymbol{f(x) = |x+2|}$
when it is reflected over the y-axis, shifted 4 units to the right and 2 down.

Main Topic	Absolute Value Equations

Password Strength Deviation

Security policies often define an **ideal entropy score** for passwords, but allow some minor deviation for usability. An absolute value equation is used to check if a new password's entropy is within this acceptable range.

Understanding the Model

Let:

- E_{ideal} be the **ideal (target) entropy score** defined by the policy (measured in bits).
- E_{new} be the **actual entropy score** of the new password.
- D_{max} be the **maximum allowed deviation** from the ideal entropy.

The policy requires that the difference between the new password's entropy and the ideal entropy must be **less than or equal to** the maximum allowed deviation. This is expressed using an absolute value inequality:

$$\boldsymbol{|E_{new} - E_{ideal}| \leq D_{max}}$$

Imagine a security policy sets an **ideal entropy** (E_{ideal}) of **80 bits** and allows a **maximum deviation** (D_{max}) of **5 bits**.

Main Topic: Solving Absolute Value Equations

$$|2x - 1| - 3 = 15$$

- Get the absolute value (the "bars") to one side of the equal sign by ITSELF.
- Write the equation TWICE, if needed.
  1st Equation: DROP the bars and write the equation as is.
  2nd Equation: DROP the bars and write the equation as is BUT change the sign of the side without the bars.
- Solve both, if needed.
- Plug the answers back into equation to check for extraneous solutions.

Remember

Absolute value **CANNOT** equal a negative number

Extraneous Solution
Occurs when answers are plugged into original equation and makes the equation false

Definition

Solve for x.

$\lvert 3x - 7\rvert + 7 = 9$	$\lvert 3x + 12\rvert + 7 = 7$
$\lvert 3x - 7\rvert + 7 = 2$	$\lvert 3x - 2\rvert = 4x + 1$

Practice

Solve for x. Make sure to check for extraneous solutions.

$\lvert 2x + 1\rvert = 9$	$\lvert 4 - x\rvert = -10$	$\lvert 2x - 5\rvert = 3x + 1$
$2\lvert x - 3\rvert = x + 1$	$\lvert 2x + 5\rvert = 7 - x$	$-8\lvert 3 - 8x\rvert = 40$
$10 - 10\lvert -8t + 4\rvert = 10$	$\lvert x - 7\rvert + 2 = 2$	$\frac{1}{4}\lvert 2x - 6\rvert + 1 = 2$

Main Topic: Absolute Value Inequalities

$$|x-2| < 5$$

- Get the absolute value (the "bars") to one side of the inequality symbol by ITSELF.
- Look and see what the absolute value is equal to (use the cheats below to help you decide).
  Case 1: If the "| |" = 0, remove the bars and solve once.
  Case 2: If the "| |" = -#, it's no solution.
  Case 3: If the "| |" = +#, follow steps on solving absolute value equations (previous lesson).

Checkpoint (Put a check mark in the box when the inequality statement is true)						
|__- 2 | < 5	|__- 2 | < 5	|__- 2 | < 5	|__- 2 | < 5	|__- 2 | < 5	|__- 2 | < 5	|__- 2 | < 5
☐	☐	☐	☐	☐	☐	☐

Cheats

"| |" ≥ 0 All real numbers "| |" < 0 No Solution

Solve for x.

$\left\|\frac{x}{4}\right\| \geq 6$	$\|2x+1\| \geq 7$
$\|3x-2\| \leq 4$	$\|x+2\| < -5$

Practice

Solve the following absolute value inequalities. Make sure to check for extraneous solutions.

$\lvert x-1\rvert \leq 0$	$\lvert 4-x\rvert < 3$
$\lvert 3x-6\rvert \geq 0$	$\lvert 2x+1\rvert > -4$
$\lvert 2x+3\rvert < 3-2x$	$2+\lvert x+1\rvert < 6$
$\lvert x\rvert + 2 \geq 5$	$9\lvert 3x-2\rvert + 6 \geq 30$
$3+4\lvert 3x+7\rvert \geq -89$	$-5\lvert 2x+2\rvert - 3 \geq -3$

Honors Level

What does the expression $|x|$ fundamentally represent geometrically on a number line, and how does that relationship dictate the two-case algebraic approach for solving $|x| = k$?

Explain the conceptual difference between the solutions for $|x| = 5$ and the solutions for $x^2 = 25$. Why do both result in two non-zero solutions?

Why is the equation $|x| = k$ guaranteed to have no solution if k is a negative number?

If the equation is $|E| = 0$, why does this simplify to a single case ($E = 0$) rather than the standard two-case approach?

When solving an absolute value equation like $|2x - 1| = 7$, why must we check the solutions in the original equation, and what conceptual error could lead to an extraneous solution?

End-of-Course Prep

1.4 Operations of Functions

Warm-Up

Perform the operations.

$(x-4)(x-4)$ $\qquad$ $3(x-4)^2-(3-5x)+10$

Main Topic **Operations of Functions**

Malware Signature Consolidation (Incident Response)

In **Incident Response**, analysts identify unique strings (signatures) of malware. These signatures often have similar components that relate to the same malicious function (e.g., file access, network command, registry modification).

Polynomial Analogy: A security system detects multiple malware variants.

$$(\mathbf{3x^2+5xz})+(\mathbf{2x^2y-xz+4y^2z})$$

Terms: $\boldsymbol{x^2y}$ represents unique file access functions
$\boldsymbol{xz}$ represents unique network connection attempts
$\boldsymbol{y^2z}$ represents unique persistence mechanisms

Combining Like Terms: Consolidating the detected functions helps create a single, simplified detection rule (a meta-signature) for the whole family of malware.

TASK 1

Simplify:

$$(2v^3 + 4v^2 - 3) + (-2v^3 + 6v^2 - 7v + 2)$$

$$(5x^2 - 6x^3 + 3x^4 - 2x - 1) + (2x^3 + 3x^2 + 12x + 7)$$

TASK 2

Simplify:

$$(x^2 + 4x + 1) - (2x^2 - 3x + 6)$$

$$(5x^3 + 2x^2 - 4x + 1) - (3x^3 - x^2 + 6x - 5)$$

TASK 3

Simplify:

$$(2x^2 + 7x - 3) + (x^2 - 4x + 2) - (3x^2 + x - 5)$$

$$(2x^3 + 5x - 7x^3 - 1) - (5x^3 + 2x^2 + 1) + (6x^2 - 4x^3 + 2x - 1)$$

TASK 4

Simplify:

$(x + 3)(x + 2)$

$4x(2x - 5)$

$(2x + 1)(3x - 4)$

$(x - 2)(x^2 + 3x + 1)$

Operations Of Functions

$f(x) + g(x) = (f + g)(x)$ $\quad f(x) \cdot g(x) = (f \cdot g)(x)$

$f(x) - g(x) = (f - g)(x)$ $\quad f(x)/g(x) = (f/g)(x)$

$\boldsymbol{f(x) = x^2 - 4x - 12}$ **and** $\boldsymbol{g(x) = x + 2}$	
$(f + g)(x)$	$(f - g)(x)$
$(f \cdot g)(x)$	$\left(\frac{f}{g}\right)(x)$

Perform the operations of functions.

$\boldsymbol{f(x) = 5x^2 + x - 3}$ **and** $\boldsymbol{g(x) = x^2 + 4x}$	
$(f + g)(x)$	$(f - g)(x)$
$(f \cdot g)(x)$	$\left(\frac{f}{g}\right)(x)$

Functions Matching

Match the operations of functions with the results in the middle by connecting the boxes

Column A

$f(x) = x^2 - 5x - 1$
and
$g(x) = 3 - 2x$

$(f + g)(x)$

$f(x) = x^2 - 5x - 1$
and
$g(x) = 3 - 2x$

$g(x) \cdot g(x)$

$f(x) = 4x^2 - 6x$
and
$g(x) = 2x^2$

$\frac{f(x)}{g(x)}$

Results

$4x^2 - 12x + 9$

$x^2 - 3x - 4$

$x^2 - 7x + 2$

$-x^2 + 3x + 4$

$x^4 - 12x^3 + 36x^2$

$\frac{2x - 3}{x}$

Column B

$f(x) = x^2 - 5x - 1$
and
$g(x) = 3 - 2x$

$f(x) - g(x)$

$f(x) = x^2 - 6x$
and
$g(x) = 3 - 2x$

$f(x) \cdot f(x)$

$f(x) = x^2 - 5x - 1$
and
$g(x) = 3 - 2x$

$(g - f)(x)$

Main Topic	Evaluating Functions

$$j(x) = x^2 + 5 \quad \text{and} \quad k(x) = 4x$$

Teacher Turn	Student Turn
$(j+k)(2)$	$(j+k)(5)$
Teacher Turn	**Student Turn**
$(j-k)(2)$	$(j-k)(5)$
Teacher Turn	**Student Turn**
$(j \cdot k)(2)$	$(j \cdot k)(5)$
Teacher Turn	**Student Turn**
$\left(\frac{j}{k}\right)(2)$	$\left(\frac{j}{k}\right)(5)$

Turn it Up!

My Turn
Solve the questions on your own

Your Turn
Have a partner check your work

Our Turn
Have the group check your work

Perform the Operations

$f(x) = 3x^2 - x - 1$

and

$g(x) = 2 - 5x$

$(f + g)(-1)$

$(g - f)(2)$

$(g \cdot g)(-2)$

$\left(\frac{g}{f}\right)(-5)$

____
Owner Name

____
Partner Name

____
Group Member Names

Honors Level

If f(x) represents the cost of producing x units and g(x) represents the revenue from selling x units, what real-world meaning does the function (g - f)(x) have, and why is this subtraction a useful operational concept?

Explain why the domain of (f + g)(x) or (f - g)(x) must be the intersection of the individual domains of f(x) and g(x).

Given two linear functions, f(x) = mx + b and g(x) = nx + c, what type of function results from $(f \cdot g)(x)$? Justify your answer.

Why must the function g(x) *not* equal zero in the domain of the quotient function $\left(\frac{f}{g}\right)(x)$.

How does this rule change the domain compared to the other three basic operations?

If f(x) is an even function and g(x) is an odd function, what can you conceptually deduce about the symmetry of the product function $(f \cdot g)(x)$?

End-of-Course Prep

1.5 Composition of Functions

Warm-Up

If $x = a + 5$, then what is $x^2 + 3x - 9$?

Main Topic Composition of Functions

Composition of Functions

$(f \circ g)(x)$ is the composition of function f with function g and it can be rewritten as $f(g(x))$. It means that $g(x)$ is plugged into $f(x)$.

Statement		Expression
The composition of function h with function k.	→	

Statement		Expression
	←	$d(e(x))$

Expression	Statement
$f(g(x))$	The composition of function f with function g.
Plug in $x - 3$ to $x^2 + 5$	$(x-3)^2 + 5$ $x^2 - 6x + 9 + 5$ $x^2 - 6x + 14$ $f(g(x)) = x^2 - 6x + 14$
Plan	**Work**

$f(x) = x^2 + 5$
and
$g(x) = x - 3$

Need Help?

Expression	Statement
$g(f(x))$	
$f(x) = x^2 + 5$ and $g(x) = x - 3$	
Plan	**Work**

Expression	Statement
	The composition of function f with function g.
$f(x) = x^2 - x + 4$ and $g(x) = x + 2$	
Plan	**Work**

Expression	Statement
$g(f(x))$	
$f(x) = x^2 - x + 4$ and $g(x) = x + 2$	
Plan	**Work**

$f(x) = x^2 - 4x - 12$ and $g(x) = x + 2$	
$(f \circ g)(x)$	$(g \circ f)(x)$
$(g \circ g)(x)$	$(f \circ f)(x)$
$f(g(x))$	$g(f(x))$

Quick Math

$f(x) = (x-3)^2$ and $g(x) = x + 5$.
What is $(f \circ g)(x)$?

Functions Matching

Match the operations of functions with the results in the middle by connecting the shapes.

Column A		Column B
$f(x) = x^2 - 5x - 1$ and $g(x) = 3 - 2x$ $f(x)+g(x)$	$4x^2 - 2x - 7$ $x^2 - 3x - 4$	$f(x) = x^2 - 5x - 1$ and $g(x) = 3 - 2x$ $f(x) - g(x)$
$f(x) = x^2 - 5x - 1$ and $g(x) = 3 - 2x$ $f(g(x))$	$x^2 - 7x + 2$ $-x^2 + 3x + 4$	$f(x) = x^2 - 5x - 1$ and $g(x) = 3 - 2x$ $(g \circ g)(x)$
$f(x) = x^2 - 5x - 1$ and $g(x) = 3 - 2x$ $(g \circ f)(x)$	$-2x^2 + 10x + 5$ $4x - 3$	$f(x) = x^2 - 5x - 1$ and $g(x) = 3 - 2x$ $g(x) - f(x)$

Honors Level

Explain the concept of function composition $(fog)(x)$ in terms of an input/output machine. Which function's output acts as the input for the other?

How does the process of evaluating $(fog)(3)$ differ from evaluating $(f \cdot g)(x)$? Describe the steps for each.

Describe the conceptual rule for finding the domain of a composite function $(fog)(x)$. Why must you consider both the domain of the inner function, g(x), *and* the domain of the resulting expression, f(g(x))?

If $h(x) = (fog)(x)$, and $h(x)$ has a domain of all real numbers, does that guarantee that the domain of g(x) is also all real numbers? Explain why or why not.

Why is the order of composition critical (i.e., $(fog)(x) \neq (gof)(x)$, in general)? Provide a simple conceptual example where the order clearly matters.

End-of-Course Prep

1.6 Inverse of Linear Functions

Warm-Up

Solve for y.

$x = y - 5$ $\qquad$ $x = \frac{y}{3} - 5$ $\qquad$ $x = \frac{y-5}{3}$

Main Topic | **Inverse of Linear Functions**

The concept of the **inverse of a function** is essential in cybersecurity, particularly in processes that must be reversible, such as encryption/decryption, or in systems where an initial transformation must be undone to verify data.

Symmetric Encryption and Decryption

Symmetric-key cryptography is the most direct application of inverse functions. The encryption function, E, must have an inverse function, D, such that decrypting the ciphertext returns the original plaintext.

- **Function E(P) (Encryption):** Takes a **plaintext message** P and, using a shared secret key k, transforms it into a unique **ciphertext** C.

$$\boldsymbol{E_k(P) = C}$$

- **Function D(C) (Decryption):** Takes the **ciphertext** C and, using the *same* secret key k, transforms it back into the original plaintext P.

$$\boldsymbol{D_k(C) = P}$$

Inverse Relationship: D(E(P)) = P

Decryption D is the **inverse function** of encryption E. The entire point of a working symmetric cipher (like AES) is that the decryption process perfectly undoes the encryption process. If D were not the exact inverse of E, the original message could not be recovered, and the system would be useless for confidentiality.

Inverse of Functions

An **inverse function** reverses the process of a function.
It undoes the process of a function. It is denoted by f^{-1} or y^{-1}.

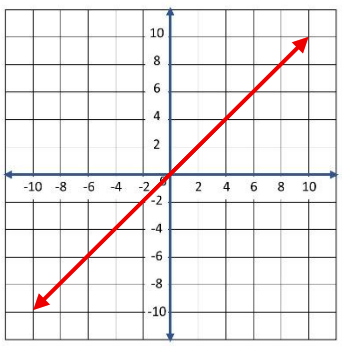
RECALL
An equation is only a function if the x-values do not repeat. It also needs to pass the Vertical Line Test (VLT).

Not all functions have inverses.

They must be one-to-one.

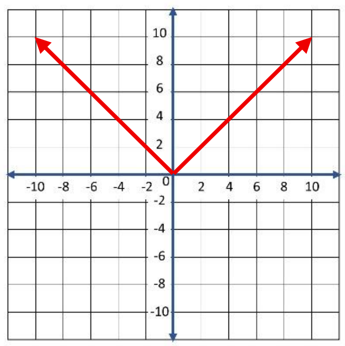
One-to-One
The y-values cannot repeat. In graphs, it needs to pass the Horizontal Line Test (HLT).

TASK 1



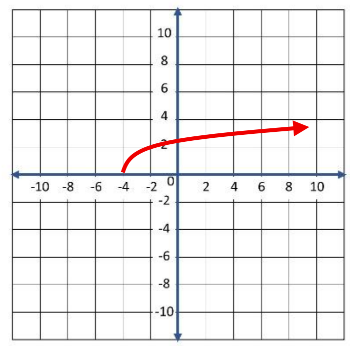
Function?
YES / NO

One-to-One?
YES / NO



Function?
YES / NO

One-to-One?
YES / NO



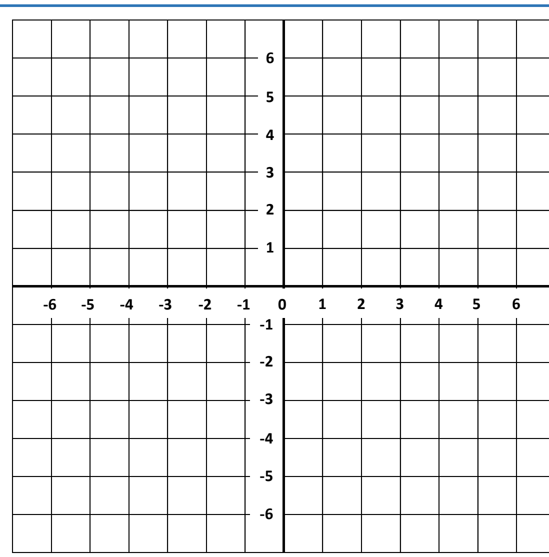
Function?
YES / NO

One-to-One?
YES / NO

TASK 2

- Graph: $f(x) = \frac{5}{2}x - 2$
- Reflect the graph over $y = x$.
- List ordered pairs in the table.

$f(x)$		$f^{-1}(x)$	
x	y	x	y



Turn-Taking Dynamics

Teacher Turn	Student Turn
$f(x) = x - 2$	$f(x) = x + 5$
Teacher Turn	**Student Turn**
$f(x) = \frac{2}{3}x + 4$	$f(x) = 7 - \frac{5}{4}x$
Teacher Turn	**Student Turn**
$f(x) = 2x + 4$	$f(x) = 3x - 6$
Teacher Turn	**Student Turn**
$f(x) = 7 - 4x$	$f(x) = 10 - 3x$
Teacher Turn	**Student Turn**
$f(x) = \frac{3x + 5}{2}$	$f(x) = \frac{7x - 2}{3}$

Agree or Disagree?

Find the inverse of each function. Find a partner to agree or disagree with your work.

$$f(x) = \frac{7}{8}x - 4$$

Disagree? Show correct work here.

$$g(x) = \frac{2 - x}{8}$$

Disagree? Show correct work here.

$$h(x) = 2(x - 3) + 9$$

Disagree? Show correct work here.

$$i(x) = \frac{2 - x}{5} - 7$$

Disagree? Show correct work here.

Inverse Functions

The functions that are inverses must satisfy the equations below.
$(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

Find the inverse.	Perform the composition of the function and its inverse.
$f(x) = 2x - 13$	
$f(x) = \frac{2}{7}x + 1$	
$f(x) = \frac{3x-5}{2} - 4$	

Need Help?

Honors Level

Why are all non-constant linear functions guaranteed to have an inverse function?
(Hint: Relate this to the Horizontal Line Test.)

If f(x) is a linear function, explain the relationship between the domain of f(x) and the range of its inverse, $f^{-1}(x)$.

Explain why the line y = x is crucial to understanding the graphical relationship between a function and its inverse.

If a linear function passes through the point (a, b), what specific point must its inverse function $f^{-1}(x)$ pass through, and why?

How does the concept of function composition prove that g(x) is the inverse of f(x)?
State the two required compositional proofs.

End-of-Course Prep

1.7 Inverse of Quadratic and Square Root Functions

Warm-Up

Solve for y.

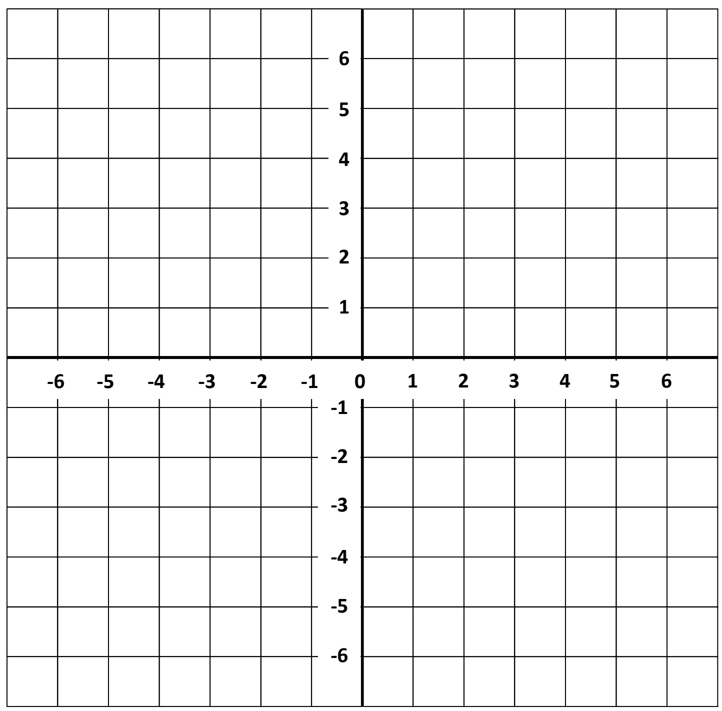
$x = y^2 - 5$ $\qquad$ $x = \frac{y^2}{3} - 5$ $\qquad$ $x = (y+3)^2 - 5$

Main Topic **Inverse of Quadratic Functions**

TASK 1

- Graph: $f(x) = x^2 - 2,\ x \geq 0$
- Fill out the table.

$f(x)$		$f^{-1}(x)$	
x	y	x	y
0	-2		
1	-1		
2	2		
3	7		



Write your observations.

____

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____

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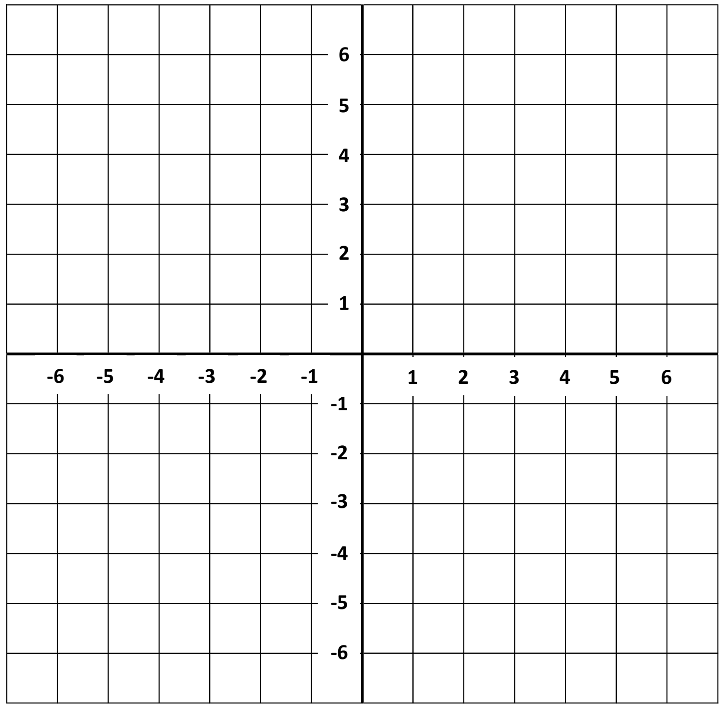
Need Help?

Main Topic	Inverse of Square Root Functions

TASK 2

- Graph: $f(x) = \sqrt{x} - 2$
- Reflect the graph over $y = x$.
- Fill out the table.

$f(x)$		$f^{-1}(x)$	
x	y	x	y
-2			
-1			
0			
1			
2			
3			
4			



State the domain of the function and find its inverse.

Write your observations.

____

____

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Need Help?

State the domain of the given function and find its inverse.

$y = \sqrt{x-5}$	$y = \sqrt{x+5}$
$y = \sqrt{3x-1}$	$y = \sqrt{1-3x}$
$y = 5\sqrt{x-1}$	$y = \dfrac{\sqrt{x-1}}{5}$
$y = 7 - 5\sqrt{x-1}$	$y = \dfrac{\sqrt{x-1}}{5} + 4$

Main Topic	Applications of Inverse of Functions in Cybersecurity

Secure Data Obfuscation and De-obfuscation in a Restricted Range

Imagine a system where sensitive data, like a specific range of internal port numbers or resource IDs, needs to be temporarily "scrambled" for transmission or storage within a very controlled environment.

- **Scenario:** A critical system component generates internal resource IDs, x, which fall within a known, small, positive integer range (e.g., $x \in [1, 100]$). To prevent simple enumeration or guessing if a packet is intercepted, these IDs are transformed before transmission.

- **Function f(x) (Obfuscation):** A quadratic function like $f(x) = x^2 + c$ (where c is a small offset and the domain is restricted to positive integers) could be used to generate an obfuscated ID y.
  For example, $f(x) = x^2 + 5$.

  If $x = 10, y = 10^2 + 5 = 105$

  Inverse Function $f^{-1}(y)$ (De-obfuscation): At the receiving end, the system needs to reverse this transformation to get the original ID x.

  Find the inverse of f.

  If the system receives $y = 105$, what is the value of x?

Data Obfuscation

Data obfuscation in cybersecurity is the technique of intentionally making data confusing, misleading, or difficult to understand to unauthorized parties, while still allowing the authorized systems or users to access and process the underlying information

It is a defensive measure used to hide the true meaning or value of data without necessarily resorting to full, reversible cryptographic encryption.

Honors Level

Explain why the inverse of the basic square root function, $f(x) = \sqrt{x}$, is not the entire parabola $y = x^2$, but rather only half of it.

What is the necessary domain restriction on the original square root function, f(x), that guarantees its inverse will be a function?

Describe the relationship between the domain and range of a square root function and the domain and range of its inverse.

What is the smallest necessary domain restriction you must apply to a quadratic function to ensure its inverse exists? How is this restriction related to the vertex?

Conceptually, how is the graph of the inverse of a quadratic function (after domain restriction) related to the graph of the original function?

End-of-Course Prep

1.8 Linear and Piecewise Functions

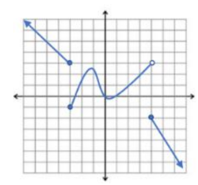
Warm-Up

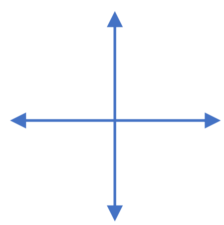
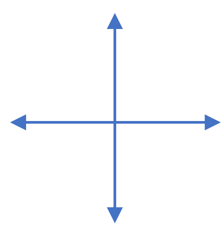
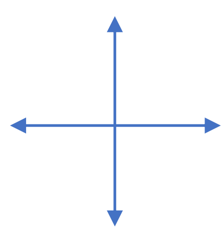
If $g(x) = 5x^2 - 2x + 3$, evaluate $g(-3)$.

Main Topic **Piecewise Functions**

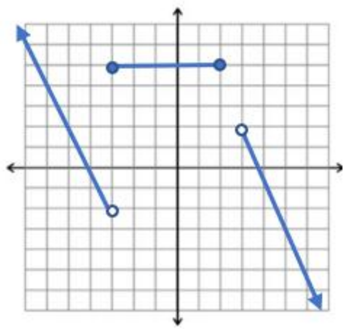
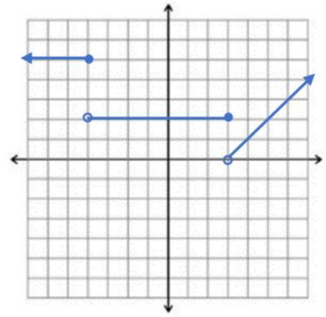
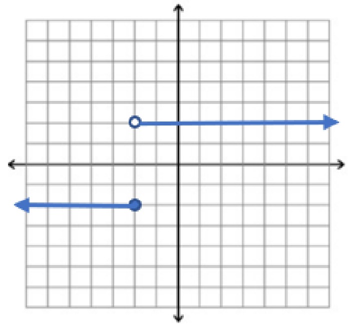
Piecewise Function

A **piecewise function** is a function composed of several functions that are cut into pieces. Each piece applies different interval of the input domain.



Function	Key Features	Graph	Evaluate
$f(x) = \begin{cases} 2x+1, & x \le -2 \\ 1, & -2 < x < 3 \\ (x-3)^2 - 3, & x \ge 3 \end{cases}$	Domain: Range:		$f(-2)$ $f(1)$ $f(6)$
$f(x) = \begin{cases} -\frac{1}{2}x + 2, & x < -2 \\ -(x-1)^2 + 3, & -2 \le x \le 3 \\ x+1, & x > 3 \end{cases}$	Domain: Range:		$f(-2)$ $f(1)$ $f(4)$
$f(x) = \begin{cases} -x+3, & x > 2 \\ x^4 - 2x^2 + 8x + 1, & x \le 2 \end{cases}$	Domain: Range:		$f(12)$ $f(0)$ $f(2)$

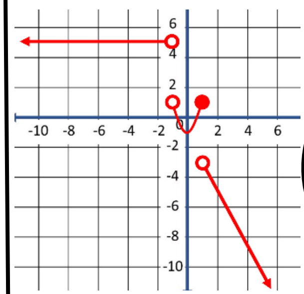
Fill in the missing information given the graphs of the piecewise functions.

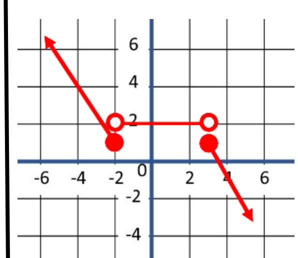
Function	Key Features	Graph	Evaluate
$f(x) =$	Domain: Range:		$f(5)$ $f(0)$ $f(-3)$
$f(x) =$	Domain: Range:		$f(0)$ $f(-15)$ $f(-4)$
$f(x) =$	Domain: Range:		$f(12)$ $f(0)$ $f(-2)$

Making Connections

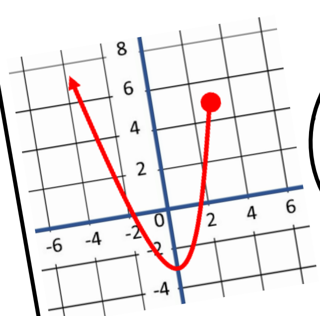
Match the piecewise functions to their graphs by drawing a line to connect the pieces.

$$f(x) = \begin{cases} -3x - 5, x \le -2 \\ 2, -2 < x < 3 \\ -2x + 7, x \ge 3 \end{cases}$$





$$f(x) = \begin{cases} 5, x \le -1 \\ 2x^2 - 1, -1 < x \le 1 \\ -2x - 1, x > 1 \end{cases}$$



$$f(x) = \begin{cases} x^2 - 3, x \le 3 \\ 2x + 1, x > 3 \end{cases}$$

Honors Level

Explain, in your own words, the difference between a single, continuous function and a piecewise function. What fundamental characteristic defines the domain of a piecewise function?

What does it mean for a piecewise function to be continuous at a boundary point (or *seam*)? Describe the algebraic condition that must be met by the two adjacent linear rules.

A piecewise linear function has a jump discontinuity. Explain what this visually implies about the limits of the two linear segments as they approach the boundary point.

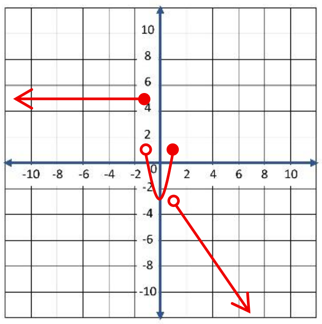
Given the graph of a piecewise linear function, how can you immediately determine the domain restrictions (the x-values where the rules change)?

If a piecewise function consists of three linear segments, how many distinct slopes can the function have, and what does each slope represent in a contextual scenario?

End-of-Course Prep

1.9 Piecewise Functions (Square Root, Radical, & Rational)

Warm-Up



Domain:

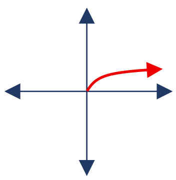
Range:

End Behavior:

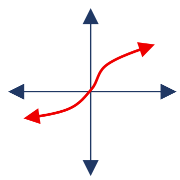
Increasing: Decreasing:

Main Topic Piecewise Functions (Square Root, Radical, & Rational)

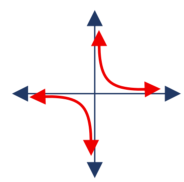
Square Root

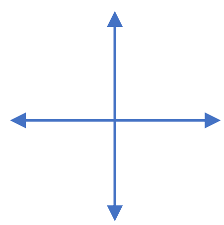
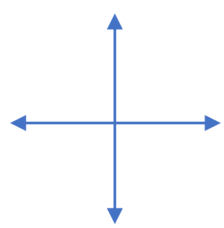
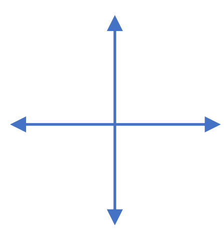


Radical

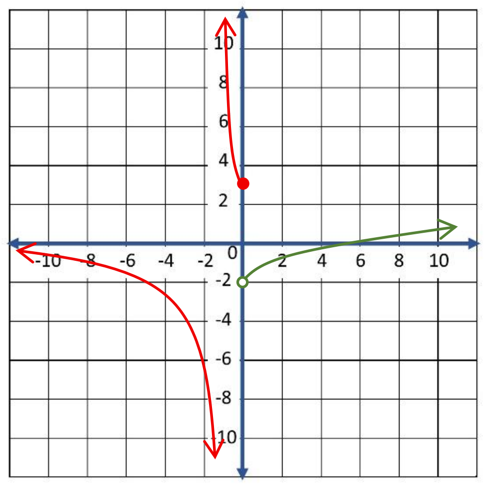
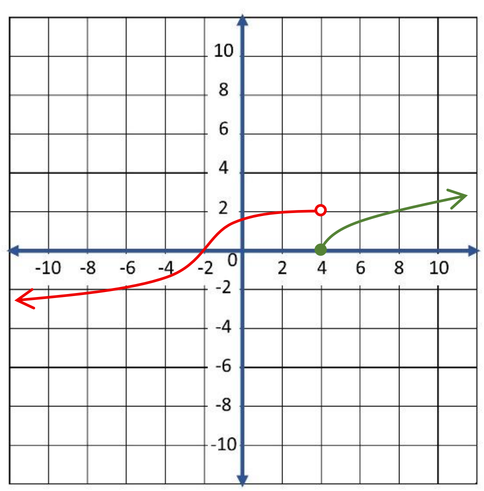


Rational



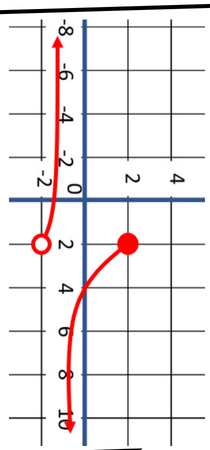
Function	Key Features	Graph	Evaluate
$f(x) = \begin{cases} \sqrt[3]{x} - 2, & x < 1 \\ \sqrt{x-1} + 2, & x \geq 1 \end{cases}$	Domain: Range:		$f(-2)$ $f(1)$ $f(10)$
$f(x) = \begin{cases} \dfrac{2}{x-4}, & x \leq 3 \\ \sqrt{x-3} + 4, & x > 3 \end{cases}$	Domain: Range:		$f(-1)$ $f(3)$ $f(28)$
$f(x) = \begin{cases} \dfrac{3}{x+1}, & x \leq 0 \\ \sqrt{x+1} - 3, & x > 0 \end{cases}$	Domain: Range:		$f(15)$ $f(0)$ $f(-1)$

Fill in the missing information given the graphs of the piecewise functions.

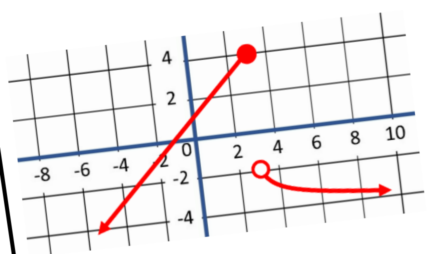
Function	**Key Features**	**Graph**	**Evaluate**
$f(x) =$	Domain: Range:		$f(-2)$ $f(0)$ $f(3)$
$f(x) =$	Domain: Range:		$f(4)$ $f(9)$ $f(1)$

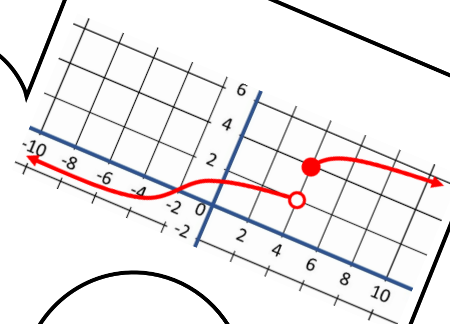
Making Connections

Match the piecewise functions to their graphs by drawing a line to connect the pieces.



$$f(x)=\begin{cases}\sqrt[3]{x+2}, x<4\\ \sqrt{x}+2, x\geq 4\end{cases}$$



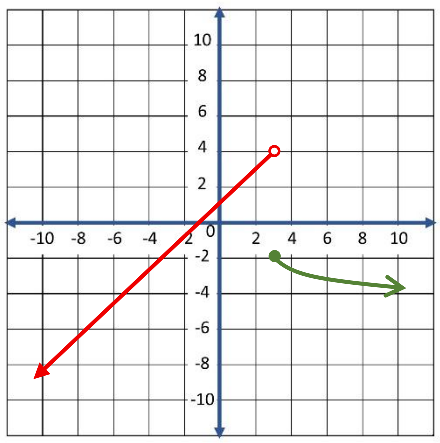
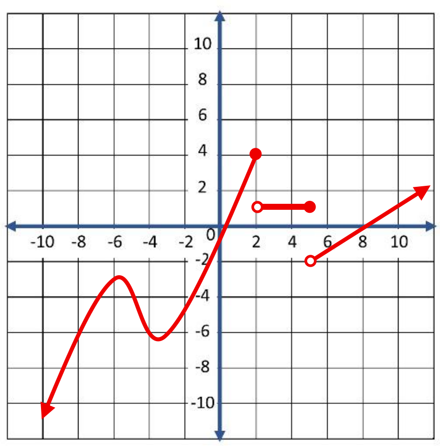
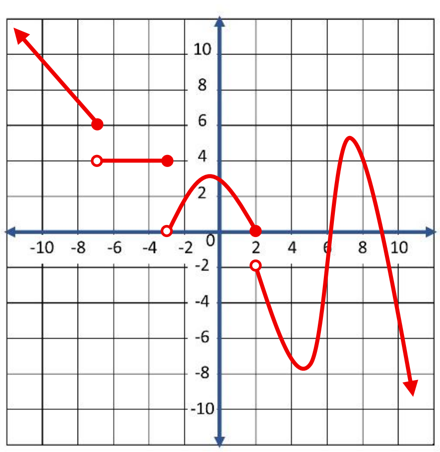


$$f(x)=\begin{cases}\dfrac{x^2-4}{x+2}+3, x\leq 3\\ -\sqrt{x-2}-1, x>3\end{cases}$$

$$f(x)=\begin{cases}\dfrac{x-3}{2}, x<2\\ -\sqrt{x}+2, x\geq 2\end{cases}$$

Main Topic **Piecewise Functions and their Key Features**

Let's see what we can remember about the key features of Piecewise Functions.

Piecewise Functions	Key Features	
	Domain Range Increasing Intercepts	Positive Negative Decreasing End Behavior
	Domain Range Increasing Intercepts	Positive Negative Decreasing End Behavior
	Domain Range Increasing Intercepts	Positive Negative Decreasing End Behavior

Honors Level

What inherent domain restriction of the square root function must be considered *in addition* to the interval restrictions when defining a piecewise radical function?

Define continuity in the context of a piecewise function. What two specific conditions must be met at the boundary point x = c where a radical piece meets another function (radical or otherwise)?

Can a piecewise radical function ever pass the Horizontal Line Test (and thus have an inverse function) if one of its pieces is not strictly monotonic? Explain.

Explain how the presence of vertical asymptotes in one piece of a piecewise rational function could affect the overall domain and range of the function, even if the asymptote occurs outside that piece's defined interval.

If a piecewise rational function is defined such that the boundary point is a hole (removable discontinuity) for the piece that approaches it, how must the other piece be defined at that point to "plug" the hole and achieve continuity?

End-of-Course Prep

1.10 Applications of Piecewise

Warm-Up

Use the piecewise function to fill in the table of values.

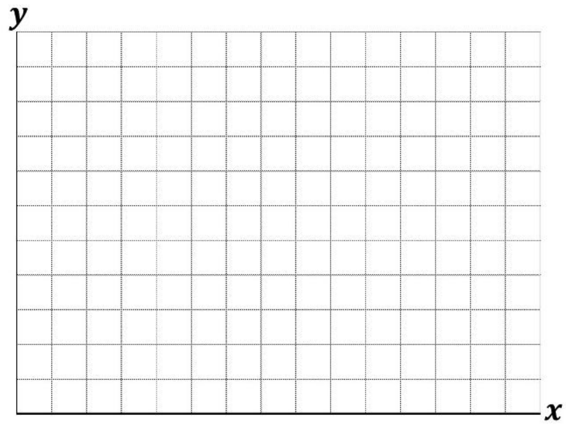
$$f(x) = \begin{cases} -x^2 + 1, & x \leq 3 \\ \sqrt{x-2} - 1, & x > 3 \end{cases}$$

x	$f(x)$
-2	
-1	
	-3
6	
	2

Main Topic | **Applications of Piecewise Functions**

TASK 1

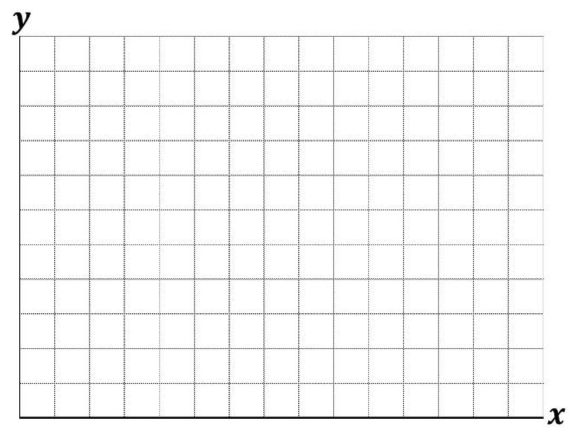
Nadia made the golf team and wants to make t-shirts to help raise money for a set of new golf clubs (shown below). Her mom can make the shirts and sell them for the following prices listed below. Write a piecewise function to represent the cost per shirt, y, in dollars and the number of t-shirts, x. Then graph it.



$$f(x) = \begin{cases} \\ \\ \\ \end{cases}$$

TASK 2

The post office uses the weight of packages to determine the cost of shipping and delivery. The charges are listed below. Write a piecewise function to represent the cost to ship, y, in dollars and the weight of the package, x. Then graph it.

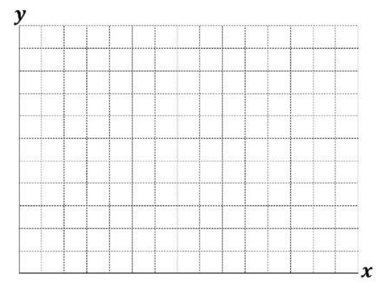
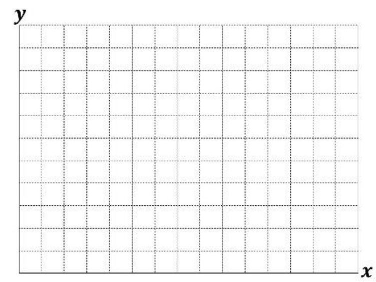
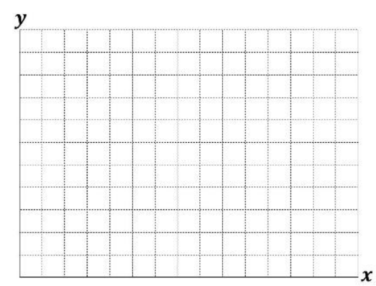


$$f(x) = \begin{cases} \\ \\ \end{cases}$$

Think – Pair – Share

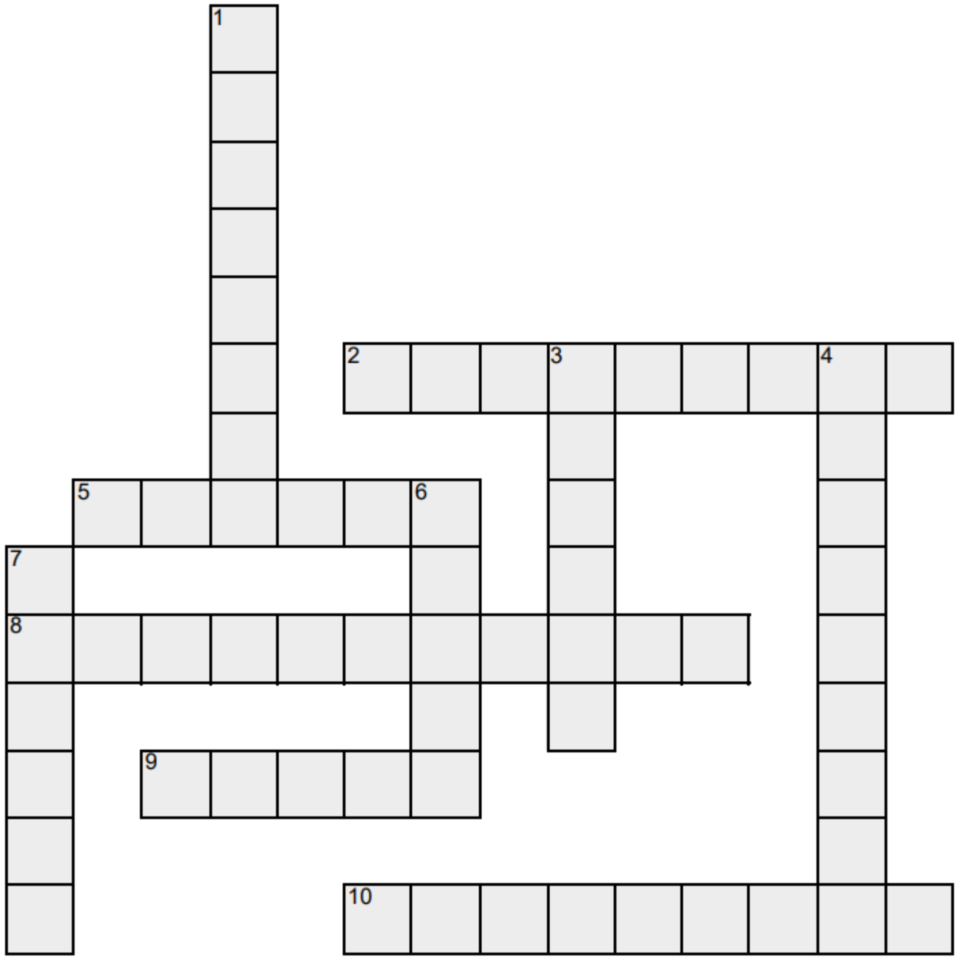
Piecewise functions often exhibit discontinuities. What do these discontinuities represent in real-world applications (e.g., a sudden price change, a switch in policy), and what are the practical implications of these abrupt changes for analysis or prediction?

Main Topic | Applications of Piecewise Functions

Problem	Function	Graph
While shopping at Target you buy a bag of chips. You notice the more you buy, the cheaper the price. A bag of chips costs \$4.50, but if you buy 3 or more bags, they only cost \$4.00 per bag.	$f(x) = \begin{cases} \\ \end{cases}$	
A car rental agency charges a flat fee of \$50 to rent a vehicle. If you rent the car for 2 days or less, it costs \$75 per day. If you rent the car 3 days or more, it only costs \$50 per day.	$f(x) = \begin{cases} \\ \end{cases}$	
You plan to sell t-shirts for your baseball team fundraiser. The printing company charges you \$15 a shirt for the first 25 shirts. If you order 26 to 50 shirts, you can get them for \$10 per shirt. Any order over 50 will be \$8 per shirt.	$f(x) = \begin{cases} \\ \end{cases}$	

End-of-Course Prep

Functions and their Inverses



Across	Down
2. A function in the 2nd degree form, and it is expressed as $f(x) = ax^2 + bx + c$. 5. A function whose rate of change is constant. 8. A function that represents a relationship where the rate of change is proportional to the current value of the function. 9. The steepness of a line and it describes the direction of a line. It is also called a rate of change. 10. A straight line where a graph continues to approach it but will never intersect it	1. A relationship between two quantities such that every input there is only one output. 3. The x-values of a relation. It is also known as the independent value or the input value in a relation. 4. A point of intersection between the graph and an axis. 6. The y-values of a relation. It is also known as the dependent value or the output value in a relation. 7. A point where the graph changes direction.

Unit 2 | Polynomials

Microbiology

High School Mathematics 3

North Carolina Math 3 Standards

Unit 2

The Complex Number System *Use complex numbers in polynomial identities and equations.*	
NC.M3.N-CN.9	Use the Fundamental Theorem of Algebra to determine the number and potential types of solutions for polynomial functions.

Seeing Structure in Expressions Write expressions in equivalent forms to solve problems.	
NC.M3.A-SSE.3	Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.

Arithmetic with Polynomial and Rational Expressions *Understand the relationship between zeros and factors of polynomials.*	
NC.M3.A-APR.2	Understand and apply the Remainder Theorem.
NC.M3.A-APR.3	Understand the relationship among factors of a polynomial expression, the solutions of a polynomial equation and the zeros of a polynomial function.

Creating Equations *Create equations that describe numbers or relationships.*	
NC.M3.A-CED.1	Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.
NC.M3.A-CED.2	Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.
NC.M3.A-CED.3	Create systems of equations and/or inequalities to model situations in context.

Reasoning with Equations and Inequalities *Understand solving equations as a process of reasoning and explain the reasoning.*	
NC.M3.A-REI.1	Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.
NC.M3.A-REI.2	Solve and interpret one variable rational equations arising from a context, and explain how extraneous solutions may be produced.

Interpreting Functions *Interpret functions that arise in applications in terms of the context.*	
NC.M3.F-IF.4	Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.
Interpreting Functions *Analyze functions using different representations.*	
NC.M3.F-IF.7	Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Building Functions *Build a function that models a relationship between two quantities.*	
NC.M3.F-BF.1	Write a function that describes a relationship between two quantities.
NC.M3.F-BF.1a	a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).
NC.M3.F-BF.1b	b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

2.1 Operations of Polynomials

Warm-Up

Perform the operations using a calculator.

$6 - 7 + 2 - 1$ $\qquad -5^2$ $\qquad 6 \div 2(1+2)$ $\qquad 6 \div 2(1+2)^2$

Main Topic | **Operations of Polynomials**

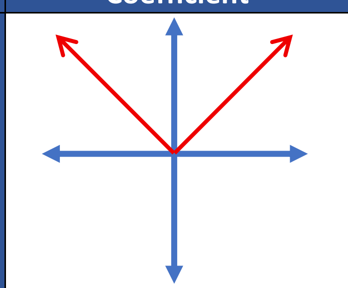
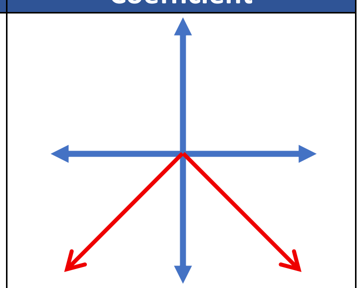
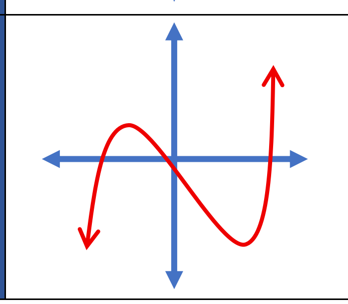
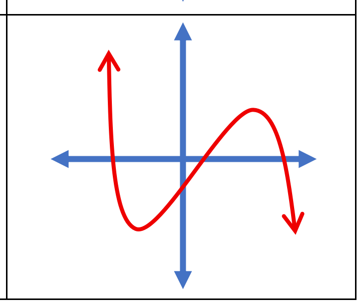
Combining
$2x + 3y + 5x - y + 4$
$7x + 2y + 4$

Important Vocabulary
Standard Form, Degree, Monomial, Binomial, Trinomial, POLYNOMIAL, Leading Coefficient, End Behavior

Adding/Subtracting
$(x^2 + 3x) - (4 - x) + 7$
$x^2 + 3x - 4 + x + 7$
$x^2 + 4x + 3$

Multiplying
$x^3 \bullet x^2 = x^5$
$3x(x^2 - 4) = 3x^3 - 12x$
$5x^2(4xy) = 20x^3y$

Dividing
$$\frac{x^7y^3}{x^5y^{10}z^2} = \frac{x^2}{y^7z^2}$$

	Positive Leading Coefficient	Negative Leading Coefficient
Even Degree		
Odd Degree		

TASK 1

$$\boldsymbol{y = 2x + 4x^2 - 3}$$

Standard Form:

Degree:

Classify by degree:

Classify by number of terms:

Leading Coefficient (LC):

LC positive or negative?

End Behavior:

Main Topic **Applications of Polynomials in Microbiology**

In microbiology, adding polynomials is useful for combining models that represent different biological populations, resources, or rates over time. Since polynomials offer a simple way to model growth, decay, or consumption in a laboratory setting, their addition allows researchers to create composite models of complex systems.

Modeling Total Toxin Production from Two Viral Strains

A lab is studying two strains of a virus that both produce a specific toxin in a host cell culture.

Scenario: A researcher is measuring the cumulative concentration of a harmful protein toxin produced by two co-existing viral strains (Strain X and Strain Y) over a 24-hour period.

Strain X Toxin Production ($T_X(h)$): Modeled as a cubic function of time *h*:

$$T_X(h) = 0.05h^3 - 0.5h^2 + 2.0h$$

Strain Y Toxin Production ($T_Y(h)$): Modeled with a different quadratic relationship:

$$T_Y(h) = 0.6h^2 - 1.5h + 0.1$$

Find the total toxin concentration, $T_{total}(h)$, at any given hour.

Microbiology Outcome: By adding the polynomials, the researcher obtains a single, composite cubic model that accurately predicts the overall safety profile of the cell culture over time. This helps in determining the critical time points when the toxin concentration exceeds safe limits.

TASK 1

$$f(x) = -2x^3 + 4x^2 - 7 - 5x^5$$

Standard Form | Degree | Classify by degree | Classify by # of terms

Leading Coefficient (LC) | LC positive or negative? | End Behavior

TASK 2

$$f(x) = -2x^3 + 4x^2 - 7 - 5x^5 + 3 + 9x^3$$

Standard Form | Degree | Classify by degree | Classify by # of terms

Leading Coefficient (LC) | LC positive or negative? | End Behavior

TASK 3

$$f(x) = (3 + x)(3 - x) + 2$$

Standard Form | Degree | Classify by degree | Classify by # of terms

Leading Coefficient (LC) | LC positive or negative? | End Behavior

Problems	Standard Form	Degree	Classify by Degree	Classify by no. of terms	L. C.	End Behavior (use arrows)
Ex. $y = 6 - 3x + 2x^2$	$y = 2x^2 - 3x + 6$	**2**	**Quadratic**	**Trinomial**	**+2**	
1. $y = 4 - 3x$						
2. $y = 2x^2 - x^3 + (x^3 + x^2)$						
3. $y = (2x)^2 + 4x - 2$						
4. $y = (3 + x)(2 + x)$						
5. $y = -x(2x + 1)(x - 2)$						
6. $y = x^2(x^2 + 3)$						
7. $y = -(2x + 3)(4x + 1)$						
8. $y = 3x^2 + 5x - 2x^5 + 6$						
9. $y = -x(x^2 + 2)(x - 1)$						
10. $y = -2 + x$						

Honors Level

When adding or subtracting polynomials, why is it necessary to combine only like terms? How does this rule relate to the distributive property and the structure of polynomials?

If $P(x)$ is a cubic polynomial and $Q(x)$ is a quadratic polynomial, what is the maximum possible degree of the resulting polynomial $(P + Q)(x)$? What is the minimum possible degree?

Explain the conceptual connection between multiplying two binomials using the FOIL method and multiplying two polynomials using the Distributive Property (or a rectangular area model).

When multiplying $(x + a)(x - a)$, why do the "middle terms" cancel out? Explain this result both algebraically and conceptually (in terms of symmetry or difference of squares).

How is polynomial multiplication used conceptually in the process of finding a polynomial given its roots?

End-of-Course Prep

2.2 Dividing Polynomials

Warm-Up

Perform the operations.

$3x^6y \cdot -5x^4y^8z$ $\qquad \frac{x^9}{x^2}$ $\qquad \frac{x^2}{x^9}$ $\qquad \frac{5x^2+10x}{5x^3-5x}$

Main Topic **Dividing Polynomials**

Simplifying Fractions

$$\frac{15}{12} = \frac{5}{4}$$

Dividing Fractions

$$\frac{15x^3y^7z}{12x^{10}y^2} = \frac{5y^5z}{4x^7}$$

Write your observations from the examples

____

____

____

Write the rules in performing the operations of monomials.

Operations of Monomials

Addition	Multiplication
Subtraction	Division

Simplify.

$\frac{2x^7 - 5x^4 + 3x}{x}$	$\frac{40x^9 - 5x^8 + 15x^4}{5x^4}$	$\frac{3x^5 - 5x^3 + 2x}{x^{-4}}$

Long Division

Perform a long division on $532 \div 2.$	Perform a long division on $532 \div 3.$

Pay attention to your teacher's instruction, list the steps, and show complete work.

$(x^2 - x - 6) \div (x - 3)$	
Steps	**Work**

Pay attention to your teacher's instruction, list the steps, and show complete work.

$(x^2 - 9) \div (x - 3)$	
Steps	**Work**

Pay attention to your teacher's instruction, list the steps, and show complete work.

$(x^3 - 4x^2 + 2x - 5) \div (x + 3)$	
Steps	**Work**

Pay attention to your teacher's instruction, list the steps, and show complete work.

$(x^4 - 4x^2 + 5) \div (x - 3)$	
Steps	**Work**

Synthetic Division

Pay attention to your teacher's instruction, list the steps, and show complete work.

$(x^2 - x - 6) \div (x - 3)$	
Steps	**Work**

$(x^3 - 4x^2 + 2x - 5) \div (3x + 12)$	
Steps	**Work**

Need Help?

$(x^4 - 4x^2 + 5) \div (20 - 5x)$ **Note: Add "0" as a placeholder for missing term.**	
Steps	**Work**

$(7 - x + x^4) \div (6x + 12)$ **Note: Add "0" as a placeholder for missing term.**	
Steps	**Work**

Practice

Divide each problem by using long or synthetic division.

$\dfrac{2x^3+7x^2-10x-24}{x+4}$	$\dfrac{2x^3+5x^2-21x-36}{x-3}$	$\dfrac{2x^3+11x^2-21x-90}{x+6}$
$\dfrac{x^3+6x^2+3x-10}{x-1}$	$\dfrac{x^3+2x-3}{x-1}$	$\dfrac{2x^3-5x^2-25x}{2x+5}$
Is $(x-1)$ a factor of $2x^4-5x^2-25x+1$?	Is $(x-3)$ a factor of $x^4-2x^3-5x^2+6x$?	Is $(3x-5)$ a factor of $3x^3+2x^2-11x-10$?

Main Topic	Applications of Dividing Polynomials in Microbiology

Polynomial division, especially synthetic division, is conceptually useful in microbiology for determining factors or breaking down complex models into simpler, rate-limiting components. This often occurs when analyzing growth rates or nutrient consumption models to find specific intercepts, saturation points, or equilibrium conditions.

Decomposing Toxin Kinetics into Rate Components

In virology or bacterial pathogenesis, the overall rate of toxin production or viral replication can be mathematically complicated. Division helps isolate different phases or mechanisms.

Scenario: A researcher models the overall rate of viral particle shedding, $S(t)$, as a function of time t. They hypothesize that the rate is controlled by a known latency factor modeled by a simple polynomial, $L(t)$. They use division to determine the residual function, $Q(t)$, which represents the unknown kinetic mechanism.

Overall Shedding Rate $\boldsymbol{S(t)}$: A high-degree polynomial representing the total rate:

$$\boldsymbol{S(t) = 2t^3 - 5t^2 + t - 3}$$

Latency Factor $\boldsymbol{L(t)}$**:** A known linear factor representing the time needed for the virus to become active:

$$\boldsymbol{L(t) = t - 3}$$

The division $\frac{\boldsymbol{S(t)}}{\boldsymbol{L(t)}}$ yields the unknown kinetic component, $\boldsymbol{Q(t)}$.

Microbiology Outcome: The quotient ( ) becomes the new, simpler kinetic model ($S(t)$) that the researcher can focus on. The complex overall model, $S(t)$, has been successfully factored into the known latency component and the unknown quadratic kinetic component (plus a remainder). The remainder of _____ indicates that the latency factor $L(t)$, is not an exact divisor and contributes to the remaining complexity.

Connecting the Dots

Perform each operation and connect it to the shape that contains its simplified form.

$$\frac{(x-2)^3}{(x-2)}$$

Answer: $\frac{3x^8-5}{2x^3}$

$$\frac{x^2+10x+25}{x+5}$$

Answer: x^2-4x+4

$$\frac{8x^{12}-16x^4}{24x^3}$$

Answer: $4x-y$

$$\frac{15x^{12}-25x^4}{10x^7}$$

Answer: $\frac{x^9-2x}{3}$

$$\frac{24x-6y}{6}$$

Answer: $x+5$

Honors Level

What is the fundamental conceptual difference between the result of dividing two polynomials (which often yields a quotient and a remainder) and the result of multiplying two polynomials?

Explain why, in polynomial long division, you only focus on dividing the leading terms of the dividend and the divisor at each step.

Why must the dividend polynomial be written with placeholders (zero coefficients) for any missing powers of x when setting up polynomial long division?

If a polynomial P(x) is divided by a linear divisor (x - c), and the remainder is zero, what is the *relationship* between P(x), the divisor, and the quotient?

Describe, in terms of degree, why it is impossible to divide a degree 3 polynomial by a degree 5 polynomial using the standard division algorithms (long or synthetic).

End-of-Course Prep

2.3 Factoring Polynomials

Warm-Up

Multiply.

$3x(4x - 7)$ $(x + 5)(x + 7)$ $(x + 6)(x - 6)$

Main Topic **Factoring Polynomials**

List the pairs of factors		
10	$4x$	$-5x$

Find the common factor		
$10x - 5$	$6x^2 - 4x$	$x^3 + 5x^2$

Factor		
$10x - 5$	$6x^2 - 4x$	$x^3 + 5x^2$

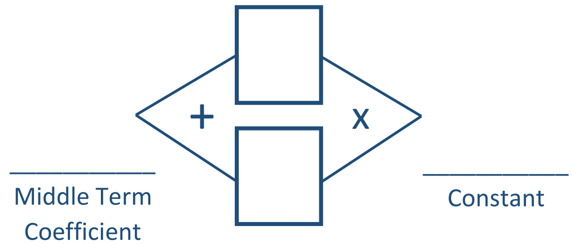
Quick Math

Factor $8x^4 - 6x^3 - 10x^2$.

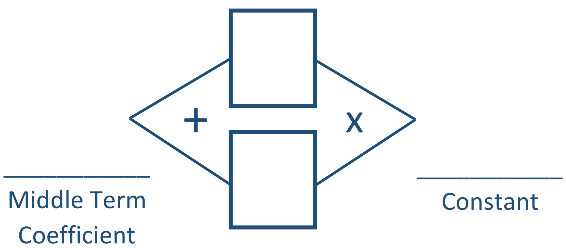
Factoring Polynomials

Sarah decided to build a rectangular garden with an area of $x^2 + 7x + 10$. What are the dimensions?

- ☐ Write the coefficient of the middle term.
- ☐ Write the constant.
- ☐ Pick two integers with a sum of 7 and a product of 10.
- ☐ Write the integers in the boxes.
- ☐ Write the factors in this form: $(x \pm a)(x \pm b)$.



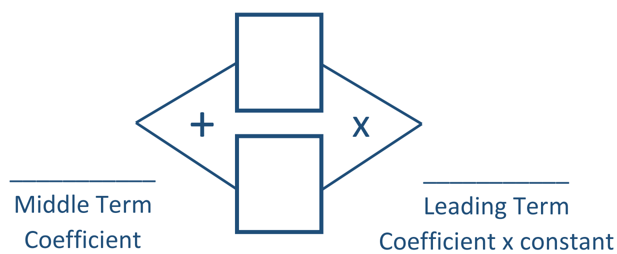
Miguel realized that a rectangular pool that he built has an area of $x^2 - 9$. What are the dimensions?



- ☐ Write the coefficient of the middle term.
- ☐ Write the constant.
- ☐ Pick two integers with a sum of 0 and a product of -9.
- ☐ Write the integers in the boxes.
- ☐ Write the factors in this form: $(x \pm a)(x \pm b)$.

Find the factors of $2x^2 + x - 15$

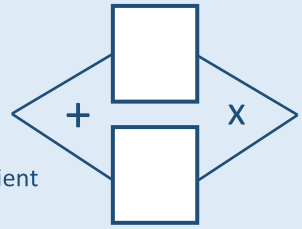
- ☐ Write the coefficient of the middle term.
- ☐ Multiply the coefficient of the leading term and the constant.
- ☐ Pick two integers with a sum of 1 and a product of -30.
- ☐ Write the integers in the boxes.
- ☐ Write the factors in this form: $(x \pm a)(x \pm b)$.
- ☐ Divide a and b by the coefficient of the leading term.



Need Help?

Factoring Polynomials

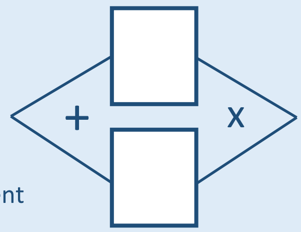
$x^2 - 11x - 26$



Middle Term Coefficient

Constant

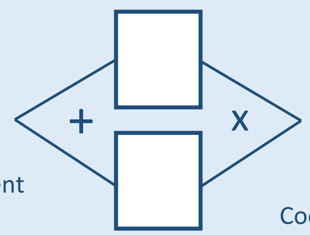
$x^2 - 121$



Middle Term Coefficient

Constant

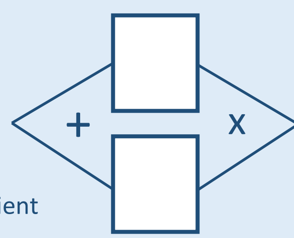
$2x^2 + 7x - 15$



Middle Term Coefficient

Leading Term Coefficient x Constant

$3x^2 + 3x - 6$



Middle Term Coefficient

Leading Term Coefficient x Constant

Practice

Factor the following polynomials. Show all work.

$x^2 - x - 6$	$2x^2 - 9x - 18$	$6x^2 - 22x + 20$
$x^2 - 12x + 36$	$2x^2 + 11x - 21$	$7x^2 - 33x - 10$
$x^2 + 12x + 27$	$x^2 - x - 72$	$10x^2 - 11x - 6$

Sum and Difference of Cubes

Pay attention to your teacher's instruction and list the steps.

$x^3 + 8$		$x^3 - 8$	
Steps	Work	Steps	Work

Factor.

$x^3 + 27$	$x^3 - 27$
$27 - x^3$	$64 - x^3$
$2x^3 + 54$	$2x^3 - 54$
$192 - 3x^3$	$4x^3 + 500$

Main Topic | **Applications of Factoring Polynomials in Microbiology**

Factoring polynomials in microbiology often helps to identify critical points, determine roots (e.g., equilibrium points, conditions for zero growth/death), or isolate contributing factors within a complex biological model. It's particularly useful when dealing with polynomial equations that represent rates, concentrations, or population dynamics.

Finding Critical Time Points for Microbial Growth/Death

When modeling bacterial growth and death rates, factoring can reveal the exact time points when the net rate changes from positive (growth) to negative (death), or vice versa.

Scenario: A microbiologist develops a polynomial function, $R(t)$, to model the net growth rate of a bacterial population in a batch culture, where t is time in hours. A positive rate means growth, a negative rate means death, and a zero rate means a stable population (or a transition point). The polynomial is:

$$\boldsymbol{R(t) = -t^3 + 12t^2 - 35t}$$

Factor the polynomials to find when the net growth rate is zero $R(t) = 0$.

Microbiology Outcome: Factoring reveals the **roots** of the polynomial: t = ___, t =___, and t =___.

- t = ___ represents the start of the observation.
- t = ___ hours and t = ___ hours are **critical time points** where the net growth rate is momentarily zero. This could indicate:
  - The transition from an exponential growth phase to a stationary phase.
  - The onset of a death phase where limiting nutrients cause the population to decline.
  - Points where the culture needs intervention (e.g., nutrient replenishment or waste removal) to prevent decline. Analyzing the signs of R(t) in the intervals (0,5), (5,7), and (7,∞) would further reveal when the population is growing or shrinking.

Honors Level

How is the process of factoring a polynomial conceptually related to the division of integers?

Explain why factoring out the Greatest Common Factor (GCF) should always be the first step in any factoring problem.

How does the Zero Product Property bridge the conceptual gap between an algebraic factorization and finding the roots of a polynomial equation?

If a polynomial cannot be factored into polynomials of lower degree with integer coefficients, what is the term for that polynomial, and what does that imply about its roots?

What does it mean for a polynomial to be factored "completely"? Give a conceptual example where a student might stop factoring too soon.

End-of-Course Prep

2.4 Remainder and Factor Theorems

Warm-Up

Evaluate.

$$\boldsymbol{a = 5, \quad b = -1, \quad c = 3}$$

$2a^2 - 5a + 7$ $\qquad$ $7(3a - 2b) + c$ $\qquad$ $a^2 + 8ab - b^2$

Main Topic	Remainder Theorem

Perform long division. $\qquad (x^5 + 3x^4 + 7x - 5) \div (x - 2)$ $\qquad$ What is the remainder?

Perform synthetic division. $\qquad (x^5 + 3x^4 + 7x - 5) \div (x - 2)$ $\qquad$ What is the remainder?

Remainder Theorem

If a polynomial p(x) is divided by the binomial $x \pm a$, the remainder obtained is p(a), which is the value of the polynomial when x is replaced with a.

Find the remainder when $f(x)$ is divided by $g(x)$. $f(x) = (x^5 + 3x^4 + 7x - 5)\ and\ g(x) = (x - 2)$	
Steps	**Work**
• Equate $g(x)$ to 0, then solve for x. • Plug in 2 to every x variable of $f(x)$. • The result is the remainder.	

Ready - Set - Go

Find the remainder.
Show complete work.

Ready	Set	Go
$x^2 - 7x + 10 \quad \div \quad x - 2$	$x^3 + x - 8 \quad \div \quad x + 7$	$x - 4 + x^3 \quad \div \quad 10 - 5x$

Need Help?

Main Topic **Factor Theorem**

Factor Theorem

If a polynomial p(x) is divided by the binomial $x \pm a$, then p(a) = 0.

Determine if $g(x)$ is a factor of $f(x)$. $f(x) = (x^3 + 2x^2 - 4x - 8)\ and\ g(x) = (x + 2)$	
Steps	**Work**
• Equate $g(x)$ to 0, then solve for x. • Plugin -2 to every x variable of $f(x)$. • If the result is zero, then $g(x)$ is factor.	

Ready - Set - Go

Determine if the $g(x)$ is a factor of $f(x)$.
Show complete work.

Ready	Set	Go
$f(x) = -x^3 + 5x^2 + 6x - 30$ $g(x) = x - 5$	$f(x) = 2x^2 - 7 + x^3$ $g(x) = 3x + 6$	$f(x) = 2x^2 - 11x + 15$ $g(x) = 12 - 4x$

Need Help?

Remainder Theorem Domino

Find the remainder of the following polynomials. Cross out pairs that do not match.

Start				
$x^2 - 3x - 4 \div (x - 4)$	0	$5x^3 - 4x^2 + x \div x - 1$	2	$x^3 + 2x - 4 \div x + 4$
				76
				$5 - x^2 + 2x^5 \div x - 1$
				6
Finish				$4x^2 - 8 \div x - 3$
1	$x^4 - x + 1 \div x - 1$	13	$3x^3 + 2x - 1 \div x + 7$	28

Main Topic **Applications of Remainder and Factor Theorems in Microbiology**

The Remainder and Factor Theorems are algebraic tools that are highly useful in computational microbiology when analyzing polynomial models of biological processes. They allow researchers to quickly test hypotheses about critical points and factors that control a system's behavior.

Predicting Nutrient Residue at a Critical Time Point

Scenario: A bioprocess engineer models the remaining nutrient concentration (in mg/L), $C(t)$, in a fermenter as a polynomial function of time t (in hours). They want to know the exact nutrient concentration remaining at the precise moment t = 8 hours, which is the planned harvest time. Performing the full division by (t - 8) to find the remainder is time-consuming.

Nutrient Concentration Model $C(t)$: $\boldsymbol{C(t) = 0.5t^3 - 7t^2 + 20t + 150}$

Microbiology Outcome: The remainder is _______. This means the remaining nutrient concentration at the 8-hour mark is _______ mg/L. This quick calculation confirms that there will be a sufficient, non-limiting amount of nutrient left for the culture to remain viable until harvest, validating the scheduled harvest time.

Decomposing Multistage Viral Replication

Scenario: A virologist models the **cumulative yield of viral particles** (Y) over time t using a high-degree polynomial. The model must account for the **eclipse phase**—a period where no infectious particles are detected. The virologist knows that the virus is non-infectious during the first t = 2 hours. They hypothesize that the eclipse phase is represented by the factor (t - 2) and want to use the Factor Theorem to ensure this factor is correctly represented in the model.

Viral Yield Model ($Y(t)$): $\boldsymbol{Y(t) = 3t^4 - 6t^3 - 12t^2 + 24t}$

Microbiology Outcome: Since Y(2) = _____, the Factor Theorem confirms that (t - 2) is ____________of the yield polynomial. The researcher can now use polynomial division (or synthetic division) to find the remaining polynomial:

$$Y(t) = (t - 2)(3t^3 - 12t)$$

The remaining cubic polynomial, ______________, represents the subsequent exponential replication phase, now cleanly separated from the initial eclipse phase.

Honors Level

The Remainder Theorem states that when a polynomial $P(x)$ is divided by $(x - c)$, the remainder is $P(c)$. Explain this concept by relating it to the functional evaluation of the polynomial.

The Remainder Theorem works when dividing by a linear expression $(x - c)$. Explain why it *does not* easily apply when dividing by a quadratic expression, such as $(x^2 + 4)$.

State the Factor Theorem precisely.
How is it conceptually a special case of the Remainder Theorem?

If $(x - 5)$ is a factor of a polynomial $P(x)$, what must the value of $P(5)$ be?
How does this result relate to the zeros or roots of the polynomial?

If $P(c) = 0$, what are the three related statements that must be true regarding the factor, the remainder, and the graph of $P(x)$?

End-of-Course Prep

2.5 Zeros of Polynomial Functions

Warm-Up

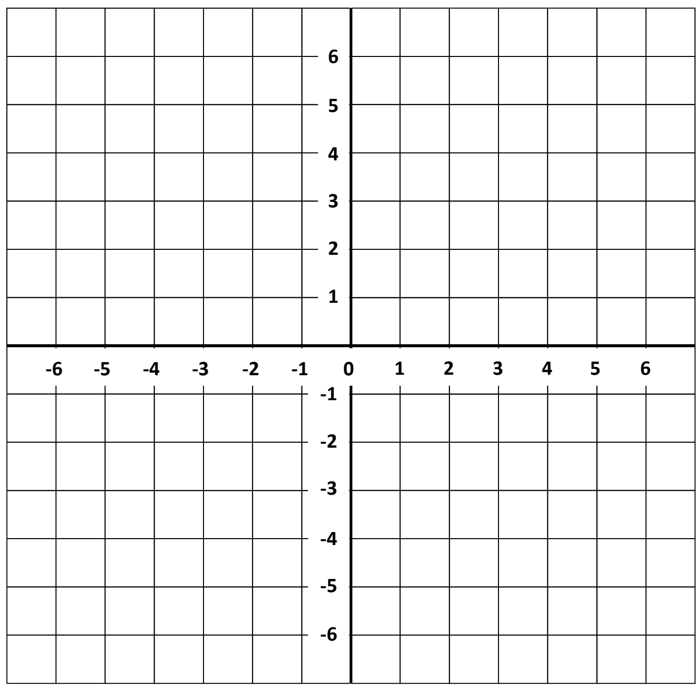
Find the solution.

$3x(4x-7)=0$ $\quad$ $x^2+7x+10=0$ $\quad$ $x^2+36=0$

Main Topic	Zeros of Polynomial Functions

TASK 1

- Graph: $f(x)=2x-6$
- What is the x-intercept?
- Prove algebraically to confirm the x-intercept.

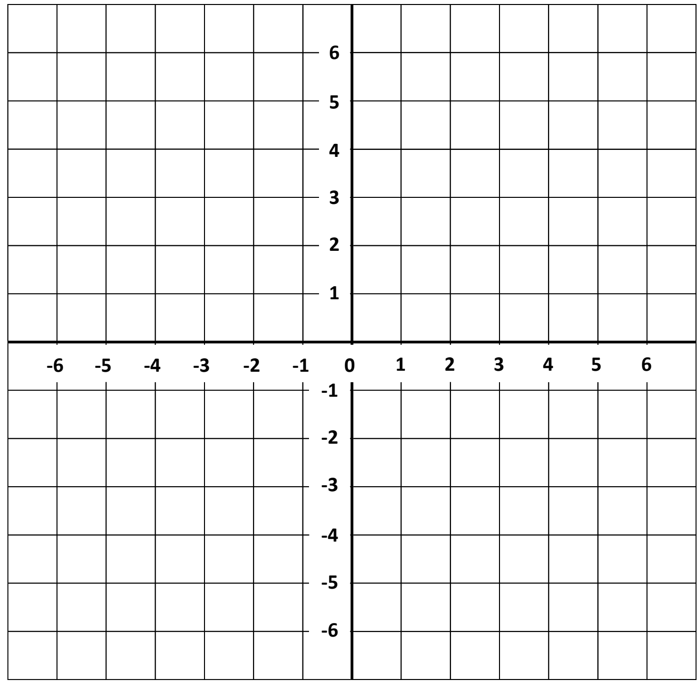


Evaluate		
$f(-5)$	$f(3)$	$f(5)$

Need Help?

TASK 2

- Graph: $f(x) = x^2 - x - 6$
- What are the x-intercepts?
- Prove algebraically to confirm the x-intercept.



Evaluate		
$f(-3)$	$f(3)$	$f(-2)$

Zeros

The **zeros** of a function are the values of the variable that make the function equals 0. The zeros are also called the x-intercepts of the graph of a function.

<table>
<tr><th colspan="2">Identify the zeros of the functions</th></tr>
<tr><td>$\{(2,3), (4,0), (-5,9), (3,0)\}$</td><td>

x	$f(x)$
0	7
4	3
2	0
-5	0

</td></tr>
</table>

Find the zeros	
$f(x) = (x+2)(2x-5)(8-4x)$	$f(x) = (2x-8)^3(x^2-x-6)$
$f(x) = x(x-5)(x-8)^2$	$f(x) = (x^2-5)(x^2-4)(3-x)$

Complex Numbers

Imaginary number is denoted by the symbol $\boldsymbol{i}$, with value of $\sqrt{-1}$.

Examples

$\sqrt{-5} = \sqrt{-1}\sqrt{5} = i\sqrt{5}$

$\sqrt{-25} = \sqrt{-1}\sqrt{25} = i5 \text{ or } 5i$

Solve for x.

$x^2 + 36 = 0$

$x^2 + 1 = -99$

$x^2 + 12 = 0$

Operations of Complex Numbers

$(3i-7) + (i+2)$

$(3i-7) - (i+2)$

$i \cdot i$

$(3i-7)(i+2)$

$\frac{10i}{2i}$

i^3

Need Help?

Main Topic	Zeros Polynomial Functions

Find the zeros of $f(x) = x^2 - 6x + 13$.	
Quadratic Formula	Completing the Square

Find the zeros of $f(x) = x^2 - 9$.	
Quadratic Formula	Completing the Square

Find the zeros of $f(x) = x^2 + 9$.	
Quadratic Formula	Completing the Square

Main Topic **Writing Polynomials**

Find the equation of a function with zeros -2, 3, and 5.

Find the equation of a function with zeros $\sqrt{2}$, $-\sqrt{2}$, and 5 multiplicity

Find the equation of a function with zeros $3i$, $-3i$, and 2 multiplicity 3.

Find the equation of a function with zeros $4i$, $-4i$, and 5 multiplicity 2.

mathBlast3

Need Help?

Main Topic | **Applications of Zeros Polynomial Functions in Microbiology**

The zeros (or roots) of a polynomial function are the values of the independent variable that make the function's output equal to zero. In microbiology, these zeros are often used to identify critical equilibrium points where a process stops, a population stabilizes, or a rate hits a baseline of zero.

Finding Equilibrium Points in a Batch Culture Growth Model

The net growth rate of a microbial population changes over time, sometimes starting and ending at zero. The zeros of the growth rate polynomial mark these key transitions.

Scenario: A microbiologist models the **net growth rate** (new cells per hour), $R(t)$, of a bacterial population in a batch fermenter over a 10-hour period. The goal is to find the exact times when the population is momentarily stable (neither growing nor declining).

Growth Rate Function ($\boldsymbol{R(t)}$): A quartic polynomial is used to model the lag, exponential, and stationary phases:

$$\boldsymbol{R(t) = t^4 - 10t^3 + 35t^2 - 50t + 24}$$

Finding the Zeros: The microbiologist sets $\boldsymbol{R(t)}$**=0** and finds the roots by factoring or using a numerical solver:

Microbiology Outcome: The zeros are______, ______, ______, and ______ hours. These time points represent moments of **zero net growth rate**.

- This indicates transitions between phases (e.g., t = 1 could be the end of the lag phase, t = 4 could be the start of the stationary phase).
- Between these zeros, the sign of $R(t)$, determines the population behavior: the population is growing when R(t) > 0 and declining when R(t) < 0. For example, between t = 4 and t = 10, the rate may become negative, signaling the onset of the death phase.

Honors Level

What is the fundamental difference between a zero of a polynomial P(x) and an x-intercept of the graph of P(x)? When are these two terms numerically equivalent?

Explain the conceptual link between a zero r of a polynomial P(x) and the corresponding factor (x - r). How does the Factor Theorem formalize this connection?

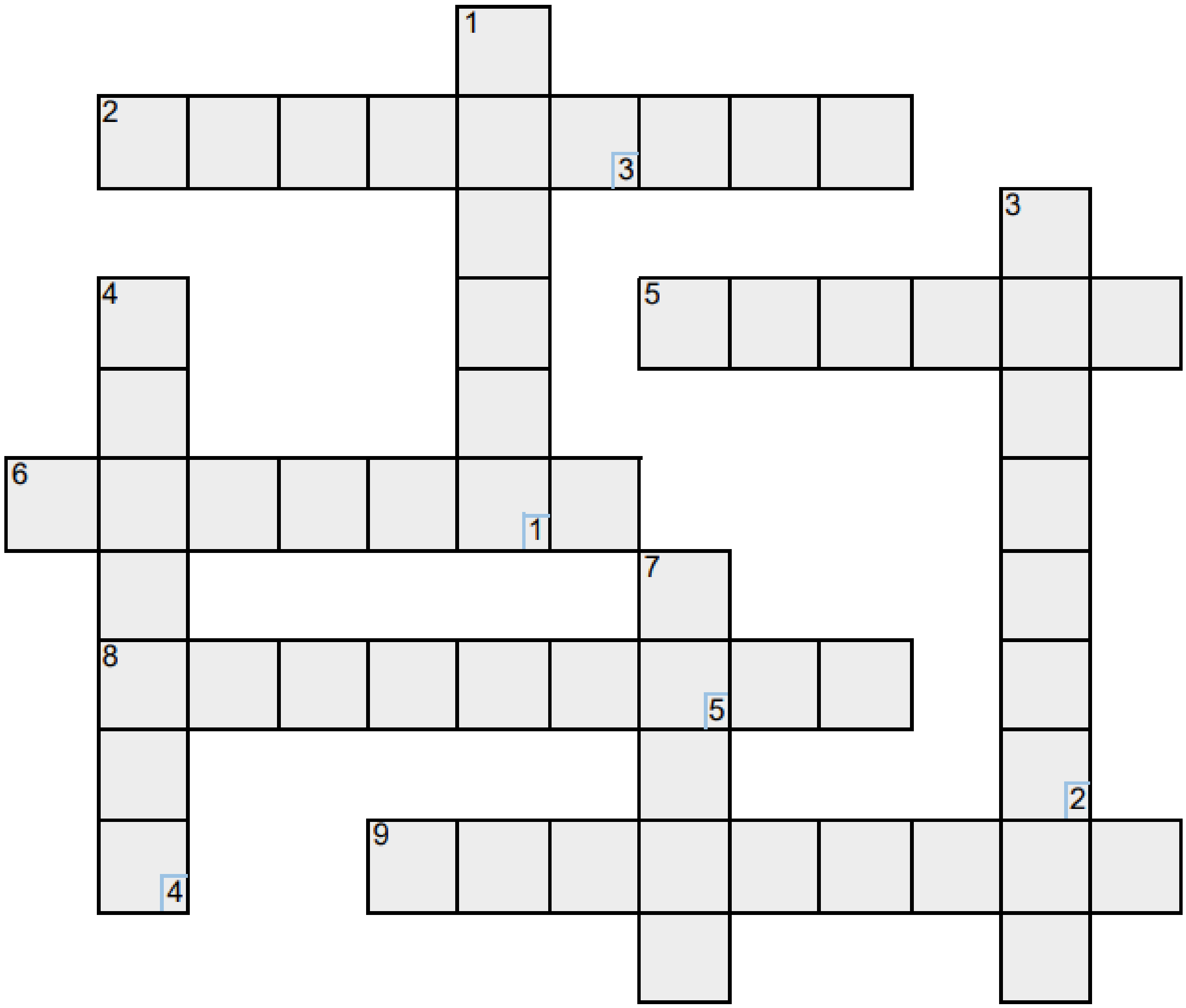
If a polynomial function P(x) has a degree of n, what does the Fundamental Theorem of Algebra guarantee about the number of complex zeros, and why is this a maximum?

How can the graph of a polynomial function visually indicate a real zero with an even multiplicity versus a real zero with an odd multiplicity? (Focus on the behavior at the x-axis).

Given a polynomial with a single real zero and two non-real complex zeros, what can you definitively conclude about the polynomial's minimum degree?

End-of-Course Prep

Polynomials



Across	Down
2. The leftover after performing a division operation. 5. The set of all possible x-values. 6. Expressions used in performing a multiplication operation 8. The point where the graph intersects the x-axis or the y-axis. 9. A polynomial with one term.	1. A function whose slope (rate of change) is constant. 3. Polynomials with two terms. 4. A term with the highest exponent in an expression. 7. Values that make the function equal to zero.

Unit 3 | Rational Functions and Expressions

Pharmacology

High School Mathematics 3

North Carolina Math 3 Standards

Unit 3

Arithmetic with Polynomial and Rational Expressions *Understand the relationship between zeros and factors of polynomials.*	
NC.M3.A-APR.2	Understand and apply the Remainder Theorem.
NC.M3.A-APR.3	Understand the relationship among factors of a polynomial expression, the solutions of a polynomial equation and the zeros of a polynomial function.

Creating Equations *Create equations that describe numbers or relationships.*	
NC.M3.A-CED.1	Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.
NC.M3.A-CED.2	Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.
NC.M3.A-CED.3	Create systems of equations and/or inequalities to model situations in context.

Reasoning with Equations and Inequalities *Understand solving equations as a process of reasoning and explain the reasoning.*	
NC.M3.A-REI.1	Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.
NC.M3.A-REI.2	Solve and interpret one variable rational equations arising from a context, and explain how extraneous solutions may be produced.

Interpreting Functions *Interpret functions that arise in applications in terms of the context.*	
NC.M3.F-IF.4	Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Interpreting Functions *Analyze functions using different representations.*	
NC.M3.F-IF.7	Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

3.1 Adding and Subtracting Rational Expressions

Warm-Up

Factor.

$5x^2 - 10x$ $\quad$ $x^2 - x - 12$ $\quad$ $x^2 - 25$ $\quad$ $7 - x$

Main Topic | **Adding and Subtracting Rational Expressions**

Rational Expressions

A **rational expression** has the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

State if an expression is linear, quadratic, exponential, radical, or rational expression.

2^{x-1}	$5x^2 - 3x + 1$	$3x - 2$	$5\sqrt{x-3}$	$\frac{x-5}{x+2}$
______________	______________	______________	______________	______________

Simplify rational expressions.

$\frac{3x-15}{3x+6}$	$\frac{x^2-5x}{x^2}$	$\frac{x^2-5x+6}{(x-3)^2}$
$\frac{x^2-x-5}{x^2-16}$	$\frac{x^2+7x+10}{x^2+10x+25}$	$\frac{x^2+x-20}{x^2+11x+30}$

Need Help?

Similar Fractions

Similar fractions are fractions with the same denominators.

Examples: $\frac{1}{5}$ *and* $\frac{3}{5}$ $\frac{7}{x}$ *and* $\frac{y}{x}$ $\frac{ab}{x+5}$ *and* $\frac{c+d}{x+5}$

Elementary Math Review

Perform the operations.

$\frac{2}{5}+\frac{1}{5}=$ $\frac{2}{5}-\frac{1}{5}=$

$\frac{6}{x}+\frac{3}{x}=$ $\frac{8}{y}-\frac{10}{y}=$

List the rules in adding and subtracting similar fractions.

____

____

____

____

Add/Subtract rational expressions.

$\frac{x+1}{2x-4}+\frac{x+5}{2x-4}$	$\frac{4x+9}{3x-1}-\frac{x-1}{3x-1}$	$\frac{x^2}{x-4}-\frac{x+12}{x-4}$
$\frac{x+10}{2x+8}+\frac{x-2}{2x+8}$	$\frac{x^2+7x}{3x+15}+\frac{10}{3x+15}$	$\frac{x^2}{2x-10}-\frac{25}{2x-10}$

Need Help?

Dissimilar Fractions

Dissimilar fractions are fractions that do not have same denominators.

Examples: $\frac{1}{5}$ *and* $\frac{3}{4}$ $\frac{7}{x}$ *and* $\frac{y}{z}$ $\frac{ab}{x-5}$ *and* $\frac{c+d}{x+5}$

Elementary Math Review

Perform the operations.

$$\frac{2}{5}+\frac{1}{4}=$$

$$\frac{2}{5}-\frac{1}{4}=$$

$$\frac{6}{x}+\frac{3}{y}=$$

$$\frac{8}{y}-\frac{10}{z}=$$

List the rules in adding and subtracting dissimilar fractions.

____

____

____

____

Add/Subtract rational expressions.

$\frac{x+1}{4}+\frac{5}{x}$	$\frac{4x}{x+1}-\frac{2}{x-1}$	$\frac{x-5}{x-4}-\frac{x+4}{x+5}$
$\frac{x+10}{x+8}+7$	$\frac{x+2}{x-3}-7$	$\frac{2x+3}{x-10}-\frac{26}{x-10}+\frac{5}{x+10}$

Need Help?

Rational Expression Maze

Perform the operations and show complete work on a separate sheet of paper to receive full credit. Connect the shapes by passing through the simplified form of the expressions.

Start

$$\frac{x+y}{y}+\frac{25}{y}$$

$$\frac{x+y}{y}+25$$

$$\frac{3}{x+3}-\frac{x-4}{x-7}$$

$$\frac{x+y+25}{y}$$

$$\frac{3x^2-2y^2}{3xy}$$

$$-\frac{x^2-2x+33}{(x+3)(x-7)}$$

$$\frac{x}{y}-\frac{2y}{3x}$$

$$x+12$$

$$\frac{x^2}{x+3}-\frac{x+12}{x+3}$$

$$\frac{x^2-2y^2}{xy}$$

$$x-4$$

$$x-12$$

$$\frac{1}{4}+\frac{9-3x}{x-6}$$

$$-\frac{11x-30}{4(x-6)}$$

End

Ask your teacher for an extra credit.

Rational Expression Matching

Match column A with a correct answer in Column B by connecting the boxes.

Column A	Column B
$\frac{x-5}{x+2}-\frac{x}{x+5}$	$\frac{2x^2-29}{x^2+7x+10}$
$\frac{x-5}{x+2}+\frac{x-2}{x+5}$	$\frac{2x^2+2x-25}{x^2+7x+10}$
$\frac{x+5}{x+2}-\frac{x-5}{x-2}$	$\frac{6x}{x^2-4}$
$\frac{x-5}{x+2}+\frac{x}{x+5}$	$\frac{2(x^2-10)}{(x+2)(x-2)}$
$\frac{x+5}{x+2}+\frac{x-5}{x-2}$	$-\frac{2x+25}{x^2+7x+10}$

Main Topic | **Applications of Rational Expressions in Pharmacology**

Rational expressions (fractions with polynomials in the numerator and/or denominator) are vital in pharmacology for combining rates, calculating equivalent concentrations, and modeling drug bioavailability when multiple factors are at play.

Combining Clearance Rates from Multiple Excretion Pathways

The rate at which a drug is eliminated (cleared) from the body often depends on multiple organs (like the liver and kidneys) working simultaneously.

Scenario: A pharmacologist models the clearance rate of a new antibiotic. The **renal (kidney) clearance rate** (R_{renal}) is modeled as a rational expression of the drug's plasma concentration (C), and the **hepatic (liver) clearance rate** ($R_{hepatic}$) is modeled similarly. The total clearance is the sum of these individual rates.

Renal Clearance Rate:

$$R_{renal}(C) = \frac{10}{C+5}$$

Hepatic Clearance Rate:

$$R_{hepatic}(C) = \frac{2C}{C^2+10C+25}$$

Adding Rational Expressions: The Total Clearance Rate (R_{total}) is the sum of the two pathways, requiring a common denominator:

Pharmacology Outcome: The simplified rational expression provides a single, combined model to predict how efficiently the body clears the drug at any given plasma concentration. This is critical for establishing appropriate **dosing schedules** to maintain therapeutic levels.

Honors Level

Conceptually, why is finding a Least Common Denominator (LCD) the single most crucial step when adding or subtracting rational expressions, but not when multiplying or dividing them?

If you use a common denominator that is *not* the LCD, what is the conceptual consequence for the resulting expression?

Explain why the domain restrictions of the final sum or difference must include the restrictions from *all* the original denominators, even if those factors canceled out during the simplification process.

Conceptually, what does it mean when the sum of two rational expressions simplifies such that all factors in the final numerator and denominator cancel out, leaving a constant?

How does the concept of a common factor in the numerator and denominator of the *simplified* answer relate to the presence of a hole in the graph of the resulting rational function?

End-of-Course Prep

3.2 Multiplying and Dividing Rational Expressions

Warm-Up

Factor.

$2x + 6$ $\quad$ $x^2 - x$ $\quad$ $9x^2 - 25$ $\quad$ $x^2 - x - 12$

Main Topic **Multiplying Rational Expressions**

Multiplying Fractions

Multiply across. Example: $\frac{2}{3} \bullet \frac{6}{5} = \frac{12}{15}$ *or* $\frac{4}{5}$

Perform the operations.

$\frac{3xy}{2x} \bullet \frac{15y}{6xy}$	$\frac{3x^2 - x}{7x} \bullet \frac{x+1}{3x-1}$	$\frac{x}{x+5} \bullet \frac{x^2+10x+25}{x^2}$
$\frac{3}{x+7} \bullet (x^2 + 14x + 49)$	$\frac{x+3}{x} \bullet \frac{5x}{2x^2-18}$	$\frac{x-4}{x+1} \bullet \frac{x^2+x}{x^2-16}$

Need Help?

Fill in the blank

Fill out all the necessary information to complete the steps of the operation.

$$\frac{2x^2y^5}{3x^3} \bullet \frac{(x^9y^3)^{-1}}{4}$$

$$\frac{2x^2y^5}{3x^3} \bullet \frac{\qquad}{4x^9y^3}$$

$$\underline{\qquad\qquad}$$

$$\frac{x^2+3x-18}{x+4} \bullet \frac{5}{x-3}$$

$$\frac{(x+6)(\qquad)}{\qquad} \bullet \frac{5}{x-3}$$

$$\underline{\qquad\qquad}$$

$$\frac{8x^3y^{12}}{12xy^{15}} \bullet \frac{(x^2y^4)^{-3}}{(xy^2)^{-2}}$$

$$\frac{8x^3y^{12}}{12xy^{15}} \bullet \underline{\qquad\qquad}$$

$$\underline{\qquad\qquad}$$

$$\frac{x^2-64}{x^2+4x-5} \bullet \frac{x+5}{x^2+7x-8}$$

$$\frac{(\qquad)(\qquad)}{(\qquad)(\qquad)} \bullet \frac{x+5}{(\qquad)(\qquad)}$$

$$\underline{\qquad\qquad}$$

$$\frac{2x^2y^{-5}}{3x^{-3}z} \bullet \frac{2(3x^2y^5)^2}{x^5y^{-7}z^{-3}}$$

$$\underline{\qquad\qquad} \bullet \underline{\qquad\qquad}$$

$$\underline{\qquad\qquad}$$

$$\frac{2x^2-2x-60}{5x^2-5} \bullet \frac{(x+1)^2}{x^2-x-30}$$

$$\frac{2(x-6)(\qquad)}{5(x+1)(\qquad)} \bullet \frac{(\qquad)(\qquad)}{(\qquad)(\qquad)}$$

$$\underline{\qquad\qquad}$$

Main Topic: Dividing Rational Expressions

Dividing Fractions

Flip the divisor then multiply. Example: $\frac{4}{5} \div \frac{8}{3} = \frac{4}{5} \bullet \frac{3}{8} = \frac{12}{40}\,or\,\frac{3}{10}$

Perform the operations.

$\frac{x}{x+1} \div \frac{x+5}{x+1}$	$\frac{x+2}{x+5} \div \frac{x+3}{x^2-25}$
$\frac{x^2+4x-12}{x^2-4} \div \frac{x^2-36}{x^2+8x+12}$	$\frac{2x^2+8x+6}{x+1} \div \frac{2x+6}{x-1}$
$\frac{x^2+9x+14}{x+2} \div \frac{x+7}{x^2-5}$	$\frac{x^2-8x+16}{8-2x} \div \frac{x^2-16}{x+4}$

Need Help?

Fill in the blank

Fill out all the necessary information to complete the steps of the operations.

$$\frac{x^2 - x - 6}{x^2 - 4} \div \frac{x^2 - 9}{x^2 + 8x + 15}$$

$$\frac{(x-3)(\quad\quad)}{(x+2)(\quad\quad)} \bullet \frac{(x+3)(\quad\quad)}{(\quad\quad)(x-3)}$$

$$\frac{x+5}{x-2}$$

$$\frac{x^2 + 2x - 15}{x^2 - 4} \div \frac{x^2 - 9}{x^2 - 4}$$

$$\frac{(x-3)(\quad\quad)}{(x+2)(\quad\quad)} \bullet \frac{(x+2)(\quad\quad)}{(x+3)(\quad\quad)}$$

$$\frac{x+5}{x+3}$$

$$\frac{x^2 + 7x + 12}{x^2 + 5x + 4} \div \frac{x^2 - 9}{x^2 - 2x - 3}$$

$$\frac{(x+3)(\quad\quad)}{(x+4)(\quad\quad)} \bullet \frac{(x-3)(\quad\quad)}{(\quad\quad)(x+3)}$$

$$1$$

$$\frac{x^2 - 2x - 8}{3x - 6} \div \frac{2x^2 + 10x + 12}{12 - 6x}$$

$$\frac{(\quad\quad)(x+2)}{3(\quad\quad)} \bullet \frac{-6(\quad\quad)}{2(\quad\quad)(\quad\quad)}$$

$$-\frac{x-4}{x+3}$$

Use the space below for extra work.

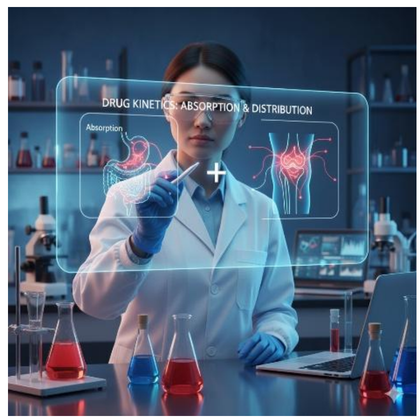
Main Topic | **Applications of Multiplying Rational Expressions in Pharmacology**

Multiplying rational expressions in pharmacology is essential when combining proportional relationships, converting units between systems, or modeling sequential processes like drug absorption and subsequent distribution. This allows pharmacologists to create a single, consolidated model from two or more rate or concentration functions.

Combining Absorption and Distribution Proportionalities

When a drug is administered, its final concentration in the target tissue depends on both the rate it's absorbed into the bloodstream and the rate it's distributed from the bloodstream into the tissue.

Scenario: A pharmacologist is modeling the concentration of a pain reliever in a patient's joint tissue (C_{joint}). This concentration is directly proportional to two factors: the fraction of drug absorbed into the plasma (F_{abs}) and the fraction of plasma drug that distributes into the joint (F_{dist}). Both factors are modeled as rational functions of the drug's lipid solubility factor (L).



Absorption Fraction (F_{abs}):

$$F_{abs}(L) = \frac{L^2+L}{L^2-1}$$

Distribution Fraction (F_{dist}):

$$F_{dist}(L) = \frac{L-1}{L+2}$$

Multiplying Rational Expressions: The Joint Concentration Factor (C_{factor}) is the product of the two fractions:

$$\boldsymbol{C_{factor}(L) = F_{abs}(L) \cdot F_{dist}(L)}$$

Pharmacology Outcome: The resulting simplified rational expression provides a direct, consolidated model to predict the drug concentration in the joint tissue based only on the solubility factor L. This allows for rapid comparison of drug candidates based on their lipid solubility for efficient tissue targeting.

Honors Level

When multiplying two rational expressions, what is the conceptual purpose of factoring the numerators and denominators *before* multiplying?

Explain the conceptual reason why dividing a rational expression $\frac{A}{B}$ by $\frac{C}{D}$ is equivalent to multiplying $\frac{A}{B}$ by the reciprocal of the divisor, $\frac{D}{C}$.

When dividing two rational expressions, why are there two distinct sets of values that must be excluded from the domain of the final quotient?

If a rational expression is divided by a polynomial (not a rational expression), what must be the conceptual "denominator" of the divisor, and why?

Why is it essential to state the restrictions on the variables for a rational expression? What conceptual problem arises if a restricted value is substituted into the expression?

End-of-Course Prep

3.3 Solving Rational Equations

Warm-Up

Solve for x.

$2x^2 - 32 = 0$ $\qquad$ $x^2 - 4x - 44 = 1$ $\qquad$ $2x^2 + 7x - 4 = 0$

Main Topic	Solving Rational Equations

List at least 5 equivalent fractions of $\frac{2}{5}$.

$\frac{2}{5} = \frac{x}{10}$	$\frac{2}{5} = \frac{8}{x}$
• What number will you multiply to 5 to get 10? • What number will you multiply to 2 to solve for x? • Solve the equation and show the complete work below.	• What number will you multiply to 2 to get 8? • What number will you multiply to 5 to solve for x? • Solve the equation and show the complete work below.

$$\frac{2}{x} = \frac{8}{x-3}$$

Steps

Work

Checkpoint

$$\frac{2}{5} = \frac{6}{x-3} + \frac{2}{x-3}$$

Steps

Work

Checkpoint

$$\frac{2}{5} = \frac{8}{x-3} + 1$$

Steps

Work

Checkpoint

$$\frac{2}{5} - \frac{4}{x} = \frac{8}{x-3}$$

Steps

Work

Checkpoint

Calculator Active

Solve for x. $\frac{1}{x+5} = \frac{x}{-2x-10}$

- Move one expression over the other side of the equation then graph it.
- Press **Y=**, enter $\frac{x}{-2x-10} - \frac{1}{x+5}$, then press **GRAPH**.
- Press **TRACE** to trace the x-intercept/zero (estimate).

Hint: Use an online graphing application for a more accurate value of x.

Pick Me

Pick the solution(s) of the equations from the shapes below.

Pick Me	Pick Me	Pick Me	Pick Me	Pick Me	Pick Me
$\frac{15-\sqrt{201}}{4}$	$\frac{42}{23}$	$\frac{17-\sqrt{481}}{8}$	$\frac{15+\sqrt{201}}{4}$	-2	$\frac{17+\sqrt{481}}{8}$

I pick

____

as the solution(s) of

$$\frac{1}{x} = \frac{5}{x} + 2$$

I pick

____

as the solution(s) of

$$\frac{x+2}{2x} - \frac{1}{3} = \frac{5}{7}$$

I pick

____

as the solution(s) of

$$\frac{x+3}{x-2} = \frac{5x}{x+4}$$

I pick

____

as the solution(s) of

$$\frac{3-2x}{4x-1} = \frac{5}{x} - \frac{8}{x}$$

Main Topic	Applications of Solving Rational Equations in Pharmacology

Solving rational equations in pharmacology is necessary to determine specific input values (like dose, time, or pH) required to achieve a target output value (like a desired concentration, a specific rate, or zero side effects).

Determining the Required Dose for a Target Concentration

A common clinical need is determining the exact dose that must be administered to achieve a specific therapeutic drug concentration in the bloodstream.

Scenario: A pharmacologist models the peak plasma drug concentration (C, in mg/L) as a function of the administered oral dose (D, in mg). They need to find the specific dose (D) required to achieve the Minimum Effective Concentration (MEC) of 15 mg/L.

Concentration Function: $$C(D) = \frac{100D}{5D+50}$$

Solving the Rational Equation: Set the function equal to the target concentration (MEC = 15):

Pharmacology Outcome: The required oral dose to reach the therapeutic concentration of 15 mg/L is _______ **mg**. This solution is essential for creating the standardized dosing guidelines for the medication.

Honors Level

What is the fundamental difference between a rational expression and a rational equation? How does this difference guide the steps you take to solve each?

Explain why the domain of a rational equation must be determined *before* solving the equation. What conceptual error does failing to do this create?

Why do the values that make the denominator zero conceptually represent vertical asymptotes on the graph of the corresponding rational function, and how does this relate to the equation having no solution at that point?

How can you conceptually use cross-multiplication as a special case of the LCD strategy when the rational equation consists of a single fraction equal to another single fraction?

Explain the conceptual reason why solving a rational equation often leads to extraneous solutions. How does the multiplication by a variable expression change the equivalence of the initial equation?

End-of-Course Prep

3.4 Discontinuity and Domain

Warm-Up

Simplify.

$$\frac{x^2-3x-40}{(x-8)(x+3)} \qquad \frac{-x^2+3x+40}{(x-8)(x+3)} \qquad \frac{49-16x^2}{4x^2+11x+7}$$

Main Topic | **Vertical Asymptotes**

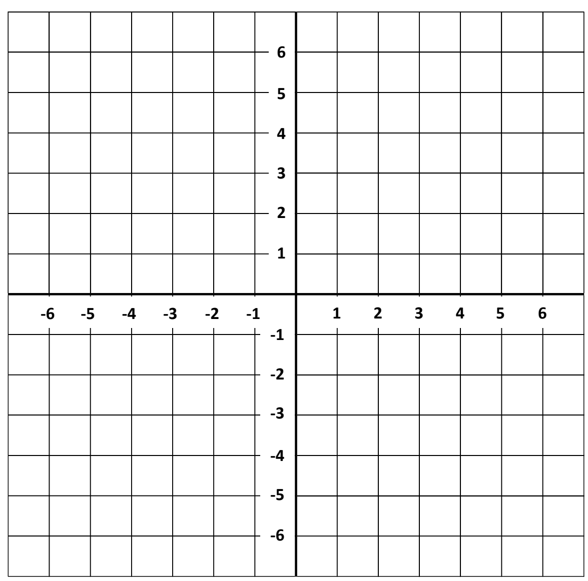
Rational Function

A **rational function** has the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

Vertical Asymptotes

$$f(x) = \frac{2}{x+3}$$

Domain	Range
-7	
-6	
-5	
-4	
-3	
-2	
-1	
0	
1	



Domain:

Vertical Asymptote is a straight line where the graph is trying to approach it but never crosses it.

Use a graphing device to verify your graph.

Need Help?

Finding Vertical Asymptotes

$$f(x) = \frac{x+7}{x-2}$$

- Can the expression be simplified?
- What x-value(s) will make the denominator equal to zero?
- What does it mean when a denominator is zero?
- What is the vertical asymptote of f?
- What is the domain of f?

$$g(x) = \frac{(x-2)(x-3)}{x^2+6x+5}$$

- Can you simplify the expression?
- What x-value(s) will make the denominator equal to zero?
- What does it mean when a denominator is zero?
- What is the vertical asymptote of g?
- What is the domain of g?

Find the vertical asymptote(s) and domain of the functions.

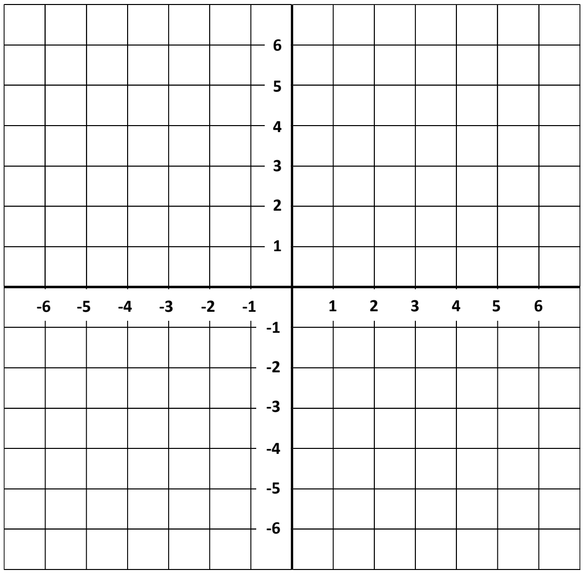
$f(x) = \frac{x+3}{x+2}$	$f(x) = \frac{x^2+x-12}{x^2+7x+6}$
$f(x) = \frac{x+4}{x^2-8x+15}$	$f(x) = \frac{x^2+6x-16}{x^2+9x+20}$

Need Help?

Point of Discontinuity

$$f(x) = \frac{(x-2)(x+3)}{x+3}$$

Domain	Range
-7	
-6	
-5	
-4	
-3	
-2	
-1	
0	
1	



Domain:

Point of Discontinuity is a hole that disrupts the continuity of the graph.

Use a graphing device to verify your graph.

Finding Point of Discontinuity

$$f(x) = \frac{(x-2)(x+3)}{x+3}$$

- Simplify the expression.
- Equate the expression that was canceled out to 0.
- Solve for x.
- Plug in the value of x to the simplified expression.
- What is the exact location of the discontinuity in the graph?
- What is the domain of the function?

$$f(x) = \frac{x^2+7x+10}{x^2-25}$$

- Simplify the expression.
- Equate the expression that was canceled out to 0.
- Solve for x.
- Plug in the value of x to the simplified expression.
- What is the exact location of the discontinuity in the graph?
- What is the domain of the function?

Need Help?

Determine the discontinuity of the function. State if it is a vertical line or a hole.

$f(x) = \dfrac{x-1}{x^2+5x-6}$	$f(x) = \dfrac{2x+10}{x^2-25}$
$f(x) = \dfrac{x^2+9x+18}{x^2+5x+6}$	$f(x) = \dfrac{x^2-2x+1}{x^2-1}$
$f(x) = \dfrac{16-x^2}{x^2+7x+12}$	$f(x) = \dfrac{2x^2-6x-20}{x^2-4}$
$f(x) = \dfrac{x+5}{x^2-25}$	$f(x) = \dfrac{x^2-6x+9}{x^2-4x+3}$

Main Topic | **Applications of Discontinuity of Functions in Pharmacology**

In pharmacology, the discontinuities (vertical asymptotes and holes) of rational functions often represent critical, non-physical, or maximum-limit conditions that a biological system cannot sustain. They highlight moments where a model breaks down, signifying a shift in physiological state, saturation, or toxicity.

Vertical Asymptotes: Modeling Toxic Saturation Limits

A vertical asymptote occurs when the denominator of a rational function is zero. In pharmacokinetics, this often signifies a concentration or dose that leads to a theoretically infinite or undefinable rate—usually indicating toxicity or saturation failure.

Scenario: A toxicologist is modeling the rate of adverse side effects (S, measured in risk units) associated with the concentration (C) of a drug in the liver. The model is a rational function where the denominator approaches zero as the concentration nears a critical maximum tolerated dose.

Risk Function:

$$S(C) = \frac{C^2+5}{C-C_{max}}$$

where C_{max} is the known toxic concentration, say 50 mg/L.

$$S(C) = \frac{C^2+5}{C-50}$$

Discontinuity: A **vertical asymptote** exists at ___________.

Pharmacology Outcome: The model predicts that as the drug concentration C approaches 50 mg/L from below, the rate of side effects (S) approaches infinity. The vertical asymptote mathematically represents the **critical, lethal dose threshold**. In reality, the body's compensatory mechanisms fail at this point, leading to acute organ toxicity and an immediate, non-linear jump in adverse outcomes, effectively rendering the system unstable or non-functional.

Honors Level

Explain the key conceptual difference between a value excluded from the domain that results in a vertical asymptote versus one that results in a hole (removable discontinuity).

Algebraically, how does the presence of a hole in the graph of a rational function relate to the common factors found in the numerator and the denominator?

Define a non-removable discontinuity in the context of rational functions. What conceptual event happens to the graph of the function at a vertical asymptote?

If the degree of the denominator is greater than the degree of the numerator, is it guaranteed that a vertical asymptote exists? Explain why or why not.

Under what single condition must a rational function have a slant (oblique) asymptote? Conceptually, what does the slant asymptote represent geometrically?

End-of-Course Prep

3.5 Horizontal Asymptotes and Range

Warm-Up

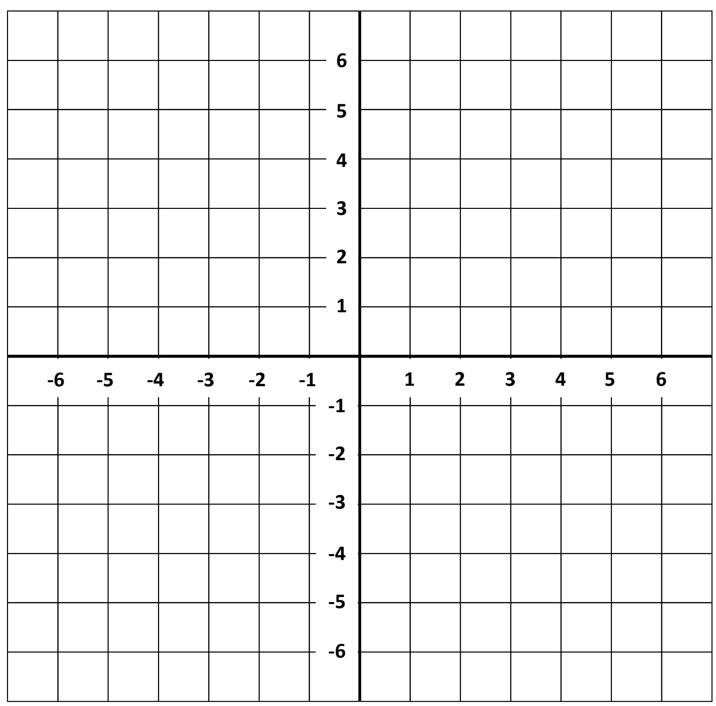
Find the asymptote.

$f(x) = 2^x + 5$ $\qquad$ $f(x) = 8 - 3^x$ $\qquad$ $f(x) = 2^x - 5$

Main Topic **Horizontal Asymptotes and Range of Rational Functions**

Horizontal Asymptotes

- Translate $f(x) = \frac{1}{x}$ 2 units left and 3 units up.
- Write the equation of the translated function.
- Rewrite the equation of the translated function in a different form.



- What do you notice about the new equation and the horizontal asymptote of the graph?

Horizontal Asymptote is a horizontal straight line where the graph is trying to approach it but never crosses it.

Use a graphing device to verify your graph.

Need Help?

Finding Horizontal Asymptotes

$$f(x) = \frac{x+7}{x-2}$$

- Divide the numerator by its denominator.
- Rewrite the equation of a function in a different form.
- What is the horizontal asymptote of the function?
- What is the range of the function?

$$g(x) = \frac{(x+2)(x-3)}{x^2+7x+10}$$

- Divide the numerator by its denominator.
- Rewrite the equation of a function in a different form.
- What is the horizontal asymptote of the function?
- What is the range of the function?

Find the horizontal asymptote and range of the functions.

$f(x) = \frac{x+4}{x^2-16}$	$f(x) = \frac{x^2+x-12}{x^2+8x+16}$
$f(x) = \frac{5-x}{x^2-8x+15}$	$f(x) = \frac{x^2+6x-16}{x^2+9x+20}$

Need Help?

Rational Functions Matching

Match column A with equivalent functions in Column B by connecting the boxes.

Column A	Column B
$f(x) = \frac{7x-33}{x-5}$	$f(x) = -\frac{7x-37}{x-5}$
$f(x) = \frac{3x-1}{x-2}$	$f(x) = \frac{3}{x-2} - 5$
$f(x) = -\frac{5x-13}{x-2}$	$f(x) = \frac{2}{x-5} + 7$
$f(x) = \frac{2}{x-5} - 7$	$f(x) = \frac{5}{x-2} + 3$
$f(x) = \frac{3}{x-2} + 5$	$f(x) = \frac{5x-7}{x-2}$

Racing to the Top

Directions:

- Start from the bottom.
- Connect the equivalent equations with their horizontal asymptote and range.

Round 1

$R: (-\infty, 0) \cup (0, \infty)$	$R: (-\infty, 5) \cup (5, \infty)$			
$y = 0$	$y = 5$	$y = \frac{11}{3}$		
$f(x) = \frac{5x - 8}{x - 3}$	$f(x) = \frac{5x + 11}{x - 3}$	$f(x) = \frac{5x - 11}{x - 3}$	$f(x) = \frac{5x + 8}{x - 3}$	
$f(x) = \frac{4}{x - 3} + 8$	$f(x) = \frac{5}{x - 3} + 4$	$f(x) = \frac{4}{x - 3} + 5$	$f(x) = \frac{x + 2}{x^2 - 5}$	$f(x) = \frac{3x + 2}{4x - 5}$

Round 2

$R: (-\infty, \frac{11}{4}) \cup (\frac{11}{4}, \infty)$	$R: (-\infty, \frac{4}{3}) \cup (\frac{4}{3}, \infty)$			
$y = 2$	$y = \frac{11}{4}$	$y = \frac{4}{3}$		
$f(x) = \frac{11x^2 - 1}{4x^2 + 1}$	$f(x) = \frac{11x^2 + 1}{4x^2 + 1}$	$f(x) = \frac{11x^2 + 1}{4x^2 - 1}$	$f(x) = \frac{4x^2 + 1}{3x^2 + 1}$	
$f(x) = \frac{3x^2 - 1}{4x^2 + 1} - 2$	$f(x) = \frac{3x^2 - 1}{4x^2 + 1} + 2$	$f(x) = \frac{3x^2 - 1}{4x^2 - 1} + 2$	$f(x) = \frac{3x^2 + 1}{4x^2 + 1} + 2$	$f(x) = \frac{4x^2 + 1}{3x^2 + 1} - 2$

Main Topic **Applications of Asymptotes of Rational Functions in Pharmacology**

In pharmacology, the horizontal asymptote of a rational function is extremely important because it represents the limiting value—the maximum or equilibrium condition—that a process (like drug concentration, absorption rate, or toxicity) approaches over time or with increasing dosage.

Maximum Steady-State Drug Concentration (Long-Term Dosing)

The horizontal asymptote models the maximum drug concentration the body can sustain, regardless of how long the medication is taken, assuming a constant dosing rate. This is known as the steady-state concentration.

Scenario: A patient starts a medication taken once daily. The concentration of the drug in the plasma, C(t), approaches a stable, maximum level over many days. The relationship between the number of days (t) and the cumulative concentration is modeled by a rational function.

Concentration Function: $$C(t) = \frac{15t+20}{t+1}$$

Finding the Horizontal Asymptote: To find the steady-state concentration as time approaches infinity ($t \to \infty$) the pharmacologist compares the degree of the numerator and the denominator. Since the degrees are equal (both are 1), the asymptote is the ratio of the leading coefficients:

Pharmacology Interpretation: The concentration approaches **15 mg/L**. This value is the **Maximum Safe Steady-State Concentration** ($C_{ss,max}$). The result confirms that the drug will not accumulate indefinitely to toxic levels; it will plateau at 15 mg/L after sufficient time.

Honors Level

Conceptually, what does the existence of a horizontal asymptote tell us about the end behavior of a rational function?

Explain the intuitive difference between a horizontal asymptote and a vertical asymptote in terms of how they relate to the domain and range of the function.

Why can a rational function sometimes cross its horizontal asymptote, but never cross its vertical asymptote?

Can a rational function have more than one horizontal asymptote? Justify your answer based on the definition of a limit at infinity.

Why must all rational functions have either a horizontal asymptote or a slant asymptote, but never both?

End-of-Course Prep

3.6 Intercepts of Rational Functions

Warm-Up

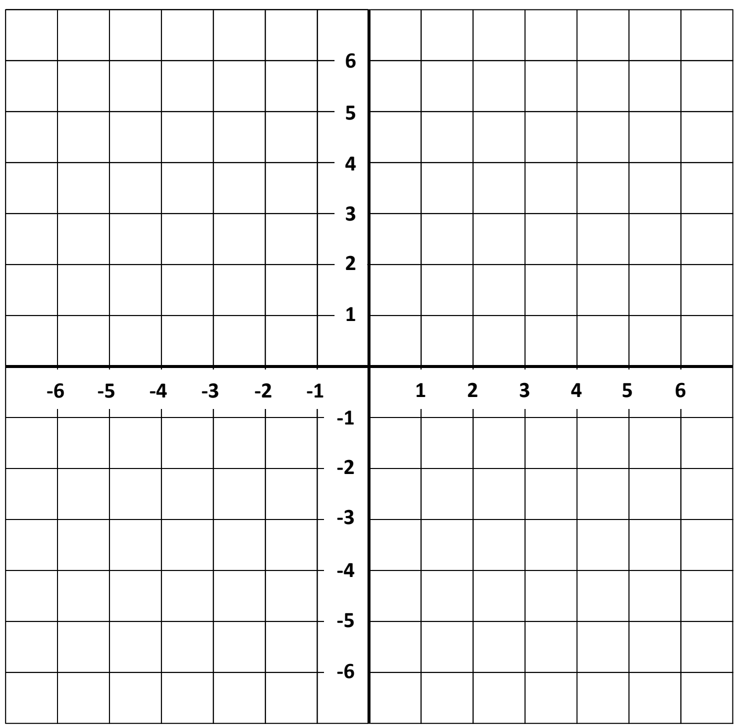
Find the intercepts.

$f(x) = 2^x + 5$ $\qquad$ $f(x) = 8 - 3^x$ $\qquad$ $f(x) = 2^x - 5$

Main Topic **x-intercepts of Rational Functions**

x-intercepts

- Translate $f(x) = \frac{1}{x}$
  2 units left and 3 units up.
- Write the equation of the translated function.
- Rewrite the equation of the translated function in a different form.
- Replace y or $f(x)$ with 0.
- Solve for x.



x-intercept is a point where the graph crosses the x-axis.

Use a graphing device to verify your graph.

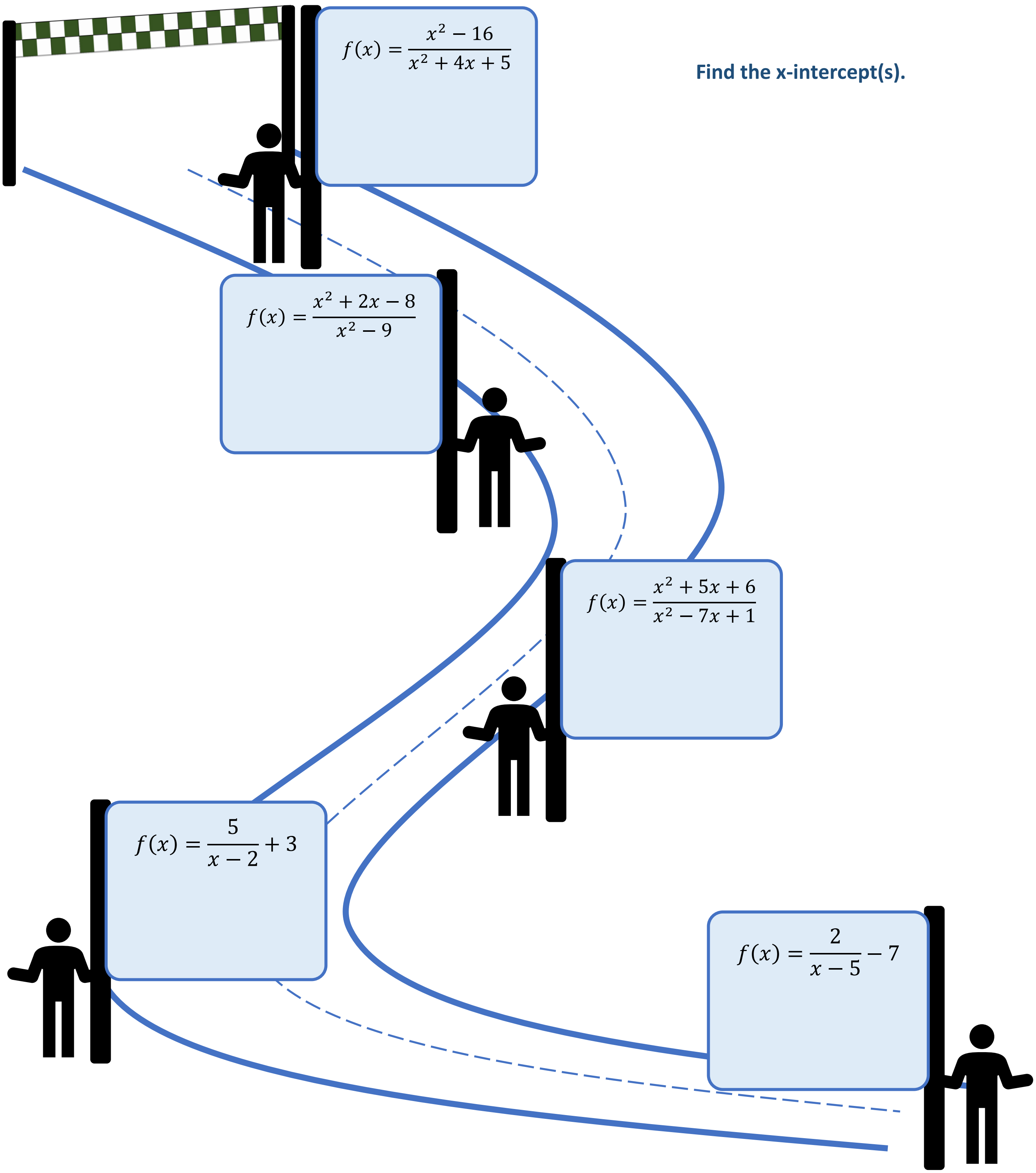
Need Help?

Find the x-intercept(s) of the functions.

$f(x) = \frac{4}{x-5} + 2$	$f(x) = 10 - \frac{7}{x+3}$
$f(x) = \frac{3x+4}{x-5}$	$f(x) = \frac{9-2x}{x+1}$
$f(x) = \frac{x+4}{x^2-16}$	$f(x) = \frac{x^2+x-12}{x^2+8x+16}$
$f(x) = \frac{5-x}{x^2-8x+15}$	$f(x) = \frac{x^2+6x-16}{x^2+9x+20}$

Reaching the Finish Line

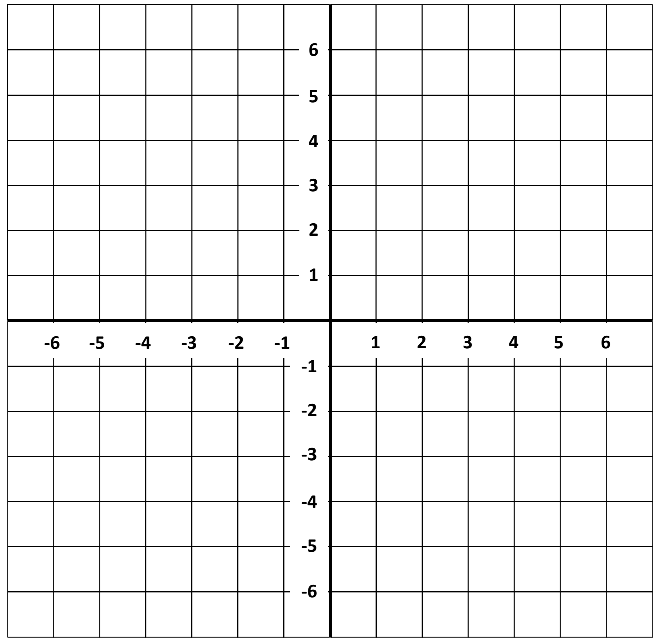
Find the x-intercept(s).



Main Topic: y-intercept of Rational Functions

y-intercept

- Translate $f(x) = \frac{1}{x}$
  5 units right and 2 units down.
- Write the equation of the translated function.
- Replace x with 0.
- Solve for y or $f(x)$.



y-intercept is a point where the graph crosses the y-axis.

Use a graphing device to verify your graph.

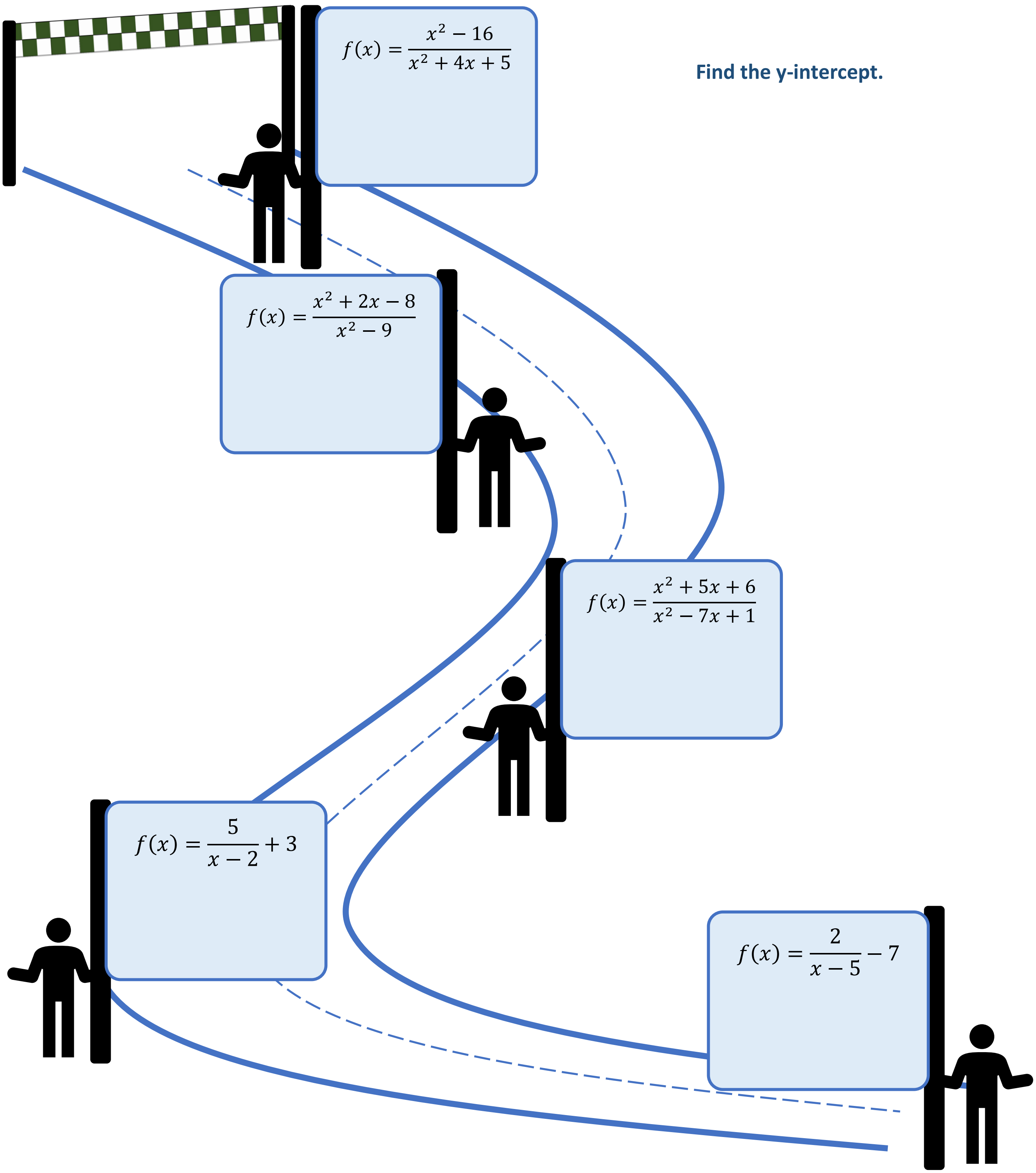
Find the y-intercept of the functions.

$f(x) = \dfrac{4}{x-5} + 2$	$f(x) = \dfrac{9-2x}{x+1}$
$f(x) = \dfrac{x+4}{x^2-16}$	$f(x) = \dfrac{x^2+x-12}{x^2+8x+16}$

Need Help?

Reaching the Finish Line

Find the y-intercept.



Main Topic	Applications of Intercepts of Rational Functions in Pharmacology

In pharmacology, the intercepts of a rational function provide crucial information about the baseline state of a system (the y-intercept) and the conditions required for a process to cease or start (the x-intercepts, or zeros).

Y-Intercept: Baseline Clearance Rate (Zero Concentration)

The **y-intercept** of a function f(x) occurs when x=0. In pharmacology, this often represents the baseline rate of a process when the concentration of the drug or substrate is zero.

Scenario: A researcher models the clearance rate (R, in mL/min) of a substance from the blood as a rational function of the substance's plasma concentration (C, in mg/L). They want to know the extrinsic (non-drug-dependent) clearance rate that exists even when the drug concentration is zero.

Clearance Rate Function: $R(C) = \frac{10C+50}{C+10}$

Finding the Y-Intercept (R(0)): Set the concentration C = 0:

Pharmacology Outcome: The **y-intercept is _____**. This means the baseline clearance rate, due to normal physiological processes (like solvent drag or non-specific filtration) that operate independent of the drug's presence, is ___ **mL/min**. This value is essential for partitioning the total clearance into drug-specific and background components.

X-Intercept (Zero): Concentration Required for Zero Effect

The **x-intercept** of a function f(x) occurs when f(x) = 0. In rate models, this often signifies the condition (time, dose, or concentration) where the net process stops or reverses.

Scenario: A pharmacologist develops a rational function, E(D), to model the **net production rate of a toxic metabolite** as a function of the administered dose (D, in mg). They want to find the **specific dose** that results in a net production rate of **zero**, ideally to guide the maximum safe dose.

Toxin Production Rate Function: $E(D) = \frac{2D-40}{D+5}$

Finding the X-Intercept (E(D)=0): Set the numerator equal to zero (since a fraction is zero only if the numerator is zero):

Pharmacology Outcome: The **x-intercept is D = _____**. This means administering a dose of _____ **mg** results in a zero net production rate of the toxic metabolite (the production rate equals the elimination rate at this dose). Doses exceeding ____mg would likely result in an accumulation of the toxic metabolite, setting a critical boundary for clinical trials.

Honors Level

What conceptual difference exists between a function being undefined at a point (leading to a vertical asymptote or hole) and a function being equal to zero at a point (leading to an x-intercept)?

What geometric feature of the graph must be true at any x-intercept of a rational function? (i.e., what is the relationship between the graph and the x-axis?)

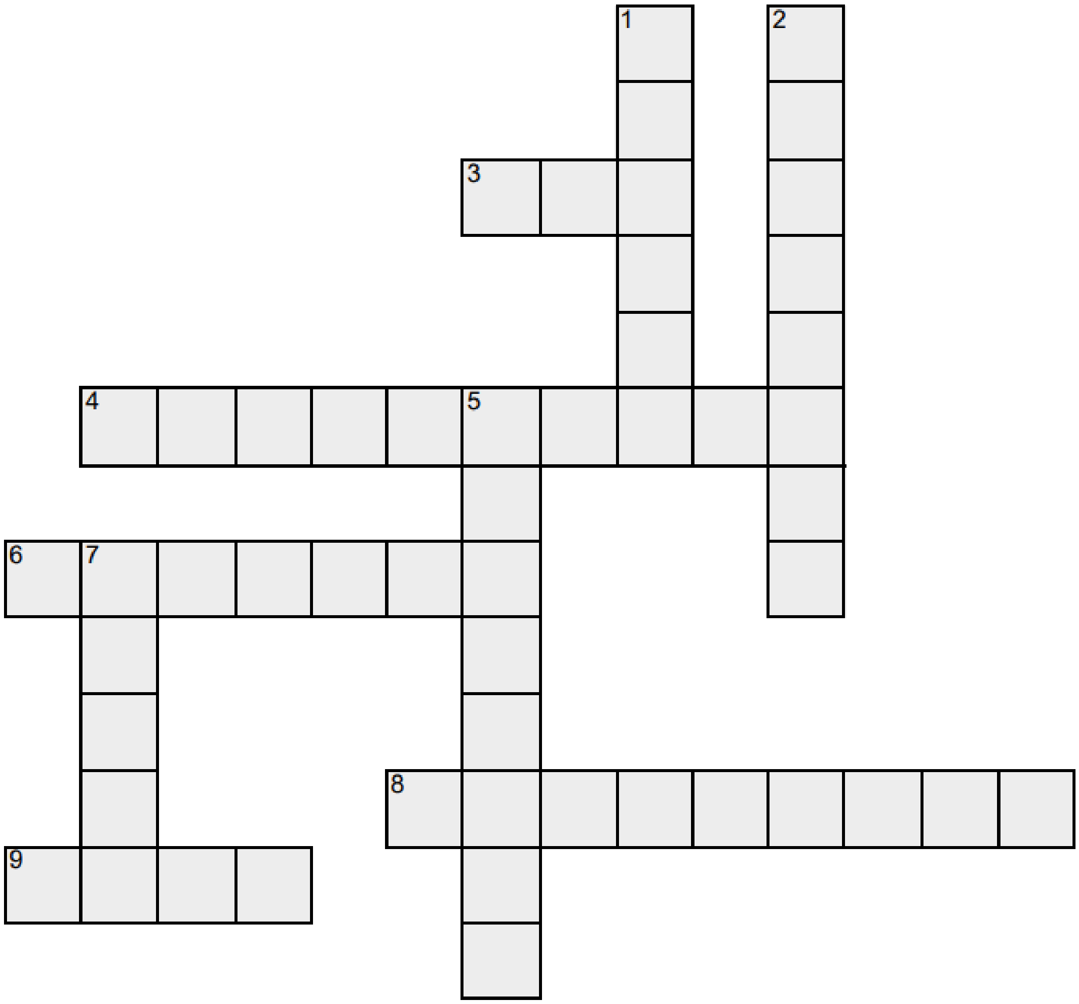
Describe how the graph of a rational function might "cross" a horizontal asymptote near the center of the graph, but it can never "cross" an x-intercept.

If a rational function has no x-intercepts, what must be conceptually true about the values of the numerator N(x) for all values of x in the domain?

Consider a rational function with a multiplicity of 2 for a zero in the numerator (e.g., $(x-c)^2$). How does this multiplicity conceptually affect the graph's behavior at the x-intercept (c, 0)?

End-of-Course Prep

Rational Functions and Expressions



Across	Down
3. The result of adding expressions.	1. The set of all possible x-values of a function.
4. The result of subtracting expressions.	2. The result of dividing expressions.
6. The result of multiplying expressions.	5. An expression that has the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.
8. A point where a graph crosses the x-axis or y-axis.	
9. A value that makes the function equal to zero.	7. The set of all possible y-values of a function.

High School Mathematics 3

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Unit 4 | Exponential and Logarithmic Functions

High School Mathematics 3

North Carolina Math 3 Standards

Unit 4

Seeing Structure in Expressions *Write expressions in equivalent forms to solve problems.*	
NC.M3.A-SSE.3	Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.
Creating Equations *Create equations that describe numbers or relationships.*	
NC.M3.A-CED.1	Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.
NC.M3.A-CED.2	Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.
NC.M3.A-CED.3	Create systems of equations and/or inequalities to model situations in context.
Interpreting Functions *Interpret functions that arise in applications in terms of the context.*	
NC.M3.F-IF.4	Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.
Interpreting Functions *Analyze functions using different representations.*	
NC.M3.F-IF.7	Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.
Building Functions *Build a function that models a relationship between two quantities.*	
NC.M3.F-BF.1	Write a function that describes a relationship between two quantities.
NC.M3.F-BF.1a	a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).
NC.M3.F-BF.1b	b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.
Linear, Quadratic, and Exponential Models *Construct and compare linear and exponential models and solve problems.*	
NC.M3.F-LE.3	Compare the end behavior of functions using their rates of change over intervals of the same length to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a polynomial function.
NC.M3.F-LE.4	Use logarithms to express the solution to $ab^{ct}=d$ where a, b, c, and d are numbers and evaluate the logarithm using technology.

4.1 Exponential Functions

Warm-Up

Express the following in exponential form then simply.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ $\quad$ $3 \cdot 3 \cdot 3 \cdot 3$ $\quad$ $4 \cdot 4 \cdot 7 \cdot 7 \cdot 7$

Main Topic Exponential Equations

Exponential Form	Simplified Form
2^{-3}	
2^{-2}	
2^{-1}	
2^{0}	
2^{1}	
2^{2}	
2^{3}	
2^{4}	
2^{5}	
2^{6}	

Express in exponential form.

9	16	81	100	1
$\frac{1}{9}$	$\frac{1}{16}$	$\frac{1}{81}$	$\frac{1}{100}$	$\frac{1}{3}$
$\frac{1}{x}$	$\frac{1}{x^{-2}}$	$\frac{1}{x^{2}}$	$\frac{1}{x^{-4}y^{2}}$	$\frac{1}{x^{4}y^{5}z^{-7}}$

Quick Math

Express in $3^{-3}a^{0}b^{-4}c^{8}$ fraction form.

Need Help?

TASK 1

$$5^x = 25$$

- Express 25 in exponential form.
- Make sure that both sides have the same base.
- Equate the exponents to each other.
- Solve for x.

Checkpoint

Use this section to confirm the solution.

Solve for x.

$7^x = 49$	$3^x = 27$
$32 = 2^x$	$216 = 6^x$
$2^x = \frac{1}{16}$	$5^x = \frac{1}{125}$
$4^x = \frac{1}{16}$	$4^x = 1$

Quick Math

Solve for x.

$4^x = \frac{1}{64}$ $\qquad$ $4^x = \left(\frac{1}{64}\right)^{-1}$

Need Help?

TASK 2

$$5^{2x-6} = 25$$

- Express 25 in exponential form.
- Make sure that both sides have the same base.
- Equate the exponents to each other.
- Solve for x.

Checkpoint

Use this section to confirm the solution.

Solve for x.

$5^{x+6} = 25$	$3^{x-5} = 27$
$32 = 2^{2x-1}$	$216 = 6^{2x-17}$
$2^{3x-31} = \frac{1}{16}$	$5^{6x-9} = \frac{1}{125}$
$4^{7-2x} = \frac{1}{16}$	$4^{10x-20} = 1$

Quick Math

Solve for x.

$$2^{x+4} = \frac{1}{256} \qquad 7^{2x-8} = \left(\frac{1}{343}\right)^{-1}$$

Need Help?

TASK 3

$$4^{x-1} = 8^{2x+7}$$

- Express 4 and 8 in exponential form.
- Make sure that both sides have the same base.
- Equate the exponents to each other.
- Solve for x.

Checkpoint

Use this section to confirm the solution.

Solve for x.

$8^{5-3x} = 16^{x+1}$	$9^{x-5} = 27^{2-x}$
$64^{3x-2} = 512^{x+1}$	$256^{3-x} = 64^{5x}$
$2^{3x-1} = \left(\frac{1}{16}\right)^{x+2}$	$5^{3x-5} = \left(\frac{1}{125}\right)^{-2}$

Quick Math

Solve for x.

$$49^{x+4} = \left(\frac{1}{7}\right)^{2-3x} \qquad 11^{x-6} = \left(\frac{1}{121}\right)^{-6x}$$

Need Help?

TASK 4

$$2^{x-1} \cdot \frac{1}{4} = 8^{2x+7}$$

- Express 2, ¼, and 8 in exponential form.
- Make sure that both sides have the same base.
- Equate the exponents to each other.
- Solve for x.

Checkpoint

Use this section to confirm the solution.

Solve for x.

$5^{x+2} \cdot 25 = 125$	$27^{5-x} \cdot \left(\frac{1}{9}\right)^{2x+4} = 9^{2x}$
$36^{-(3x-4)} \cdot 6^{x-5} = 36^{x+2}$	$16^{2-3x} \cdot \left(\frac{1}{4}\right)^{-2} = 8^{x+7}$
$\frac{9^{x+2}}{81^{3x-1}} = \left(\frac{1}{27}\right)^{x-10}$	$\frac{16^{5x}}{32^{2x-8}} = 64^{3-x}$

Quick Math

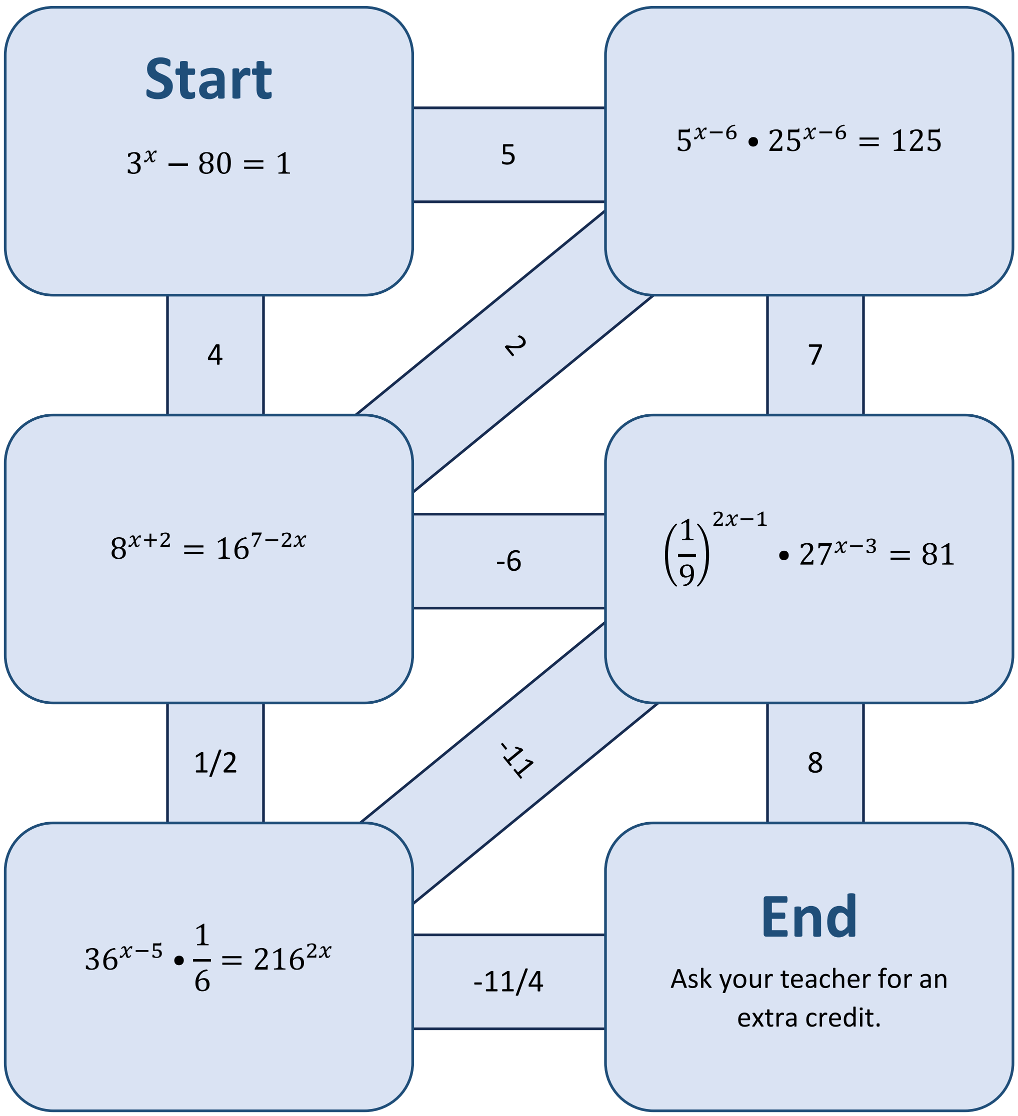
Solve for x.

$$\frac{6^{3x-1}}{36^{x-4}} = \left(\frac{1}{6}\right)^{1-2x}$$

Need Help?

Exponential Equation Maze

Connect the shapes by drawing a line passing the correct solution.



End-of-Course Prep

4.2 Characteristics of Exponential Functions

Warm-Up

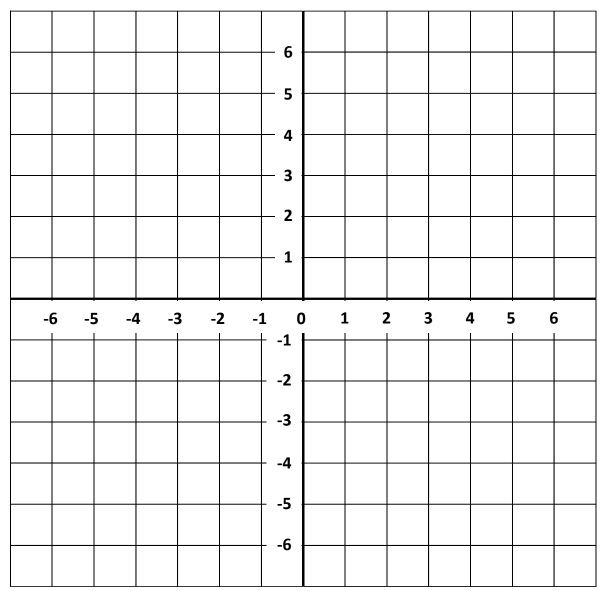
Write all you know about the following terms.

Domain	Intercept	Increasing Function
Range	Slope	Decreasing Function

Main Topic **Exponential Functions and their Graphs**

TASK 1

- ☐ Graph $f(x) = 2^x$.
- ☐ Identify the following features of the function.
  - Domain:
  - Range:
  - Asymptote:
  - x-intercept:
  - y-intercept:
  - Left End Behavior:
  - Right End Behavior:
  - Increasing or Decreasing:

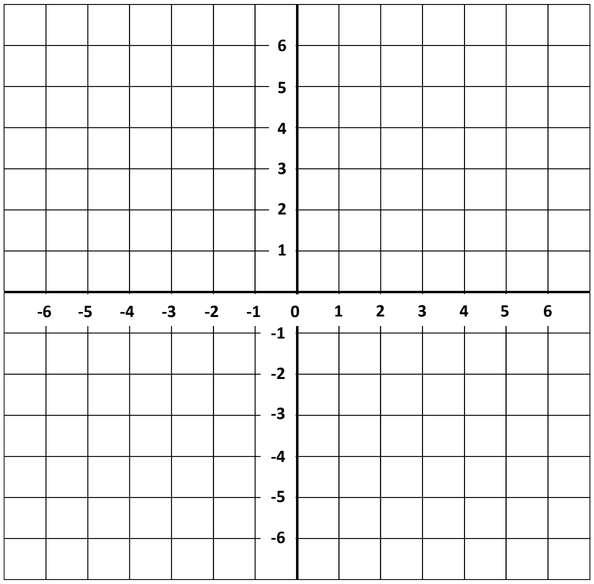


Asymptote

An **asymptote** is a straight line where a graph is trying to approach but never touches it. For $f(x) = 2^{-x} - 5$, the asymptote is -5.

Need Help?

TASK 2



- ☐ Graph $f(x) = 2^{-x} + 3$.
- ☐ Identify the following features of the function.
  - Domain:
  - Range:
  - Asymptote:
  - x-intercept:
  - y-intercept:
  - Left End Behavior:
  - Right End Behavior:
  - Increasing or Decreasing:

Complete the tables with necessary information.

$f(x) = 2^x + 3$

Domain and Range	x and y intercepts
Asymptote	Left and Right End Behavior

$f(x) = -2^x + 3$

Domain and Range	x and y intercepts
Asymptote	Left and Right End Behavior

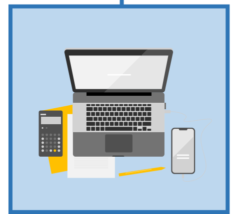
Quick Math

What is the x-intercept and y-intercept of $f(x) = 5 - 4^x$?

Transformations of Functions

Write your observations when $f(x) = 2^x$ is changed to

$f(x) = 2^x - 5$ $\quad$ $f(x) = 2^{-x} + 5$ $\quad$ $f(x) = -2^x + 5$ $\quad$ $f(x) = 2^{x-4} + 5$



Fill in the blank

Word Bank: Translates, Reflects, Horizontal, Vertical, Left, Right, Down, Up

$$f(x) = 4^x + 2 \rightarrow f(x) = -4^{-x} + 2$$

The negative sign for 4 ________________ the graph over a ________________ line then the negative sign for x ________________ the graph over a ________________ line.

$$f(x) = 6 - 3^{-x} \rightarrow f(x) = -4 + 3^{-x}$$

The negative 4 ________________ the graph 10 units down then the positive sign for 3 ________________ the graph over a ________________ line.

$$f(x) = 5^{-x+3} - 2 \rightarrow f(x) = 5^{-x-4} - 6$$

The negative 4 ________________ the graph 7 units to the ________________ then the negative sign for 6 ________________ the graph 4 units ________________.

$$f(x) = 7^{x+4} - 1 \rightarrow f(x) = 7^{x+1} + 5$$

The positive 1 ________________ the graph 3 units to the ________________ then the positive 5 ________________ the graph 6 units ________________.

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End-of-Course Prep

4.3 Logarithmic Equations

Warm-Up

Find the inverse of the functions.

$f(x) = 2x - 5$ $\quad$ $g(x) = x^2 + 7$ $\quad$ $h(x) = \sqrt{x - 4} + 1$

Main Topic Logarithmic Equations

Logarithm

A **logarithm** is the inverse of exponential function.

$5^2 = 25$	$7^{-2} = \frac{1}{49}$	$2^5 = 32$
⇕	⇕	⇕
$log_5 25 = 2$	$log_7 \frac{1}{49} = -2$	$log_2 32 = 5$
⇕	⇕	⇕
The logarithm of 25 with base 5 is 2.	The logarithm of $\frac{1}{49}$ with base 7 is -2.	The logarithm of 32 with base 2 is 5.

Quick Math

Express $log_3 27 = 3$ in exponential form. $\quad$ Express $4^2 = 16$ in logarithmic form.

Need Help?

Solve for x.

$log_9 81 = x$	$log_x 1 = 0$	$log_8 x = 2$

Rules of Logarithm

		Examples
Product Rule	$log_b(x \bullet y) = log_b(x) + log_b(y)$	$log_2(4 \bullet 5) = log_2(4) + log_2(5)$
Quotient Rule	$log_b(x\ /\ y) = log_b(x) - log_b(y)$	$log_3(5/\ 8) = log_3(5) - log_3(8)$
Power Rule	$log_b(x^y) = y log_b(x)$	$log_4(7^2) = 2 log_4(7)$
Base Change Rule	$log_b(x) = log(x)/\ log(b)$	$log_6(3) = log(3)/\ log(6)$

Expand.

$log_b(acd)$	$log_4(5\ /\ 6)$	$log_k(g^h)$
$log_8(9)$	$log_b(hk)^m$	$log_x\left(\frac{s}{t}\right)^u$

Simplify.

$-2 log_3(5)$	$log_2(a) + log_2(b)$	$log(7)/\ log(9)$
$c[log_d(a) + log_d(b)]$	$log_7(4) - log_7(5)$	$2 log_2(3) - 5 log_2(2)$

Main Topic **Exponential Equations**

Task 1

$$3^{x-1} = 9$$

- Can you express both sides having the same base?
- What is the exponential form of 9?
- What will you do with the exponents when both sides have the same base?
- What is the value of x?

Task 2

$$3^{x} = 8$$

- Can you express both sides having the same base?
- What exponential rule can you apply to transfer x in front of 3?
- What will you do with the equation to solve for x?
- What is the value of x?

Task 3

$$3^{x-1} = 8$$

- Can you express both sides having the same base?
- What exponential rule can you apply to transfer x-1 in front of 3?
- What will you do with the equation to solve for x?
- What is the value of x?

Task 4

$$3^{x-1} - 2 = 8$$

- What is the first step to solve the equation?
- Can you express both sides having the same base?
- What exponential rule can you apply to transfer x-1 in front of 3?
- What will you do with the equation to solve for x?
- What is the value of x?

Need Help?

Tic-Tac-Toe

Pick 3 questions horizontally, vertically, or diagonally to solve.
Show complete work in each box to receive full credit.

$2^x = 5$	$7^{x-2} = 12$	$2^x - 6 = 10$
$3^x - 4 = 8$	$2^x = \frac{1}{8}$	$9^{2-x} + 5 = 7$
$6^{3x} = \frac{1}{12}$	$6^{x-1} + 3 = 10$	$3^{5-2x} = 2$

Main Topic **Logarithmic Equations**

Task 1

$$\log(4x) = \log(3x + 5)$$

- What do the expressions on both sides have in common?
- How do you cancel them?
- Can you solve for x?
- What is the value of x?

Task 2

$$log_2 8 = x - 1$$

- Can you express the equation in exponential form?
- What is the exponential form of the equation?
- Can you express both sides having the same base?
- What is the value of x?

Task 3

$$-2log_2 4 = x - 1$$

- What exponential rule can you use to transfer -2?
- What is the exponential equation of the logarithmic equation?
- Can you express both sides having the same base?
- What is the value of x?

Task 4

$$log_2 (x + 1) + log_2(x) = log_2 12$$

- What exponential rule can you use to simplify the left side of the equation?
- What is the simplified form of the left side of the equation?
- What do the expressions on both sides have in common?
- How do you get rid of them?
- How do you solve for x?
- What is/are the value(s) of x?

Need Help?

Tic-Tac-Toe

Pick 3 questions horizontally, vertically, or diagonally to solve.
Show complete work in each box to receive full credit.

$log_4(x^2 - 25) - log_4(x+5) = log_4(2)$	$log_3(9) = 5 - 2x$	$log_5\left(\frac{1}{25}\right) = 3x - 2$
$log_2(2) = 3x - 2$	$log_6(1-3x) = log_6(x)$	$log_4(x-1) - log_4(x) = log_4(12)$
$2log_4 5 = 2x$	$log_4(x-1) + log_4(x) = log_4(12)$	$log_9(1-3x) = log_9(x)$

End-of-Course Prep

4.4 Logarithmic Functions

Warm-Up

List the logarithmic rules.

Main Topic **Logarithmic Functions**

Solve for x.

$5^x = 25$	$5^x = 9$

Calculator Steps

List the steps in entering $log_5 7$ on your calculator.

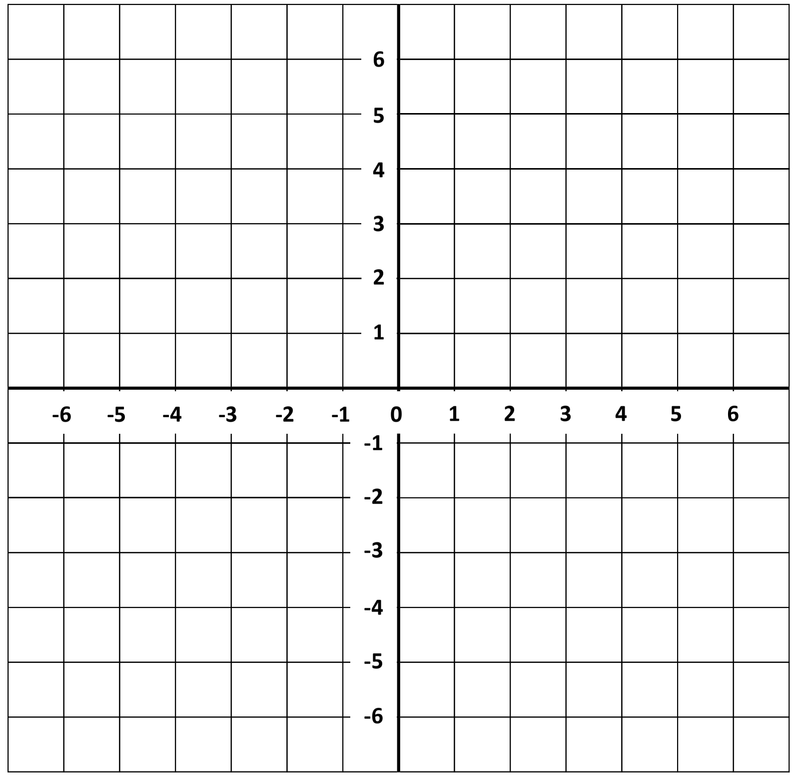
Find the inverse of the functions.

$f(x) = 3x - 7$	$f(x) = 2^x$

TASK 1

- Graph $f(x) = 2^x$.
- Fill out the table.

Domain	Range
0	
1	
2	
3	
4	



- Graph $f(x) = log_2 x$.
- Fill out the table.

Domain	Range
1	
2	
4	
8	
16	

Fill out the table.

	Domain	**Range**	**x-intercept**	**y-intercept**	**Asymptote**
$f(x) = 2^x$					
$f(x) = log_2 x$					

Write your observations between $f(x) = 2^x$ and $f(x) = log_2 x$.

____

____

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Transformations of Logarithmic Functions

Write the equation of the transformed function in the box.

Translate 4 units left Translate 2 units up	Reflect vertically
Translate 5 units right Reflect vertically	Translate 7 units down Reflect vertically

$$f(x) = log_4 x$$

Translate 4 units left Translate 2 units up	Reflect over the y-axis
Translate 5 units right Reflect vertically	Translate 7 units down Reflect over the x-axis

$$f(x) = log_4(x - 10) + 1$$

Translate 4 units left Translate 2 units up	Reflect vertically
Translate 5 units right Reflect vertically	Translate 7 units down Reflect over the y-axis

$$f(x) = -log_4(x + 7) - 3$$

End-of-Course Prep

4.5 Exponential Growth and Decay

Warm-Up

List the steps in simplifying $5(1-0.07)^3$.

Main Topic **Exponential Growth and Decay**

Matthew decided to invest $5,400 in a cryptocurrency where he is expecting to make 5% interest rate annually. What will be the value of his investment in 7 years?

Exponential Growth

$\boldsymbol{f(t) = a(1+r)^t}$

a = initial amount
r = growth rate
t = time interval

Exponential Decay

$\boldsymbol{f(t) = a(1-r)^t}$

a = initial amount
r = growth rate
t = time interval

Matthew decided to invest $5,400 in a cryptocurrency. Due to a pandemic, his investment has depreciated by 5% annually. What will be the value of his investment in 7 years?

Rose Ann inherited a property that is currently worth $15,000. She is aware that the value of the property appreciates by 3% every year. What will be the value of the property in 10 years?	Thomas and his fiancée are both from New York. In 2020, New York has a population of 2,500,000. They both found out that New York's population decreases by 5.2% yearly. What will be the population of New York in 2026?

Need Help?

State if the equation is an exponential growth or decay. Explain.

$f(x) = 5(1.03)^9$	$f(x) = 3(0.53)^4$	$f(x) = 7(1.45)^7$
$f(x) = 2(1.56)^5$	$f(x) = 100(1.53)^4$	$f(x) = 120(0.45)^{16}$

Fernando plans to invest \$5,000 at an interest rate of 3.7% compounded annually.		
What is his total investment in 3 years?	How long will it take to double his money?	How long will it take to triple his money?

Jake's business has been experiencing a 5.06% decrease in profit since 2019. Prior to 2019, he was making a profit of about \$345,000.		
How much will be the estimated profit in 2026?	How long will the profit to be about \$300,000?	How long will it take for the profit to be cut in half?

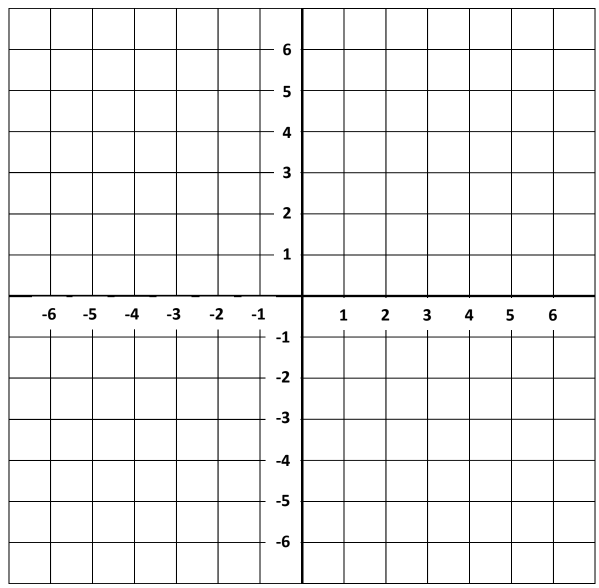
Research

Look up any information about $\boldsymbol{e}$ in Mathematics on the web.

☐ Graph $f(x) = e^x$.

☐ Identify the following features of the function.

- Domain:
- Range:
- Asymptote:
- x-intercept:
- y-intercept:
- Left End Behavior:
- Right End Behavior:
- Increasing or Decreasing:



Continuous Compounding Formula

$$A = Pe^{rt}$$

A-Final Amount **P**-Principal Amount **r**-Growth Rate **t**-Time Interval

Matthew realized that he could invest in a company at an annual interest rate of 4.3% compounded continuously. How long will it take him to triple his $2,000 investment?

Scavenger Hunt

Solve the question in each station with a partner.

Station 1

Station 6

Station 2

Show complete work to receive full credit.

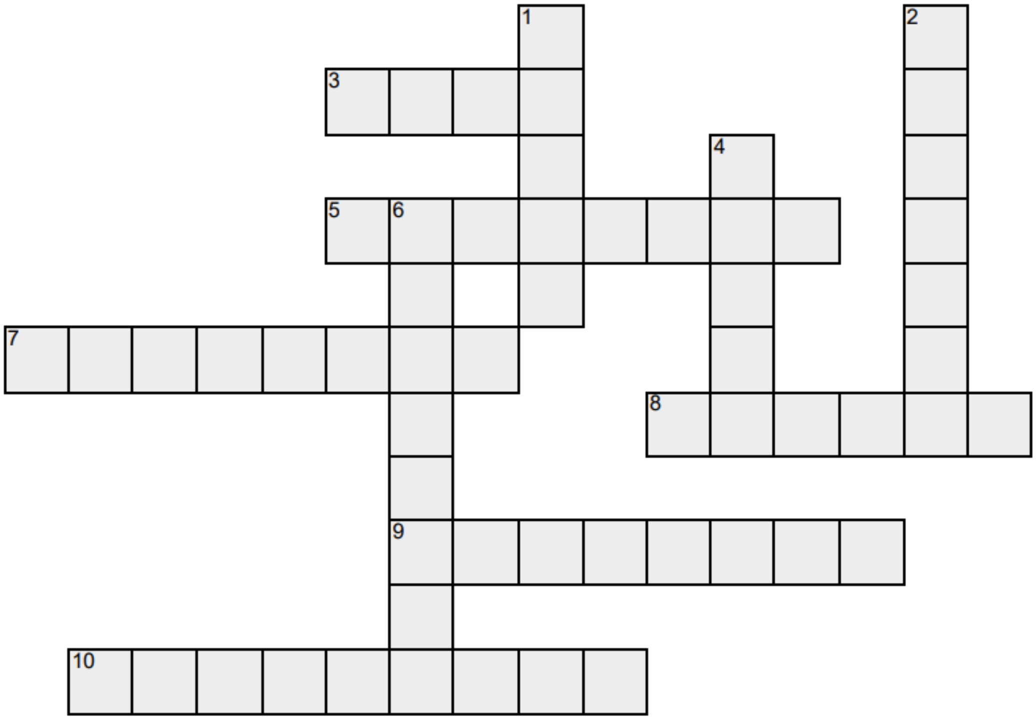
Station 5

Station 3

Station 4

End-of-Course Prep

Exponential and Logarithmic Functions



Across	Down
3. A logarithmic rule stating that $log_b(x) = log(x)/log(b)$. 5. A statement of equality between two mathematical expressions. 7. A relationship between two quantities such that every input there is only one output. 8. Exponential ________ is a process where a quantity increases at a rapid rate over time and it is represented by a formula $a = P(1 + r)^t$. 9. It determines the number of times a base is multiplied by itself. 10. A straight line where a graph continues to approach it but will never intersect it.	1. An exponential _____ is a process where a quantity decreases at a rapid rate over time and it is represented by a formula $a = P(1 - r)^t$. 2. A logarithmic rule stating that $log_b(x \bullet y) = log_b(x) + log_b(y)$. 4. A logarithmic rule stating that $log_b(x^y) = ylog_b(x)$. 6. A logarithmic rule stating that $log_b(x\,/\,y) = log_b(x) - log_b(y)$.

Unit 5 | Modeling with Geometry

High School Mathematics 3

North Carolina Math 3 Standards

Unit 5

Geometric Measurement & Dimension *Explain volume formulas and use them to solve problems.*	
NC.M3.G-GMD.3	Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

Geometric Measurement & Dimension *Visualize relationships between two-dimensional and three-dimensional objects.*	
NC.M3.G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry *Apply geometric concepts in modeling situations.*	
NC.M3.G-MG.1	Apply geometric concepts in modeling situations • Use geometric and algebraic concepts to solve problems in modeling situations: • Use geometric shapes, their measures, and their properties, to model real-life objects. • Use geometric formulas and algebraic functions to model relationships. • Apply concepts of density based on area and volume. • Apply geometric concepts to solve design and optimization problems.

Modeling with Geometry *Prove Geometric Theorems.*	
NC.M3.G-CO.10	Verify experimentally properties of the centers of triangles (centroid, incenter, and circumcenter).
NC.M3.G-CO.11	Prove theorems about parallelograms • Opposite sides of a parallelogram are congruent. • Opposite angles of a parallelogram are congruent. • Diagonals of a parallelogram bisect each other. • If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
NC.M3.G-CO.14	Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

5.1 Centers of Triangles

Warm-Up

Write all you know about the following terms.

Triangle	Bisector	Midpoint
Hypotenuse	Perpendicular	Median

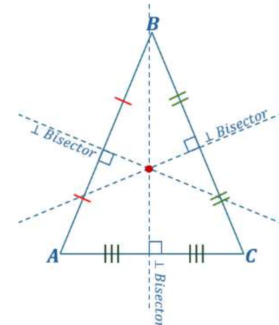
Main Topic Centers of Triangles

Circumcenter

The point where three perpendicular bisectors from the sides of the triangle intersect.

Location of Circumcenter

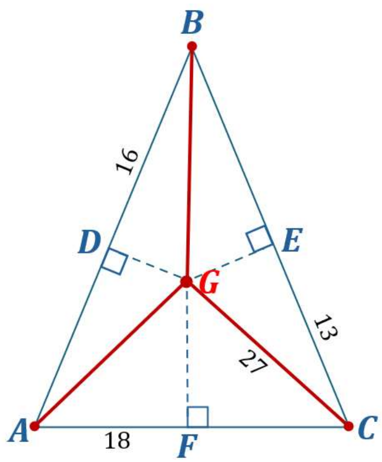
- Acute Triangle: Lies INSIDE the triangle
- Right Tringle: Lies at the MIDPOINT of the Hypotenuse
- Obtuse Triangle: Lies OUTSIDE the triangle



TASK 1

In the triangle below is created by perpendicular bisectors that meet at Point G.

Find the measures of the segments listed below.



BG: _________ $m\angle BDG$: _________

BC: __________ AG: _________

AD: __________ If AG = 2x – 12,

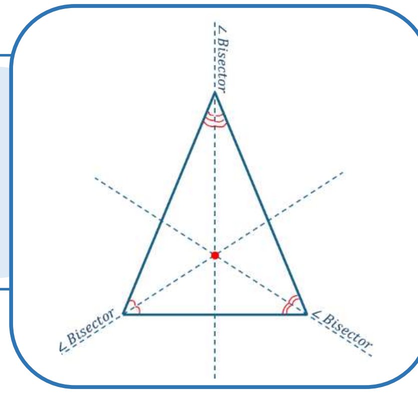
find x _________

Incenter

The point where three angle bisectors of the triangle intersect.

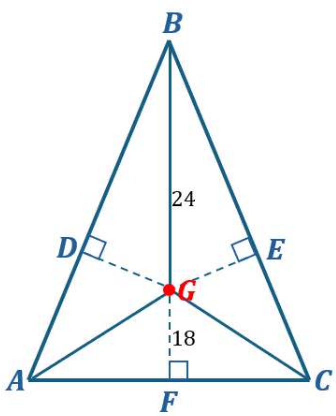
Location of Incenter

- Acute Triangle: Lies INSIDE the triangle
- Right Tringle: Lies INSIDE the triangle
- Obtuse Triangle: Lies INSIDE the triangle



TASK 2

In the triangle below is created by angle bisectors that meet at Point G. Find the measures of the segments listed below.



DG: ________

If GE = (4x-1),

find x: __________

AG: ______

If $m\angle CBG = 26°$,

find $m\angle ABG$: __________

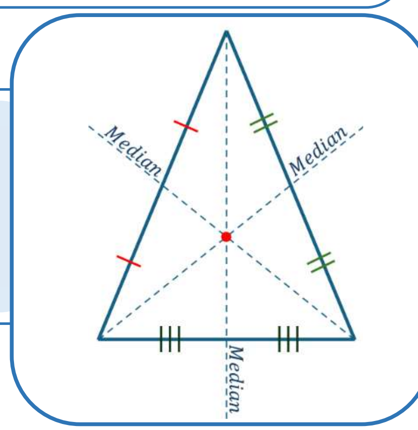
find $m\angle ADG$:___________

Centroid

The point where three medians of the triangle intersect.

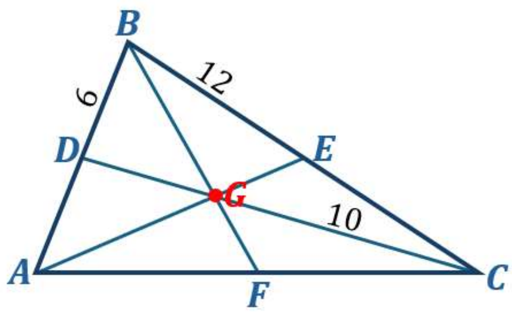
Location Centroid

- Acute Triangle: Lies INSIDE the triangle
- Right Tringle: Lies INSIDE the triangle
- Obtuse Triangle: Lies INSIDE the triangle



TASK 3

In the triangle below is created by medians that meet at Point G. Find the measures of the segments listed below.



AD: ________ BC: ___________

DC: ________ FC: ___________

If AE = 27, find GE: ______________

Honors Level

What fundamental geometric construction (involving lines and angles) is used to locate the circumcenter, and how does this construction inherently link to the concept of **perpendicular bisectors**?

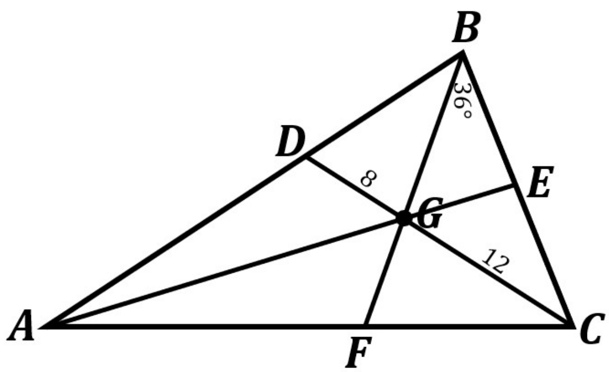
The incenter is equidistant from the sides of the triangle. How does this specific equidistance property directly relate to the incenter being the center of the largest circle (the **incircle**) that can be inscribed *within* the triangle, touching all three sides at exactly one point?

When the three medians intersect at the centroid, they divide the original triangle into six smaller triangles. Describe the relationship between the areas of these six smaller triangles and explain *why* they share this specific area relationship.

Centers of Triangles

The following triangles are created by angle bisectors intersecting at point G.

1.

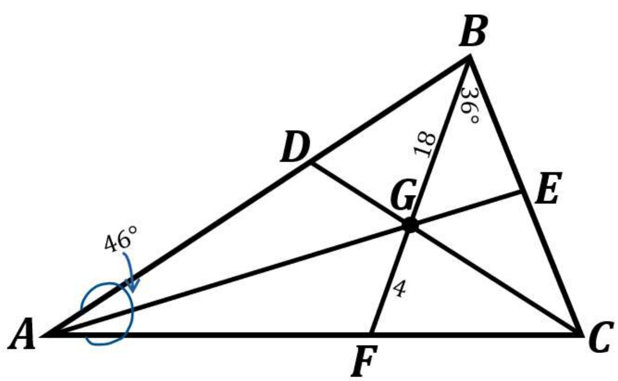


GF: ____________ AG: ____________

If BG = 2x + 1, find x: ______________

$m\angle ABG$: _______________

$m\angle AFG$: _______________



GE: ____________ GC: _____________

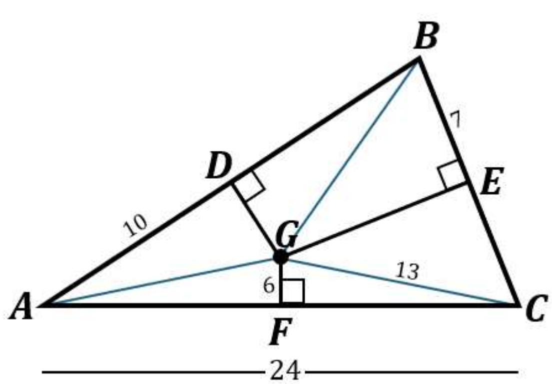
If GF = 4x + 3, find x: ______________

$m\angle BAG$: _______________

$m\angle BEG$: _______________

The following triangles are created by perpendicular bisectors intersecting at point G.

3.

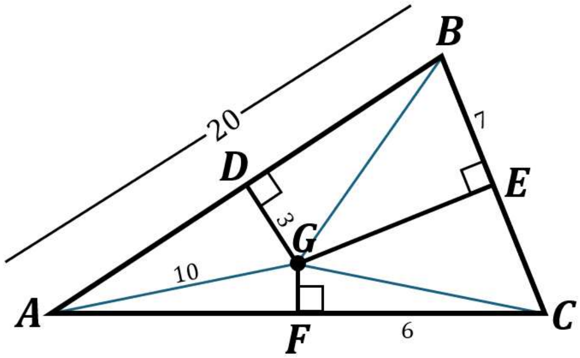


AB: ____________ BC: ______________

AF: _____________ AG: ______________

$m\angle ADG$: _________ If GC = 3x - 8, find x: ______

4.



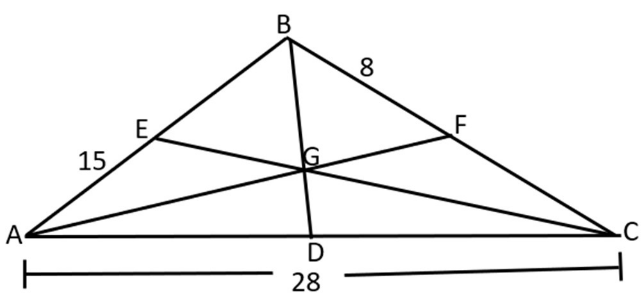
DB: ____________ AC: ____________

AF: _____________ GB: ____________

$m\angle BEG$: _________ If GA = 4x - 2, find x: ____

The following triangles are created by medians intersecting at point G.

5.

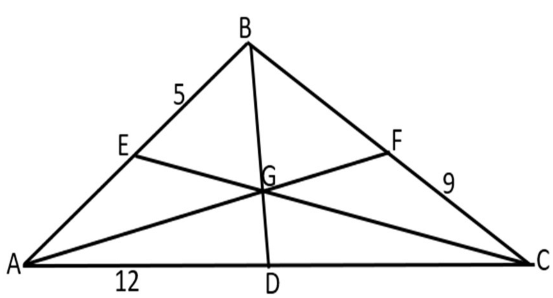


BC: ____________ EB: __________

AD: _____________ If EG = 7, find EC: ______

If AF = 33, find AG: __________

6.



BC: ____________ AB: _____________

AC: _____________ If AF = 24, find GF: _________

If EG = 12, find GC: __________

End-of-Course Prep

5.2 Midsegments of Triangles and Trapezoids

Warm-Up

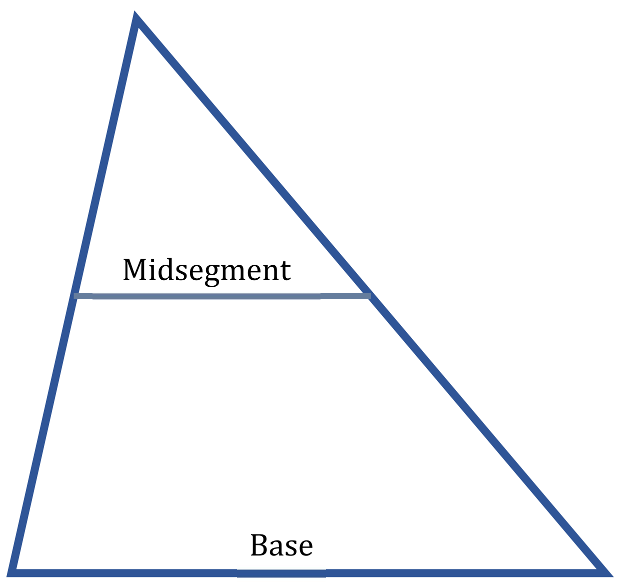
Write all you know about the characteristics of trapezoids.

Main Topic Midsegment of Triangles

Midsegment of a Triangle

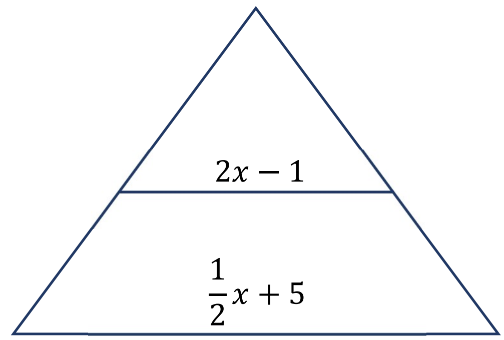
The segment that connects the midpoints of two sides of the triangle. It is parallel to the base (or third side) of the triangle.

$$\text{Midsegment} = \frac{\text{Base}}{2} \quad \text{or} \quad \text{Base} = 2\text{Midsegment}$$



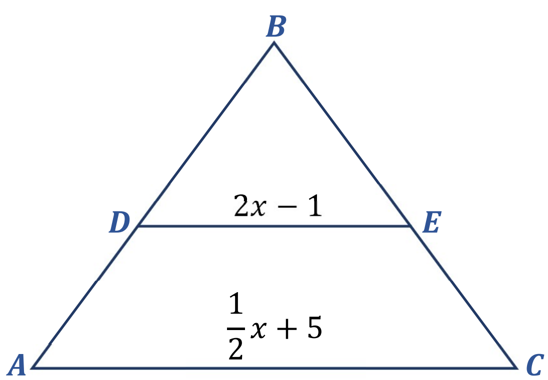
TASK 1

Find the value x.



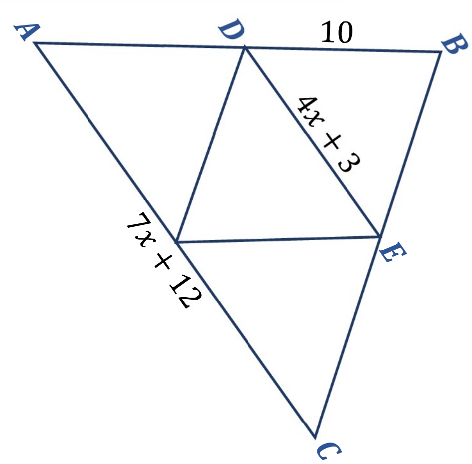
TASK 2

Find the length of segment AC.



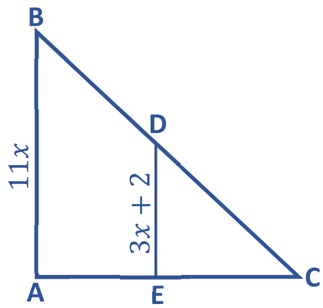
TASK 3

Find the length of segment DE.

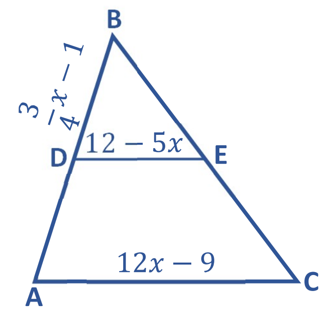


Main Topic	Midsegment of Triangles

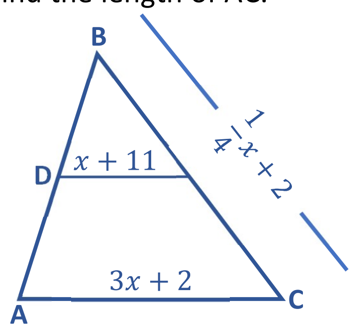
Find x.



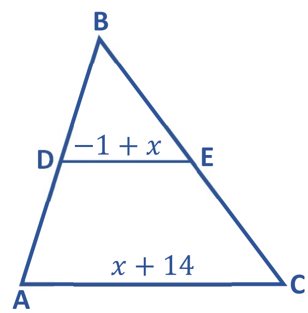
Find x.



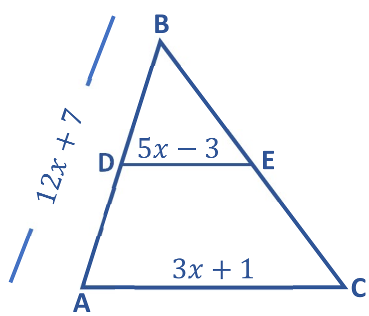
Find the length of AC.



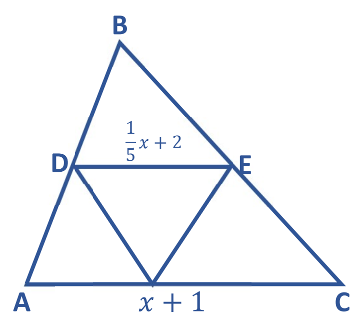
Find the length of segment DE.



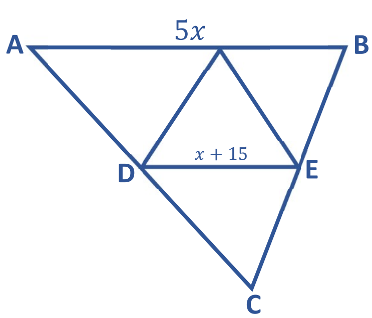
Find x.



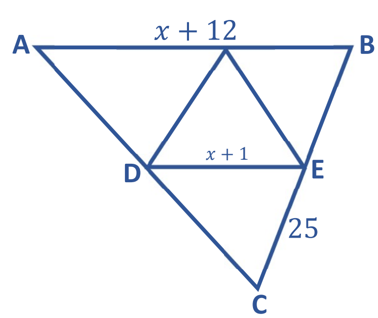
Find x.



Find the length of AB



Find the length of DE.

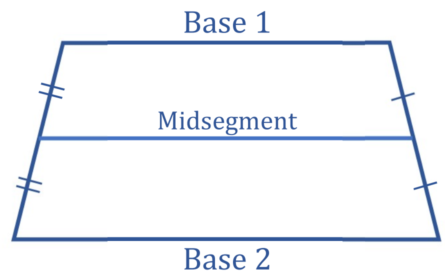


Main Topic **Midsegment of Trapezoids**

Midsegment of a Trapezoid

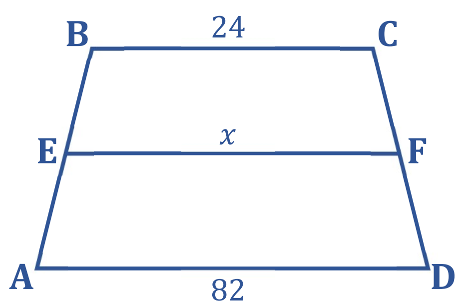
The line segment that connects the midpoints of the two sides that are NOT parallel.

$$\textbf{Midsegment} = \frac{\textbf{Base 1} + \textbf{Base 2}}{2}$$



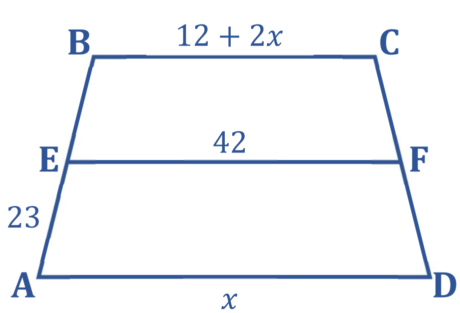
TASK 1

Find x.



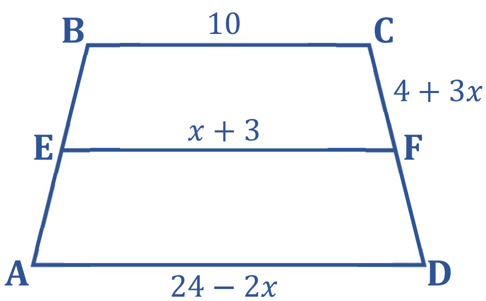
TASK 2

Find x.

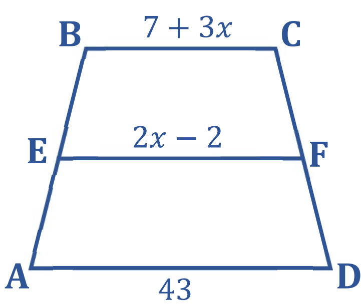
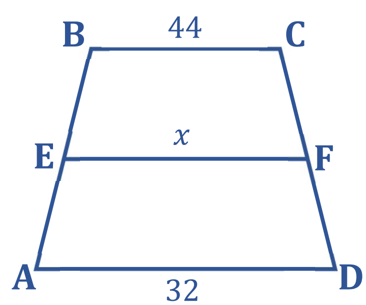
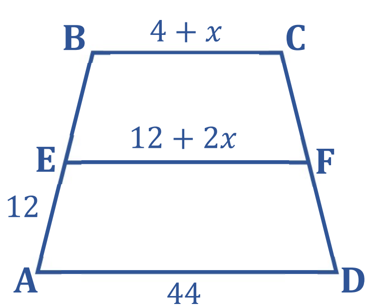
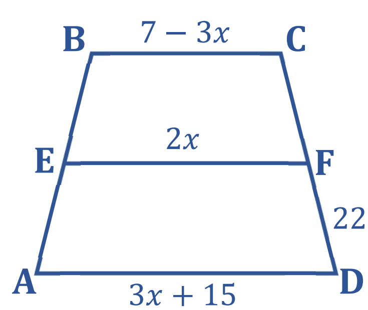


TASK 3

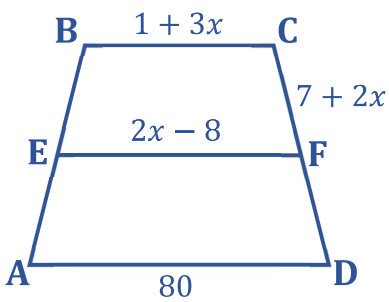
Find the length of AD.



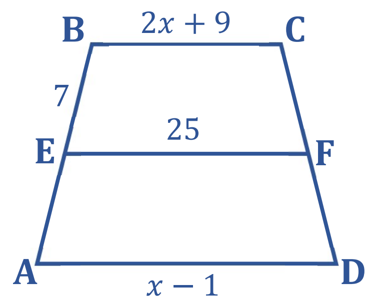
Practice

Find the length of EF. 	Find x. 
Find x. 	Find the length of AD. 

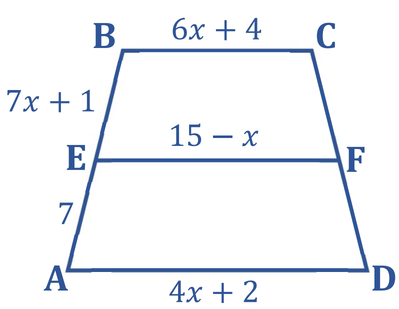
Find x.



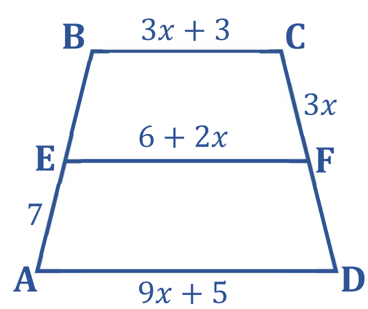
Find the length of BC.



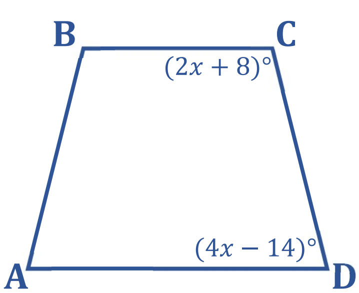
Find the length of EF.



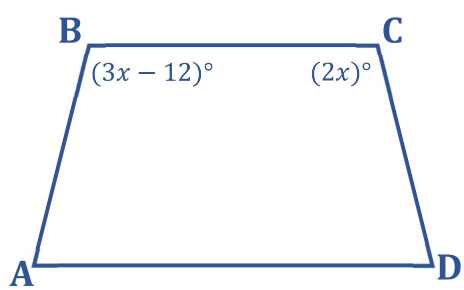
Find x.



Find the m $\measuredangle B$.



Find the m $\measuredangle A$.



End-of-Course Prep

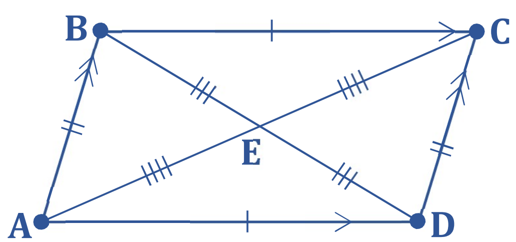
5.3 Properties of Parallelograms

Warm-Up

What is a quadrilateral? What do you know about quadrilaterals?

Main Topic Properties of Parallelograms

Properties of a Parallelogram



$$m\angle A + m\angle B = 180°$$

$$m\angle B + m\angle C = 180°$$

$$m\angle C + m\angle D = 180°$$

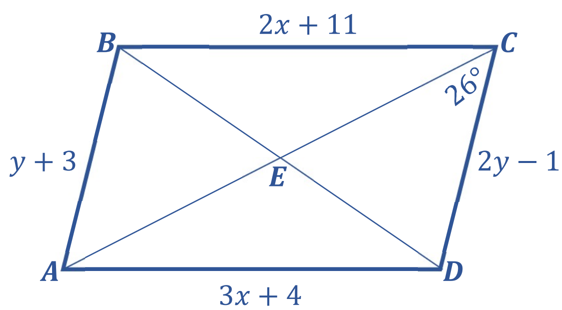
$$m\angle D + m\angle A = 180°$$

Interior angles add up to 360°

Write your observations on the properties of parallelograms.

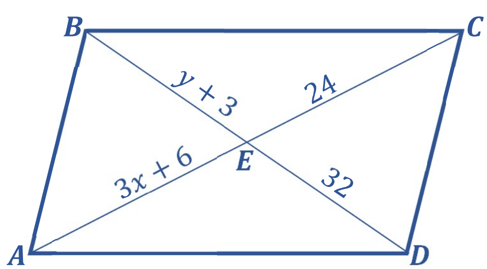
TASK 1

Find x and y.



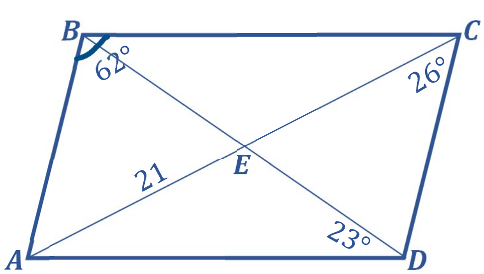
TASK 2

Find x and y.



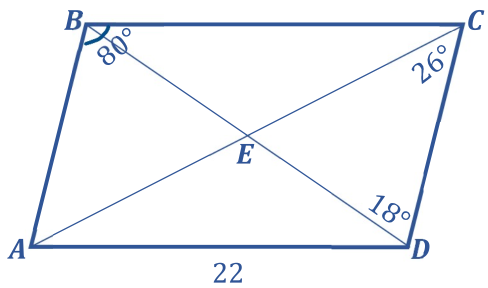
TASK 3

Find the $m\angle ABD$.



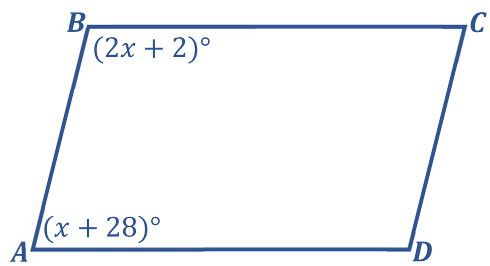
TASK 4

Find the $m\angle ABD$.



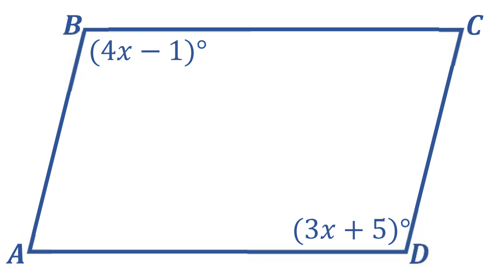
TASK 5

Find x.

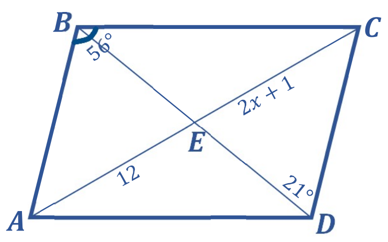
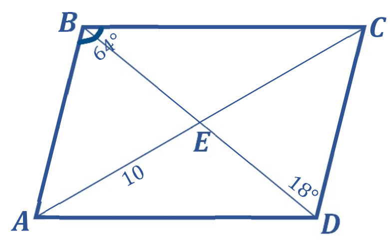
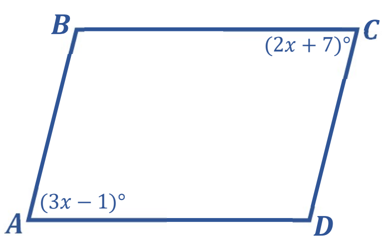
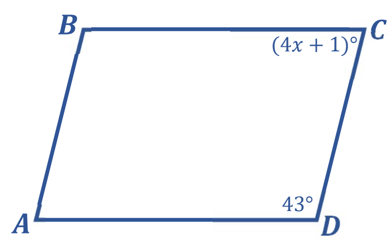
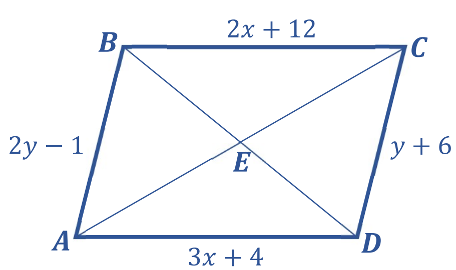
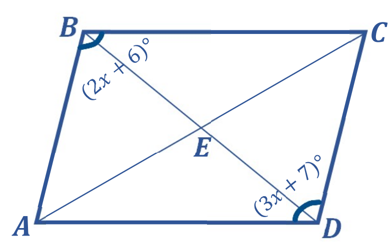


TASK 6

Find the $m\angle D$.



Solve each problem and show all steps.

Find x.  	Find the $m\angle CBD$. 
Find $m\angle C$.  	Find x. 
Find x.  	Find the $m\angle ADC$. 

Quick Math

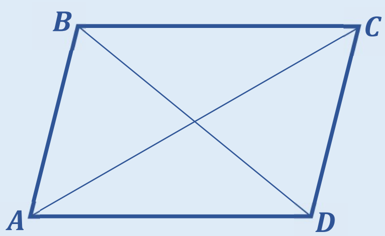
What do the following stand for?

SSS SAS ASA AAS HL

Main Topic | **Parallelogram Proofs**

Prove using a two-column proof

We Do

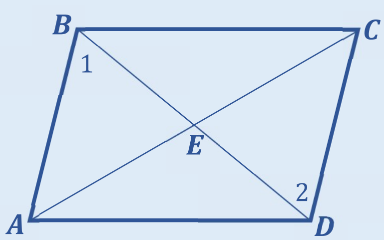


Given: Quadrilateral ABCD with segments $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ and diagonal $\overline{AC}$.

Prove: ABCD is a parallelogram.

Statements	Reasons
	Given
$\overline{AC} \cong \overline{AC}$	
	SSS Postulate
$\angle BAC \cong \angle DCA$ $\angle BCA \cong \angle DAC$	
	Alternate Interior Angles
ABCD is a parallelogram	

We Do

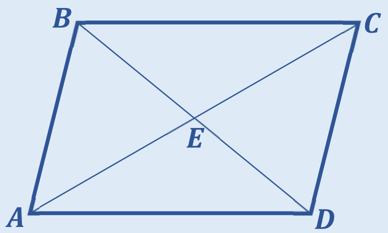


Given: $\overline{AC}$ bisects $\overline{BD}$
$\angle 1 \cong \angle 2$

Prove: ABCD is a parallelogram.

Statements	Reasons

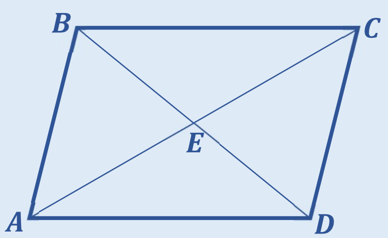
You Do



Given: $\square ABCD$
Prove: $\Delta AED \cong \Delta CEB$

Statements	Reasons

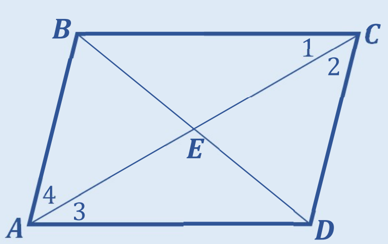
You Do



Given: $\square ABCD$
Prove: $\overline{BD}$ and $\overline{CA}$ bisect each other.

Statements	Reasons

You Do



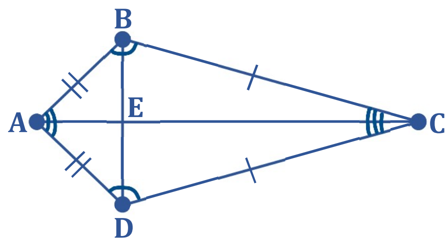
Given: $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$
Prove: $\square ABCD$

Statements	Reasons

Main Topic Properties of Kites

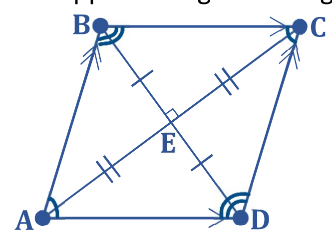
Properties of Kites

$\overline{BD}$ bisects $\overline{BD}$ creating 90° angles.
$\overline{AC}$ bisects $\angle A$ and $\angle C$ cutting them in half.



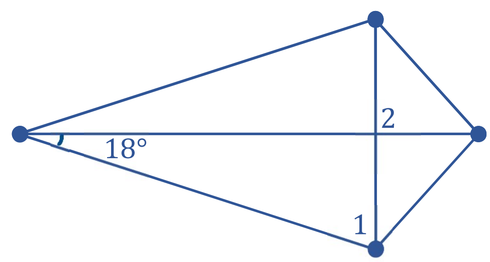
Properties of a Rhombus

$\overline{BD}$ bisects $\overline{AC}$ creating 90° angles.
Consecutive angles are supplementary.
Diagonals bisect opposite angles cutting the in half.



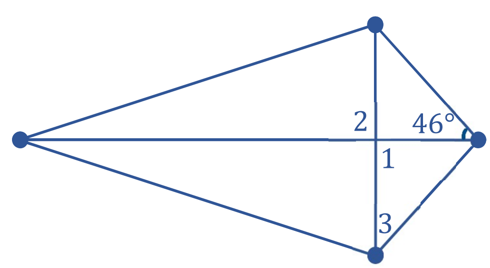
TASK 1

Find angles 1 and 2.



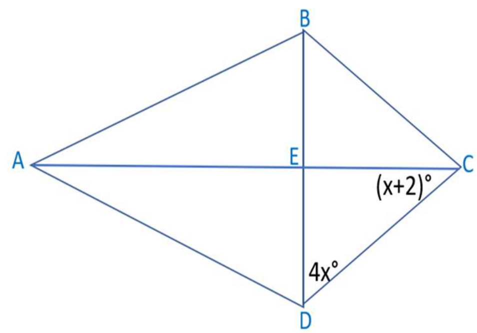
TASK 2

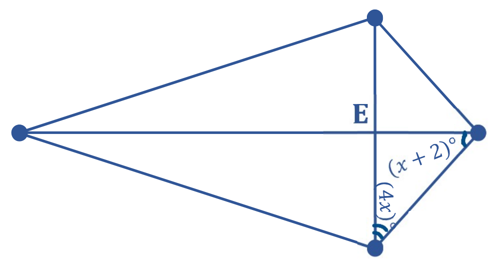
Find the measure if angles 1, 2 and 3.



TASK 3

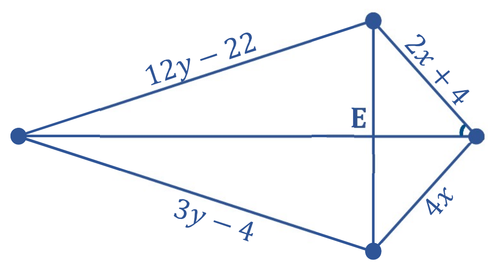
Find x.





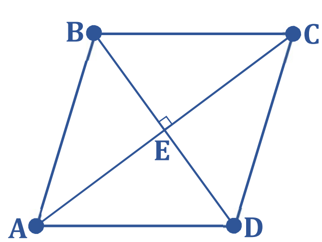
TASK 4

Find the values for x and y.



Main Topic	Properties of Rhombus

TASK 1



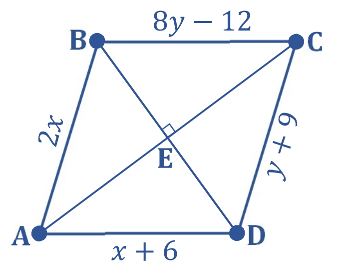
If $\overline{AE}$ = 12, find $\overline{AC}$.

If $m\measuredangle DCE$ = 44°, find $m\measuredangle DCB$:

$m\measuredangle BEA$.

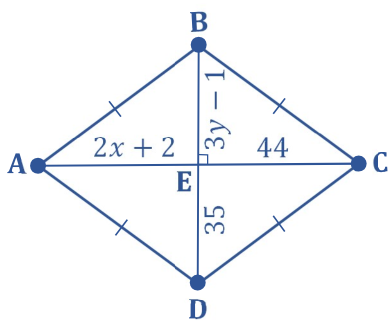
TASK 2

Find x and y.

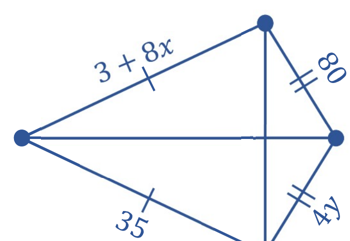


Rhombus and Kites

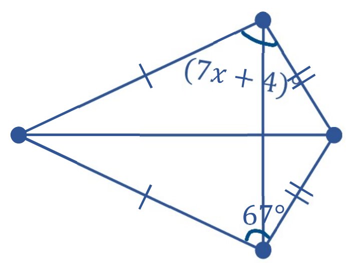
Find x and y



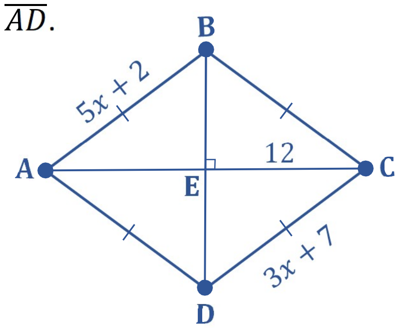
Find x.



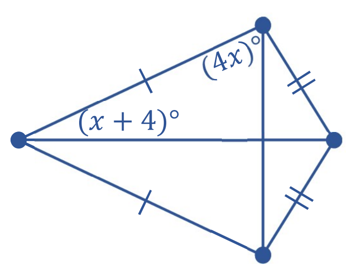
Find x.



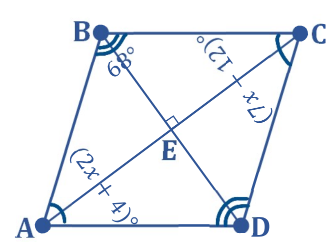
Find the length of $\overline{AD}$.



Find the m $\measuredangle ABE$.



Find the m $\measuredangle BAD$.



End-of-Course Prep

5.4 Surface Area – Part 1

Warm-Up

Write the formula of the area of the shapes.

Rectangle	Square	Triangle	Hexagon

Main Topic Surface Area of Prisms

Prism

A **prism** is a three-dimensional shape with congruent and parallel faces, referred to as bases, connected by parallelogram faces.

It's a Wrap

Materials:

Graph paper, scissors, clear tape

Instructions:

- Wrap the prism with graphing paper.
- Label faces 1 through 6.
- Label each face with the number of squares on it.
- Cut the edges of the wrapper in such a way that all faces lay flat, but one face must be connected with another face.

Fill out the table with the number of squares on the faces.

Face 1	Face 2	Face 3
Face 4	Face 5	Face 6

TASK 1

- Construct a rectangular prism with sides 5 units, 3 units and 4 units.
- Construct the net of the prism and label each side of the shapes.
- Identify the shapes in the net that are congruent.

Fill out the table with face areas.

	Face 1	Face 2	Face 3	Face 4	Face 5	Face 6
Area						
Surface Area						
Surface Area Formula						

Quick Math

Find the surface area of a rectangular prism with sides 9 in, 12 in, and 10 in.

Find the surface area of a rectangular prism with sides 13 cm, 30 cm, and 15 cm.

Need Help?

Main Topic Surface Area of Pyramids

Pyramid

A **pyramid** is a three-dimensional shape with polygonal base and triangular sides that meet at a single point (apex).

TASK 2

- Construct a rectangular pyramid whose base measures 8 units and 5 units with slant height that measures 10 units.
- Construct the net of the pyramid and label each side of the shapes.
- Identify the shapes in the net that are congruent.

Fill out the table with face areas.

	Base	Face 1	Face 2	Face 3	Face 4
Area					
Surface Area					
Surface Area Formula					

Find the surface area of the shapes.

Square Pyramid	**Regular Hexagonal Pyramid**
Each side of the base measures 7 in and the slant height measures 12 in.	Each side of the base measures 5 cm and the slant height measures 8 cm.

Need Help?

Surface Area Stations

Solve the questions in each station with a partner.

Station 1

Station 6

Station 2

Show complete work to receive full credit.

Station 5

Station 3

Station 4

End-of-Course Prep

5.5 Surface Area – Part 2

Warm-Up

Write the formula of area of the shapes.

Circle

Rectangle

Main Topic Surface Area of Spheres

Sphere

A **sphere** is a three-dimensional shape with curved surface and every point is equidistant from its center.

I'm Peeling Good

Materials:

Orange fruit, marker, paper

Instructions:

- Draw 4 circles by tracing the orange.
- Peel the orange.
- Fill the circles with orange peel.

Surface Area Formula	

Main Topic Surface Area of Cylinders

Cylinder

A **cylinder** is a three-dimensional shape with congruent and parallel circular faces, referred to as bases, connected by a curved surface.

TASK 1

- Construct a cylinder whose radius of the base is 6 units and a height of 10 units.
- Construct the net of the cylinder and label the radius and sides of the shapes.
- Identify the shapes in the net that are congruent.

	Bottom Base	Lateral Area	Top Base
Area			
Surface Area			
Surface Area Formula			

Find the surface area of the cylinders. Round to the nearest tenth.

Radius = 12 in Height = 20 in	Diameter = 18 cm Height = 10 cm
Circumference = 100 in Height = 35 in	Base Area = 84 sq in Height = 21 in

Need Help?

Main Topic Surface Area of Cones

Cone

A **cone** is a three-dimensional shape with a circular base and a curved side that tapers smoothly to a single point called apex.

TASK 2

Cone Construction

- Construct a cone whose radius of the base is 6 units and slant height of 10 units.
- Construct the net of the cone and label the radius and sides of the shapes.
- Identify the shapes in the net that are congruent.

	Base	**Lateral Area**
Area		
Surface Area		
Surface Area Formula		

Find the surface area of the cones. Round to the nearest tenth.

Radius = 12 in Slant Height = 20 in	Diameter = 18 cm Slant Height = 10 cm
Circumference = 100 in Slant Height = 35 in	Base Area = 84 sq in Slant Height = 21 in

Need Help?

Surface Area Stations

Solve the questions in each station with a partner.

Station 1

Station 6

Station 2

Show complete work to receive full credit.

Station 5

Station 3

Station 4

End-of-Course Prep

5.6 Volume of Prisms and Pyramids

Warm-Up

Describe the shapes.

Prism

Pyramid

Main Topic Volume of Prisms and Pyramids

Volume

Volume is the number of cubic units occupied in a three-dimensional space.

Cubes in 3D Shapes

Material:

Cubes manipulatives

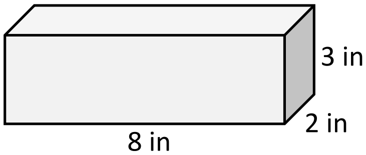
Instructions:

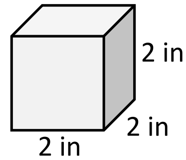
- Form a prism by connecting the cubes.

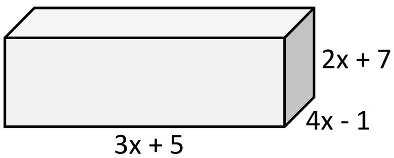
Fill out the table with the number of units (cubes) based on your prism.

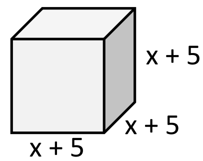
	Length	Width	Height
No. of Units/Cubes			
Volume			
Volume Formula			

Find the volume of the shapes.









The length of the base of a rectangular prism is 10 cm while its side faces are squares. If the volume is 490 cubic cm, what is the length of the width and height?

The length of the base of a rectangular prism is $5x - 2$ while its side faces are squares. If the volume is $5x^3 + 8x^2 + x - 2$, what is the length of the width and height?

Volume of a Pyramid

Materials:

Prism, Pyramid, Water

Instructions:

- Fill up the pyramid with water.
- Transfer the water to the prism.

Write your general observations about the volume of the pyramid and prism.

____

____

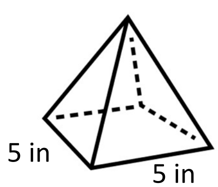
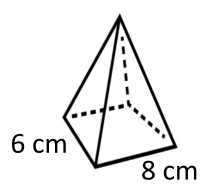
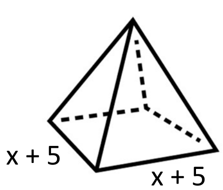
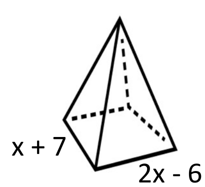
____

____

What is the formula of the volume of prism?

What is the formula of the volume of pyramid?

Find the volume of the shapes.

Vertical Height = 7 in 	Vertical Height = 12 cm 
Vertical Height = 3x + 6 	Vertical Height = 4x + 1 

End-of-Course Prep

5.7 Volume of Cylinders, Cones, and Spheres

Warm-Up

Describe the shapes.

Cylinder Cone Sphere

Main Topic **Volume of Cylinders**

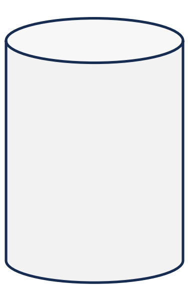
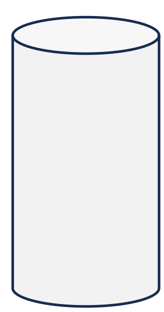
TASK 1

Cylinders

- Construct a cylinder.
- Write the formula to find the area of the base.
- Label the height, h.
- Write the formula to find the volume of a cylinder.

Hint: Use the formula of prism.

Find the volume of the shapes.

Radius = 6 in Height = 8 in	Diameter = 8 cm Height = 12 cm
	

Need Help?

Find the height of a cylinder with radius equals 10 in and a volume of 2,826 cubic inches.	Find the radius of the base of a cylinder with a height of 6 cm and a volume of 923.16 cubic centimeters.
Find the volume of a cylinder with radius equals $x + 7$ and a height equals $2x + 9$.	Find the volume of a cylinder with diameter equals $6x + 10$ and a height equals $20 - x$.
Find the volume of a cylinder with radius equals $3x - 4$ and a height equals $5x + 1$.	Find the volume of a cylinder with diameter equals $2x + 8$ and a height equals $x + 9$.

Need Help?

Main Topic	Volume of Cones

Materials:

Cylinder, Cone, Water

Instructions:

- Fill up the cone with water.
- Transfer the water to the cylinder.

Write your observations about the volume of the cone and cylinder.

____
____
____
____

What is the formula for the volume of cylinders?

What is the formula for the volume of cones?

Find the volume of the cones.

Radius = 7 in Slant Height = 12 in	Diameter = 8 cm Slant Height = 15 cm
Radius = x + 8 Slant Height = x + 10	Diameter = 10x - 20 Slant Height = 3x + 5

Main Topic	Volume of Spheres

Materials:

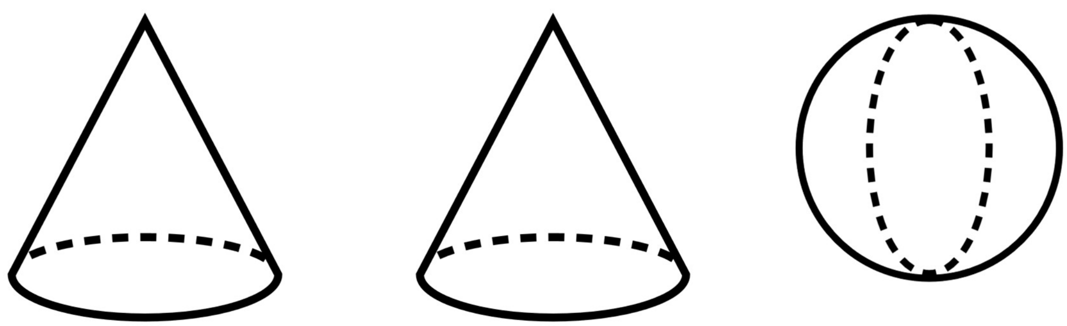
Cone, Sphere, Water

Instructions:

- Fill up the cone with water.
- Transfer the water to the sphere.

Write your observations about the volume of the cone and sphere.

____
____
____
____



Volume of Sphere Formula	

End-of-Course Prep

5.8 Cross Section of Three-Dimensional Shapes

Warm-Up

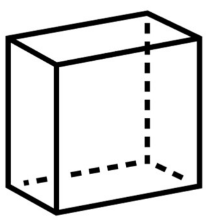
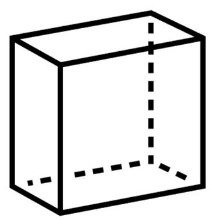
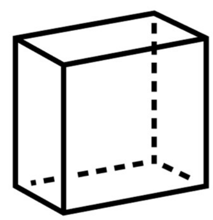
Write all the names of 3D shapes that you know.

Main Topic	Cross Section of Rectangular Prism

Cross Section

A **cross section** of a three-dimensional shape is a flat surface generated when a 3D shape is cut horizontally, vertically, or diagonally.

Cut the prism in several ways.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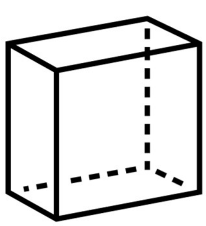	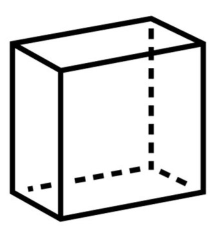	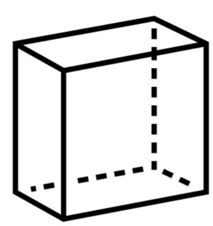
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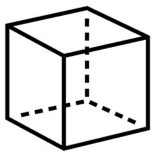
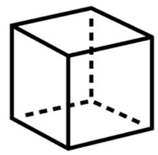
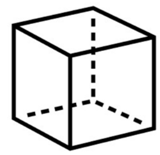
Main Topic | Cross Section of a Cube

Cut the cube in several ways.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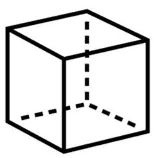	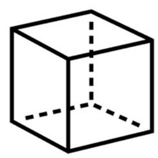	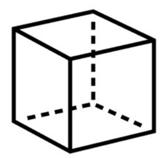
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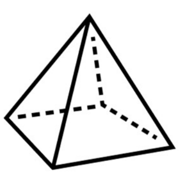
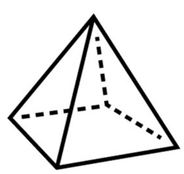
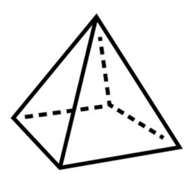
Main Topic | Cross Section of a Pyramid

Cut the pyramid in several ways.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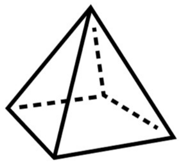	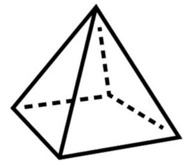	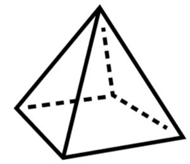
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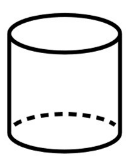
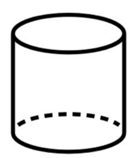
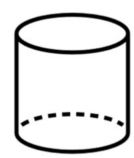
Main Topic	Cross Section of a Cylinder

Cut the cylinder in several ways.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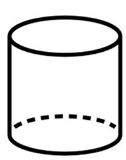	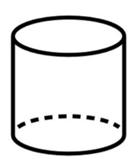	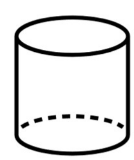
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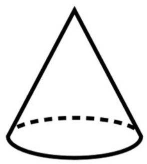
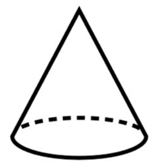
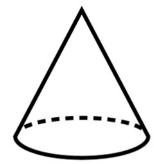
Main Topic | Cross Section of a Cone

Cut the cone in several ways.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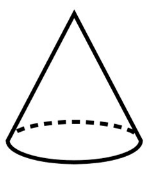
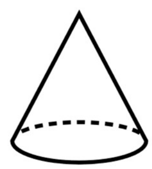
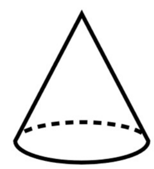
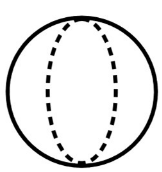
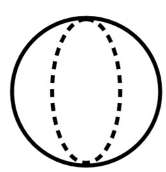
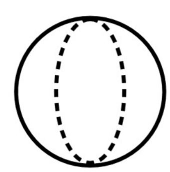
		
Description of the cross section.	Description of the cross section.	Description of the cross section.

Main Topic | Cross Section of a Sphere

Cut the sphere in several ways.

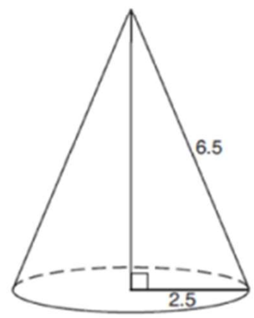
		
Description of the cross section.	Description of the cross section.	Description of the cross section.

End-of-Course Prep

5.9 Rotation of a 2-D Shape to Create a 3-D Shape

Warm-Up

Find the volume.



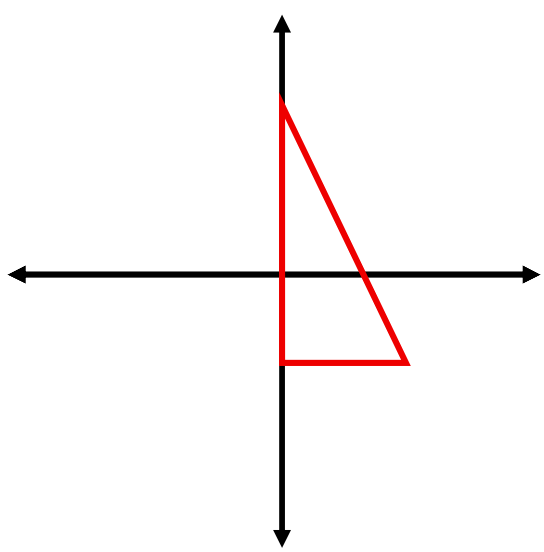
Main Topic **Rotation of a 2-D Shape**

Axis of Rotation

The line (or axis) that you rotate or "wrap" the shape around to create your 3-D shape.

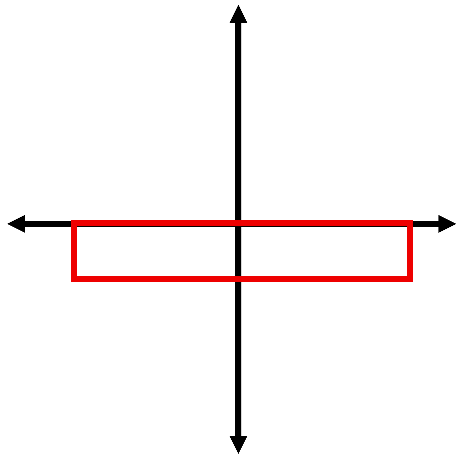
TASK 1

What 3-D shape will result in rotating a right triangle about the y-axis.



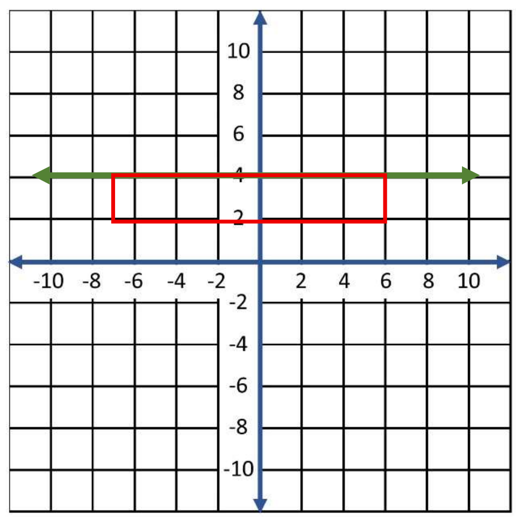
TASK 2

What 3-D shape will result in rotating a right triangle about the x-axis.

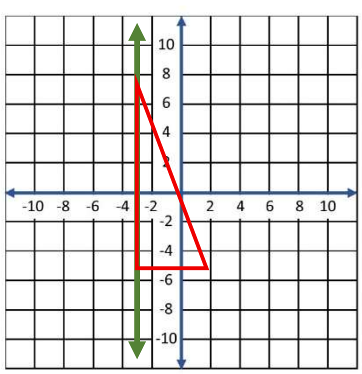
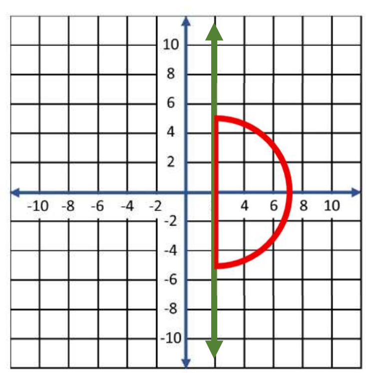


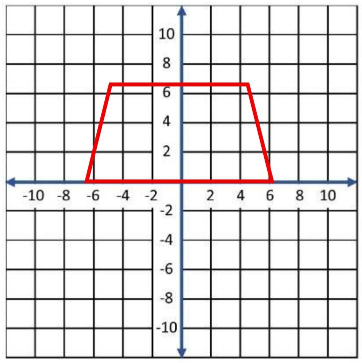
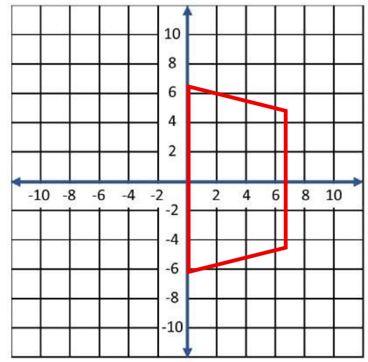
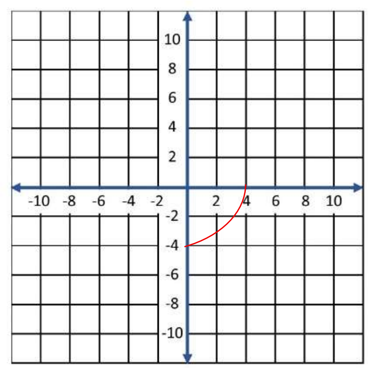
TASK 3

What 3-D shape will result in rotating a rectangle about the line y = 4?



Name the 3-D shape that results from your rotation.

Rotate around x = -3.	Rotate around x = 2.
	

Rotate about the x-axis.  	Rotate about the y-axis. 
If the rectangle below is continuously rotated about the side labeled s, what 3-D shape will be formed?  s	The quarter circle is centered at the origin. What 3-D shape will result by the quarter circle continuously being rotated about the y-axis? 

Honors Level

Discuss the following with a math buddy.

- ☐ How does the position of the axis of rotation relative to the 2D shape affect the resulting 3D shape's symmetry, overall form, and internal structure (e.g., resulting in a solid volume versus a hollowed-out shape)?
- ☐ If a 3D solid is generated by rotating a 2D profile, how can one mathematically or conceptually determine the shape of any cross-section perpendicular to the axis of rotation, and how does this relate back to the original 2D shape?
- ☐ How is the surface area of the generated 3D solid conceptually and computationally related to the arc length (or perimeter) of the original 2D shape's boundary, considering different distances from the axis of rotation (Pappus's Second Centroid Theorem)?

5.10 Density

Warm-Up

Solve the equations for the given variable.

$d = \frac{m}{v}$, solve for m

$d = \frac{m}{v}$, solve for v

Main Topic **Density**

Density $D = \frac{m}{v}$	Is a measure of how much of something is contained within a certain space or interval.

TASK 1

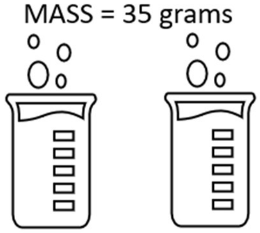
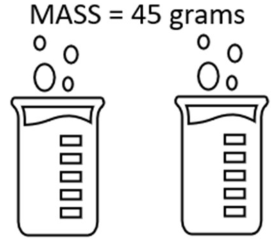
- ☐ What is the density of CO gas if 17.6 grams occupies a volume of 204 ml? Don't forget your units!

- ☐ You have a piece of solid metal in the shape of a cube with each side length being 3 cm long. It has a mass of 32 g. What is the density? Don't forget your units!

- ☐ Platinum has a density of 8.5 $grams/cm^3$ and silver has a density of 16.4 $grams/cm^3$. Which has the greater mass, 8 cm^3 of platinum or 5 cm^3 of silver?

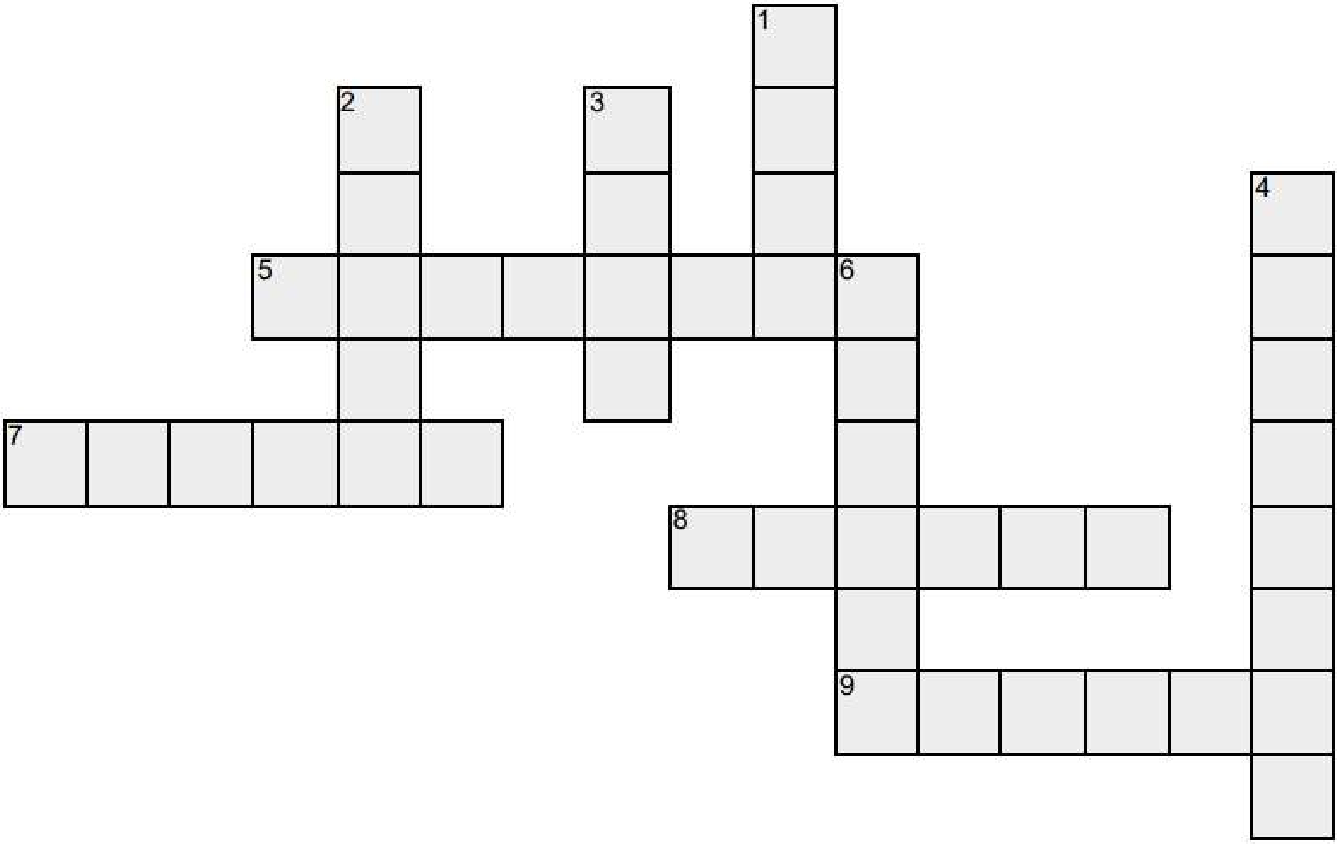
- ☐ <u>**Water Displacement:**</u> In math lab today, we dropped a stone into a beaker containing 50 ml of water. The height of the water rose to 80 ml. If the mass of the stone was 68 grams, what was the density? (HINT: Final Water Level – Starting Water Level) = Volume.

Solve each problem and show each step.

What is the density of CO gas if 0.237 grams occupies a volume of 140 ml? Don't forget your units!	You have a piece of wood in the shape of a cube with each side length being 2 cm long. It has a mass of 28 g. What is the density? Don't forget your units!
We have 8 ml of benzene and eight ml of diazinon. Benzene has a mass of 5.4 grams and diazinon has a mass of 22 grams. Which one is denser?	A sample of gold has the dimensions 4 cm x 2 cm x 4 cm. If the mass of the rectangular-shape object is 86 grams, what is the density?
Find the volume and density. MASS = 35 grams  START: 35mL FINAL: 70mL	Find the volume and density. MASS = 45 grams  START: 62mL FINAL: 98mL

End-of-Course Prep

Modeling with Geometry



Across	Down
5. The distance between two points on a circle that passes through the center.	1. A three-dimensional shape with a circular base and a curved side that tapers smoothly to a single point called apex.
7. The number of cubic units occupied in a three-dimensional space.	2. Connected by parallelogram faces.
8. The vertical distance between the base and its uppermost point.	3. The number of square units in a shape.
	4. Connected by a curved surface.
9. A three-dimensional shape with curved surface and every point is equidistant from its center.	6. The distance between the center and any point on the circle.

Unit 6 | Circles

High School Mathematics 3

North Carolina Math 3 Standards

Unit 6

Circles *Understand and apply theorems about circles.*	
NC.M3.G-C.2	Understand and apply theorems about circles. • Understand and apply theorems about relationships with angles and circles, including central, inscribed and circumscribed angles. • Understand and apply theorems about relationships with line segments and circles including, radii, diameter, secants, tangents and chords.
NC.M3.G-C.5	Using similarity, demonstrate that the length of an arc, s, for a given central angle is proportional to the radius, r, of the circle. Define radian measure of the central angle as the ratio of the length of the arc to the radius of the circle, s/r. Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations *Translate between the geometric description and the equation for a conic section.*	
NC.M3.G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

6.1 Arcs, Angles, and Chords

Warm-Up

Write all you know about the following.

Arc

Angles

Chords

Main Topic **Radius, Diameter, and Chords**

TASK 1

- Draw a circle.
- Label the center with C.
- Plot 3 points on the circle and label with X, Y, and Z.
- Connect X, Y, and Z to C.
- What do you call the segment that connects C and X?
- Connect X and Y.
- What do you call the segment that connects X and Y?
- What do you call a line segment that connects two points on a circle that passes through the center?
- $\overline{CX} = 3x + 3, \overline{CZ} = 2x + 8$. Solve for x.
- What is the length of the radius?
- What is the length of the diameter?

Need Help?

Main Topic **Arc and Central Angle**

TASK 2

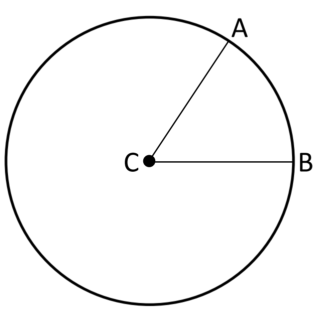
- Draw a circle.
- Label the center with C.
- Plot 3 points on the circle and label with X, Y, and Z.
- Connect X, Y, and Z to C.
- Use a protractor to measure the angles formed at the center.

- Use a protractor to measure the angles formed by the intercepted arcs.

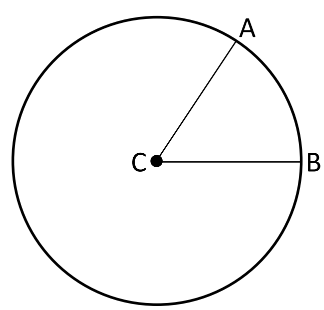
Central Angles	Intercept Arcs	Observation
$m\angle XCY =$	$m\widehat{XY} =$	
$m\angle YCZ =$	$m\widehat{YZ} =$	
$m\angle XCZ =$	$m\widehat{XZ} =$	

Solve for x.

$\angle ACB = 5x + 5$
$m\widehat{AB} = 70°$



$\angle ACB = 11x - 6$
$\widehat{AB} = 9x + 10$



Need Help?

Main Topic: Arc and Inscribed Angle

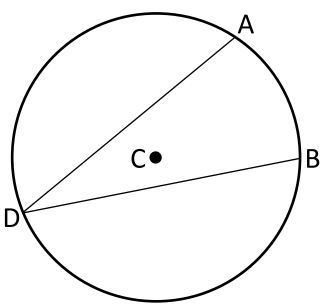
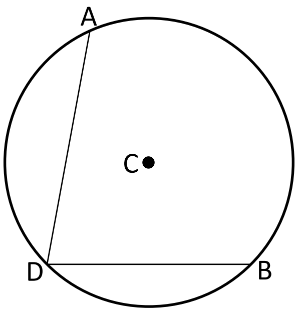
TASK 3

- Draw a circle.
- Connect two points, X and Y, on the circle that passes through the center, C.
- Plot point Z on the circle and connect it to X and Y.
- Use a protractor to measure $\angle XZY$.
- Use a protractor to measure $\widehat{XY}$.
- How does an inscribed angle measurement relate with the measurement of its intercepted arc?

____

____

Solve for x.

$\angle ADB = x + 7$ $m\widehat{AB} = 50°$ 	$\angle ADB = 4x - 12$ $\widehat{AB} = 7x + 1$ 

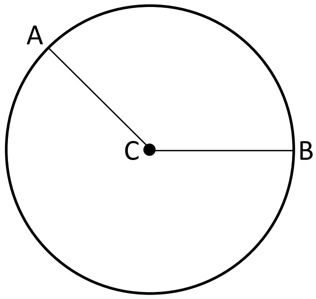

Need Help?

Fill in the blank

A _______________ is a distance between the center and all the points on the circle while a __________________ is a distance between two points on the circle that passes through the center. Any two points on the circle are connected by a ____________. An angle with vertex at the center of the circle with rays intersecting an arc is called a _______________________ and its measurement is _________________ with the angle measurement of its intercepted arc. An angle with vertex on the circle with rays intersecting an arc is called an _____________________ and its measurement is _____________________ of the angle measurement of its intercepted arc.

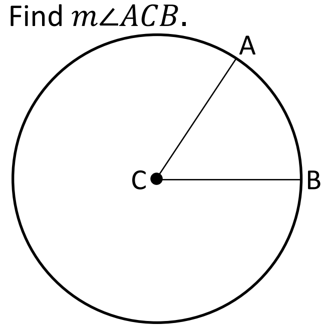
$\angle ACB = 5x - 15$ Solve for x.

$m\widehat{AB} = 160°$



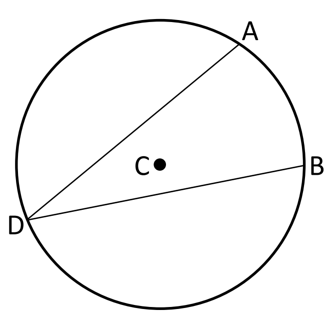
$\angle ACB = 7x + 2$ $\widehat{AB} = 5x + 22$

Find $m\angle ACB$.



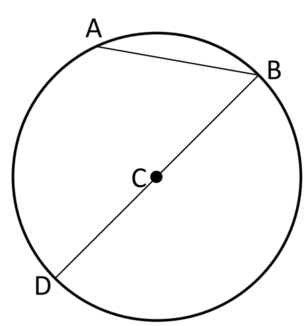
$\angle ADB = 6x + 3$ Solve for x.

$m\widehat{AB} = 54°$



$\angle ABD = 5x - 5$ $\widehat{AD} = 9x + 2$

Find $m\widehat{AD}$.



Scavenger Hunt

Solve the questions that are posted across the classroom.

End-of-Course Prep

6.2 The Secant

Warm-Up

Solve for x.

$$\frac{(5x+2)+(3x-8)}{2} = 10 - 2x \qquad \frac{(5x+2)-(3x-8)}{2} = 10 - 2x$$

Main Topic | **Secant of a Circle**

TASK 1

- Construct a circle.
- Construct $\overline{PQ}$ with endpoints on the circle.
- Construct $\overline{ST}$ with endpoints on the circle and intersecting $\overline{PQ}$ at R.
- Mark $\angle SRP$.

Secant

A line that crosses a circle at two point is called a **secant**.

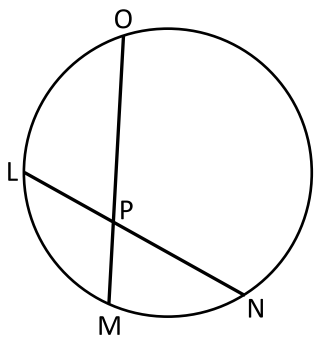
Given: $\overline{PQ}$ and $\overline{ST}$ intersect at R.

Prove: $m\angle 1 = \frac{1}{2}\left(m\widehat{PS} + m\widehat{QT}\right)$

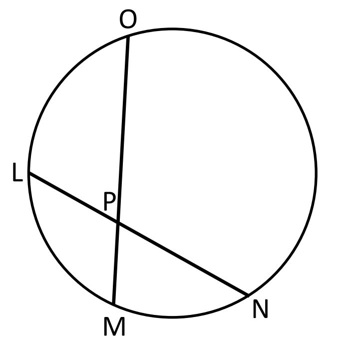
Statements	Reasons

Need Help?

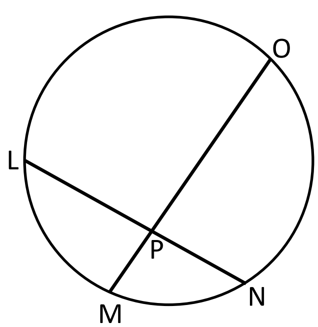
$\widehat{LM} = 60°$
$\widehat{ON} = 172°$
Find $m\angle LPM$.



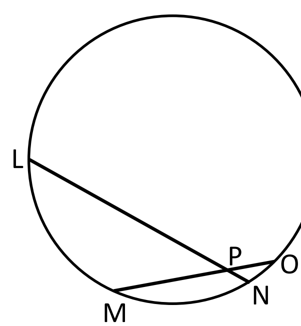
$\widehat{LO} = 100°$
$\widehat{MN} = 70°$
Find $m\angle LPO$.



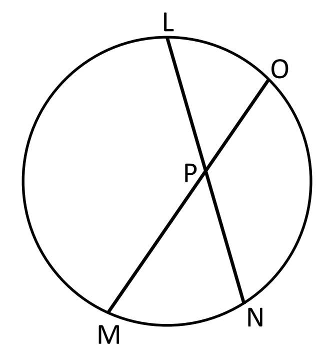
$\widehat{LM} = 9x - 1$
$\widehat{ON} = 110°$
$m\angle LPM = 77$
Solve for x.



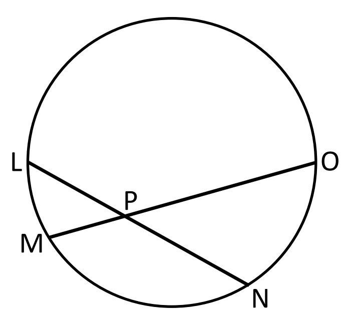
$\widehat{LO} = 210°$
$\widehat{MN} = 5x + 3$
$m\angle LPO = 129°$
Solve for x.



$\widehat{LM} = 10x + 8$
$\widehat{ON} = 8x$
$\angle LPM = 10x - 11$
Find $m\angle LPM$.



$\widehat{LO} = 13x + 11$
$\widehat{MN} = 8x + 6$
$\angle LPO = 11x - 3$
Find $m\widehat{MN}$.



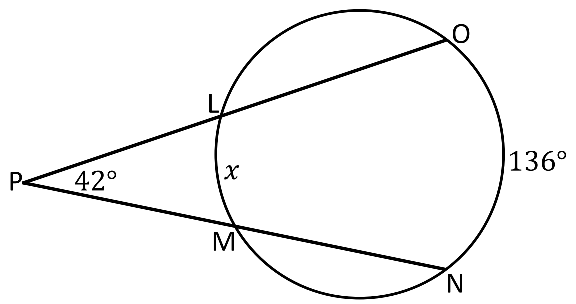
Main Topic **Secant of a Circle**

- Construct a circle.
- Construct an angle, H, with vertex outside the circle.
- The 1st ray intersects the circle at I and J.
- The 2nd ray intersects the circle at K and L.
- $m\widehat{JL} = 80°, m\widehat{IK} = 62°$
- Connect L and I
- Connect J and K
- $\overline{LI}$ and $\overline{JK}$ intersect at M.
- What is $m\angle JIL$?
- What is $m\angle IJK$?
- What is $m\angle JKL$?
- What is $m\angle ILK$?
- Create a general formula to find $m\angle IHK$ using $m\widehat{JL}$ and $m\widehat{IK}$.

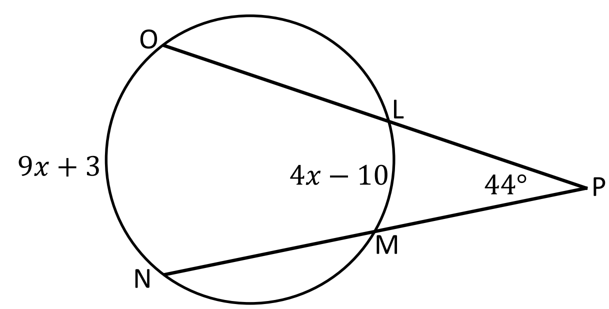
TASK 2

Need Help?

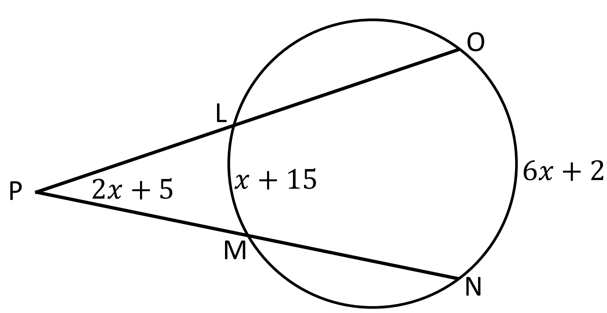
Solve for x.



Find $m\overline{ON}$, and $m\overline{LM}$.

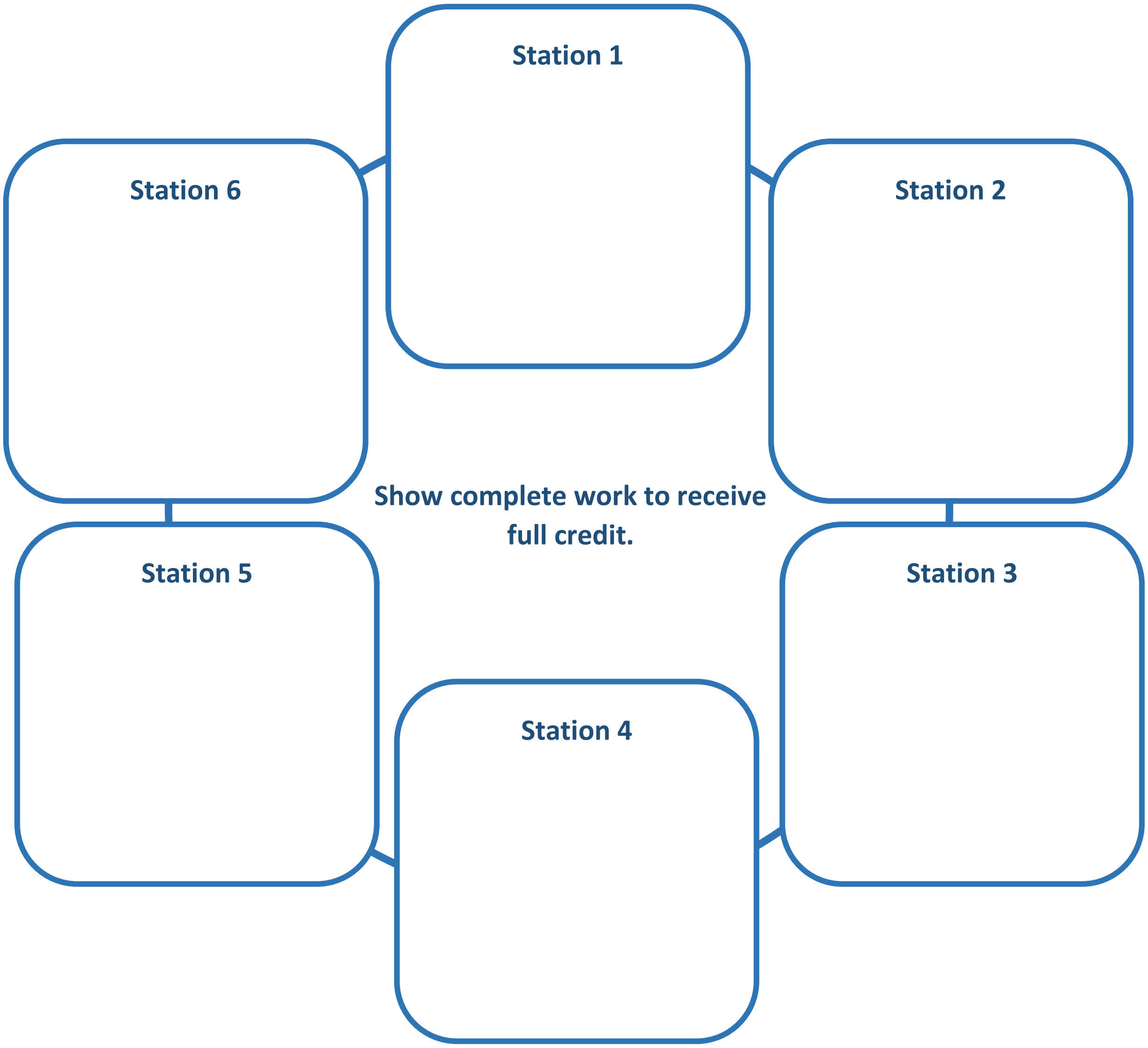


Find m$\angle LPM$, $m\overline{ON}$, and $m\overline{LM}$.



Secant Stations

Solve the questions in each station with a partner.



End-of-Course Prep

6.3 The Tangent

Warm-Up

Find the length of the missing side of a right triangle.

$a = 3, \quad b = 4, \quad c = ?$

$a = 6, \quad b = ?, \quad c = 10$

Main Topic **Tangent of a Circle**

TASK 1

- Construct a circle.
- Label the center with C.
- Construct a line , l, that intersects the circle at one point, P.
- Connect C and P.
- What do you notice about the angle formed by the radius and tangent?

Circle

A **circle** is a set of points that are equidistant from a fixed point called **center**.

Describe radius.

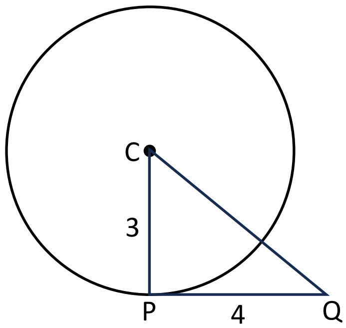
Describe diameter.

A line on the same page that crosses a circle at exactly one point is called a **tangent**.

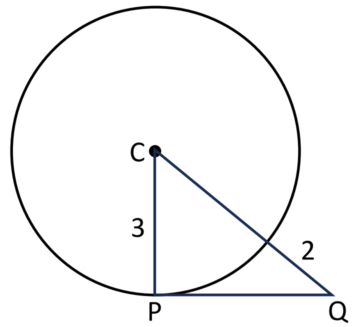
Tangent

Need Help?

Find the length of CQ.



Find the length of PQ.



TASK 2

- Construct a circle.
- Label the center with C.
- Construct two segments that share an endpoint, S, and both tangent to the circle at T and U.
- What do you notice about the length of $\overline{\mathrm{ST}}$ and $\overline{\mathrm{SU}}$?

Solve for x if $\overline{\mathrm{ST}} = 3x + 3$ and $\overline{\mathrm{SU}} = 2x + 8$.	Solve for x if $\overline{\mathrm{ST}} = \frac{x}{5} + 33$ and $\overline{\mathrm{SU}} = 2x - 3$.

Quick Math

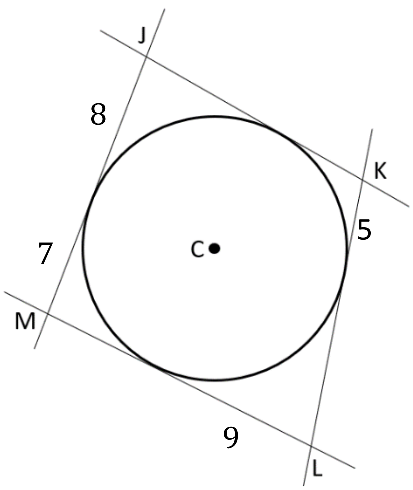
$\overline{\mathrm{PQ}} = 5x - 12$ and $\overline{\mathrm{PR}} = 2x + 9$ are both tangent to a circle.
Find the length of $\overline{\mathrm{PQ}}$ and $\overline{\mathrm{PR}}$.

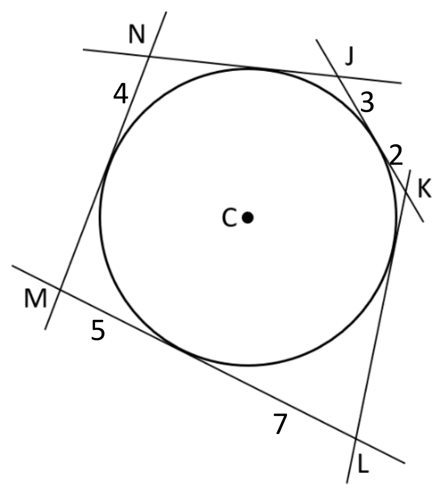
Need Help?

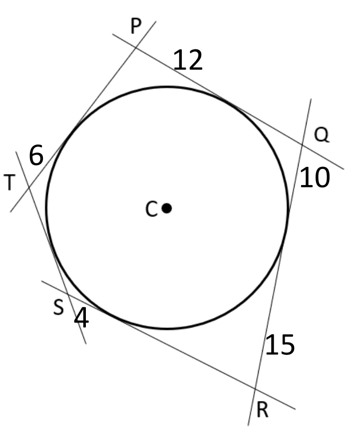
TASK 3

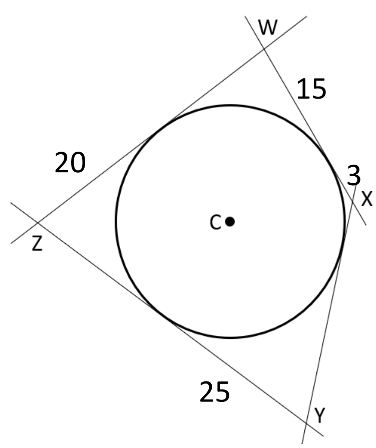
- Construct a polygon whose sides are tangent to a circle.
- Label congruent segments.

Find the perimeter of the polygon.



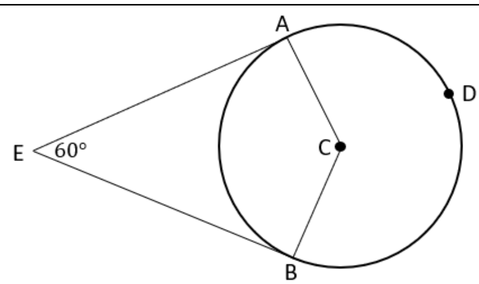




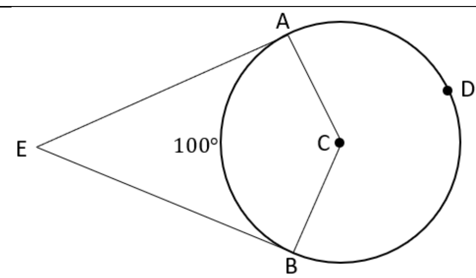


TASK 4

- Construct a circle.
- Label the center with C.
- Construct two segments that share an endpoint, S, and both tangent to the circle at T and U.
- Plot a 3rd point on the circle, V.
  (Plot V on the opposite side of S)
- Identify the minor and major arcs.



Find $m\angle C$.	Find $m\widehat{AB}$.	Find $m\widehat{ADB}$.



Find $m\angle E$.	Find $m\angle C$.	Find $m\widehat{ADB}$.

Create a general formula of the relationship between angle formed by the tangents with the major and minor arcs.

Tangent Stations

Solve the questions in each station with a partner.

Station 1

Station 6

Station 2

Show complete work

to receive full credit.

Station 5

Station 3

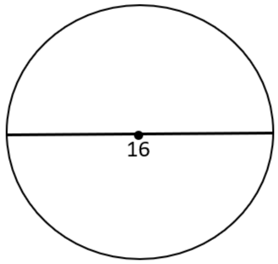
Station 4

End-of-Course Prep

6.4 Area of a Sector

Warm-Up

Find the area of the circle.



Main Topic Radians and Degrees

Important Formulas

$$Radian = \frac{\pi}{180} \cdot Degree$$

$$Degree = \frac{180}{\pi} \cdot Radian$$

TASK 1

Convert the following to radians.

a. 245°   b. 632°   c. -135°   d. -1040°

TASK 2

Convert the following to degrees.

a. $\frac{7\pi}{6}$ b. $\frac{11\pi}{6}$ c. $-\frac{\pi}{3}$ d. $-\frac{13\pi}{3}$

Convert the following.

$\frac{3\pi}{2}$	260°	$\frac{21\pi}{4}$
-960°	$-\frac{11\pi}{6}$	135°
$-\frac{14\pi}{3}$	-250°	$\frac{\pi}{2}$
1070°	$\frac{\pi}{6}$	180°

Main Topic Area of a Sector

Important Formulas

Degrees

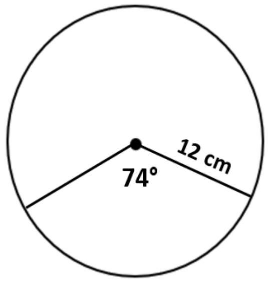
$$A = \pi r^2 \cdot \frac{\theta}{360}$$

Radians

$$A = \pi r^2 \cdot \frac{\theta}{2\pi}$$

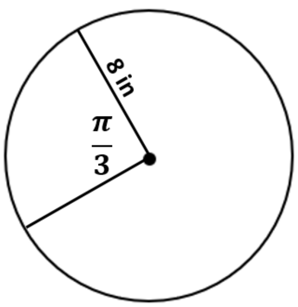
TASK 1

Find the area of the sector given the following circle. Round to the nearest tenth.



TASK 2

Find the area of the sector given the following circle. Round to the nearest tenth.



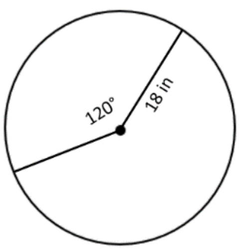
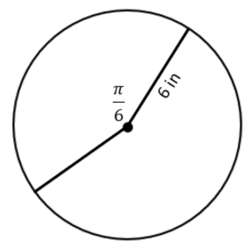
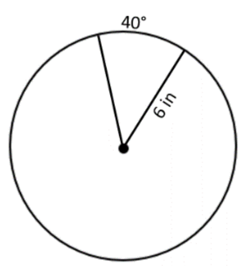
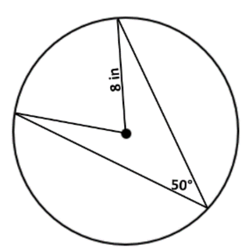
TASK 3

If the area is 26π and the radius is 6cm, find the central angle.

TASK 4

Circle G has a minor arc $\widehat{AB}$ with angle measure of 45°. Sector AGB has an area of 18π. What is the radius of Circle G?

Solve each problem and show all steps.

<table>
<tr>
<td>1. Find the area of the sector with the central angle at 120°.

</td>
<td>2. In circle O, the area of sector AOB is $42\pi\ in^2$. Find the central angle if the radius is 16 in.</td>
<td>3. Find the area of the sector with the central angle of $\frac{\pi}{6}$.

</td>
</tr>
<tr>
<td>4. Find the area of the sector if the radius is 4 in and the inscribed angle is 26°.</td>
<td>5. Find the area of the sector with the intercepted arc of 40°.

</td>
<td>6. Find the area of the sector if the radius is 8 in and the inscribed angle is $\frac{3\pi}{4}$.</td>
</tr>
<tr>
<td>7. Find the area of the sector with the central angle of $\frac{7\pi}{6}$ and a radius of 9 cm.</td>
<td>8. In circle O, the area of sector AOB is $22\pi\ cm^2$. Find the radius if the central angle is 58°.</td>
<td>9. Find the area of the sector with the inscribed angle of 50°.

</td>
</tr>
</table>

End-of-Course Prep

6.5 Arc Length

Warm-Up

What is the formula for the circumference of a circle?

How do I find the radius given the diameter?

Main Topic **Arc Length**

Important Formula

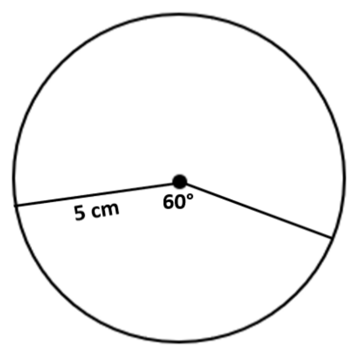
Arc Length, Radius, Angle (IN RADIANS)

$$S = r\theta$$

Theta (θ) has to be in radians!

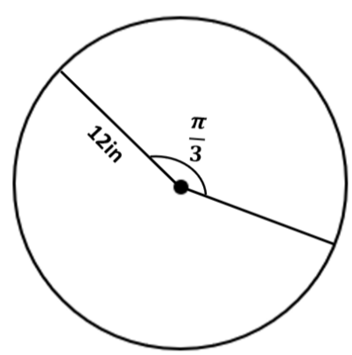
TASK 1

Find the arc length with the central angle of 60° and a radius of 5cm.
Round to the nearest tenth.



TASK 2

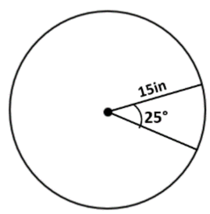
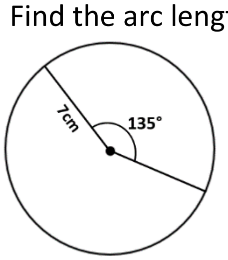
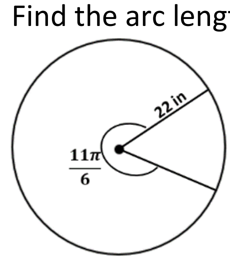
Find the Arc Length.



TASK 3

Find the arc length of a circle with a diameter of 20 meters and a central angle of 135°.

Find the Arc Length.

The radius of a circle is 8 ft and the central angle is $\frac{3\pi}{4}$.	Find the arc length.  	The radius of a circle is 16 cm and the central angle is 250°.
The arc length of a circle is 8.2m. If the central angle is 36°, what is the radius?	The diameter of a circle is 10 ft and the central angle is $\frac{7\pi}{4}$.	The arc length of a circle is 53.41 miles. If the central angle is $\frac{2\pi}{3}$, what is the radius?
Find the arc length.  	The arc length of a circle is 8.2m. If the radius is 4m, what is the central angle?	Find the arc length. 

End-of-Course Prep

6.6 Equations of Circles

Warm-Up

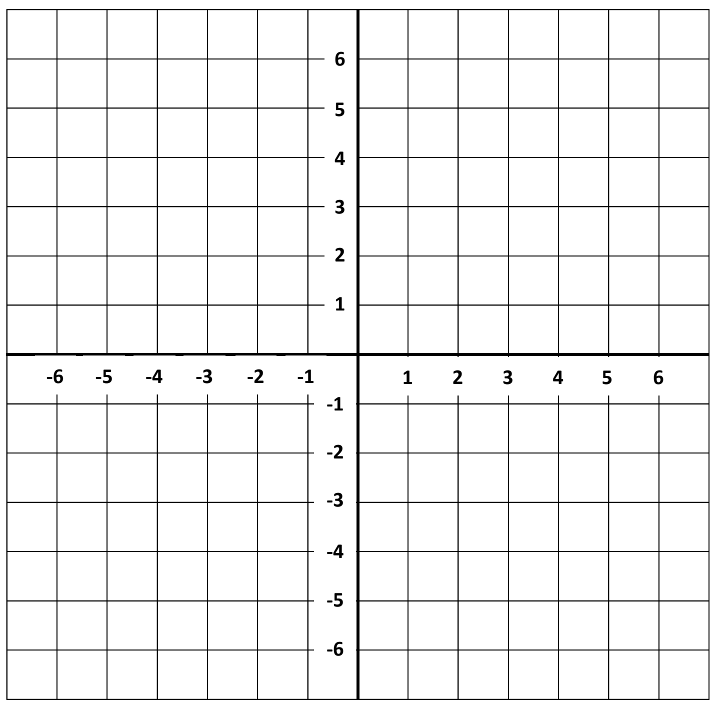
Simplify.

$(x+5)^2$ $\qquad$ $(x-5)^2+(y+4)^2$

Main Topic **Equation of Circles with Center at the Origin**

TASK 1

- Draw a circle with a center at the origin, C.
- Plot a point on the circle and label it with P(x, y).
- Connect points C and P.
- From P, draw a line perpendicular to the x-axis.
- Trace the right triangle that is formed.
- Label the sides with a, b, and c.
  (Note: Make sure that c is the hypotenuse)
- Write the general formula to find the lengths of a right triangle.
- Identify the length of a.
- Identify the length of b.



- Use Pythagorean Theorem to write the general equation of a circle with center at the origin.

General equation of the circle with center at the origin: ______________________

Need Help?

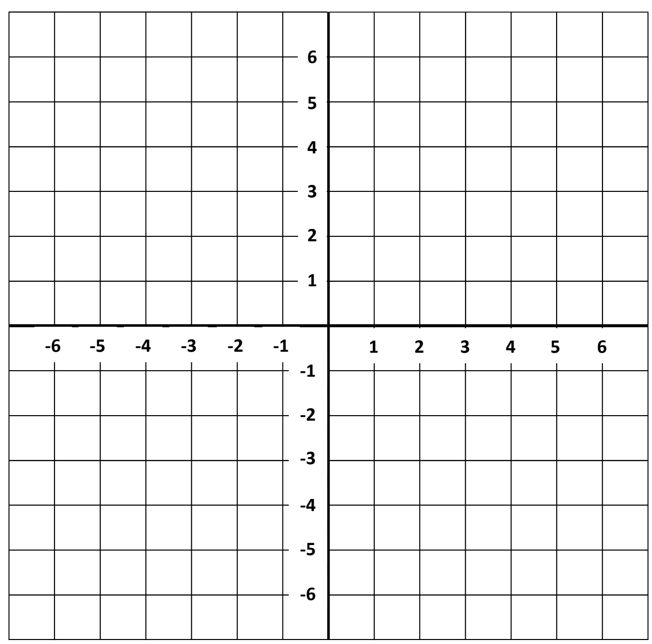
Find the equation of a circle with center at the origin and given the radius.

Radius = 7 units	Radius = 10 units
Radius = 12 units	Diameter = 18 units

Main Topic **Equation of Circles with Center at (h,k)**

TASK 2

- Draw a circle with center at C(h, k).
- Connect C with any point, P, on the circle.
- From P, draw a line perpendicular to the x-axis.
- Draw a horizontal line that passes through C.
- Trace the right triangle that is formed.
- Label the sides of the triangle with a, b, and c.
- Identify the lengths of a and b.
- Use Pythagorean Theorem to write the general formula of the equation of a circle with center at (h, k).



General equation of the circle with center at (h, k): ______________________

Need Help?

Find the equation of a circle with center at (h, k) and given the radius.

Center at (5, 2) Radius = 3 units	Center at (-7, 1) Radius = 9 units

Center at (-8, -2) Diameter = 10 units	Center at (6, -4) Diameter = 4 units

Center at (-2, 2) and tangent to x = -5 and x = 1.	Center at (4, -4) and tangent to x and y axes.

Main Topic: Center and Radius from the Equation of a Circle

TASK 3

$$x^2 + y^2 - 10x - 4y + 20 = 0$$

- Put $x^2 - 10x$ together,
  then leave a space after $-10x$.
- Put $y^2 - 4y$ together,
  then leave a space after $-4y$.
- Move the constant, 20, over the equal sign.
- Take half of -10, then square it.
- Add 25 after $x^2 - 10x$ and after -20.
- Take half of -4, then square it.
- Add 4 after $y^2 - 4y$ and after -20.
- Express $x^2 - 10x + 25$ in square of binomial form.
- Express $y^2 - 4y + 4$ in square of binomial form.
- Express the constant in exponential form.
- Equation: $(x-h)^2 + (y-k)^2 = r^2$
- **Center**: Take the values of h and k out of the parentheses and switch their signs.
- **Radius**: Take the square root of the constant.

Fill in the blank

Find the center and radius.

$$x^2 + y^2 + 14x - 2y - 31 = 0$$

$$x^2 + 14x + \_\_\_\_ + y^2 - 2y + \_\_\_\_ = 31$$

$$(x + \_\_\_\_)^2 + (y - \_\_\_\_)^2 = (\quad)^2$$

C(,) r =

$$x^2 + y^2 - 16x - 6y - 27 = 0$$

$$x^2 - \_\_\_x + \_\_\_ + y^2 - \_\_\_ + \_\_\_ = \_\_\_$$

$$(x - \_\_\_\_)^2 + (y - \_\_\_\_)^2 = (\quad)^2$$

C(,) r =

Need Help?

Find the center and radius of the circles.

$x^2 + y^2 - 49 = 0$	$x^2 + y^2 + 16x + 4y - 32 = 0$
$x^2 + y^2 - 12x + 8y + 48 = 0$	$x^2 + y^2 + 14x - 2y - 31 = 0$
$x^2 + y^2 - 24x + 12y + 176 = 0$	$x^2 + y^2 + 14x + 16y - 88 = 0$

What's the Password?

Solve the question in each station with a partner.

Station 1

____
Password

Station 6

____
Password

Station 2

____
Password

Show complete work

to receive full credit.

Station 5

____
Password

Station 3

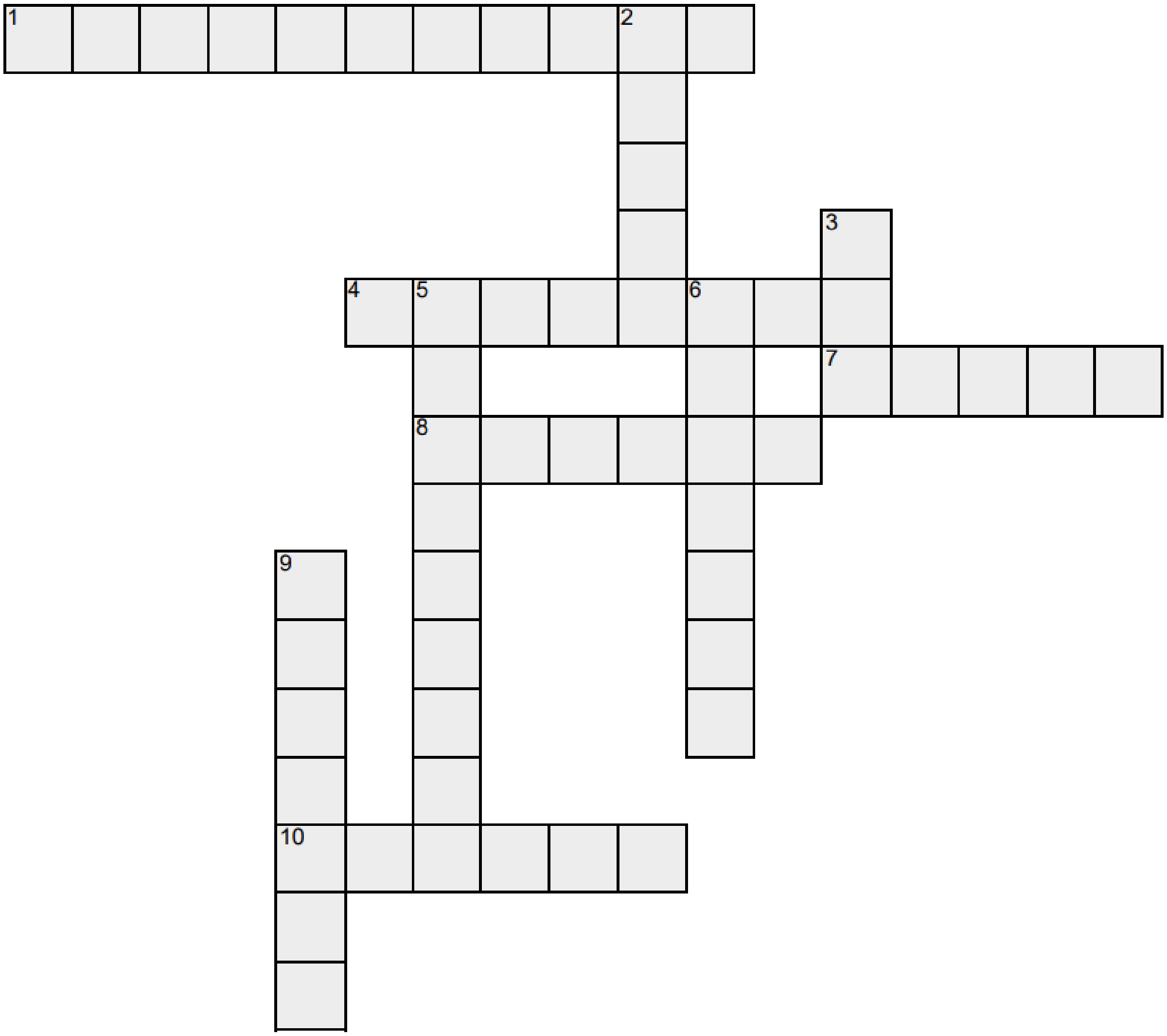
____
Password

Station 4

____
Password

End-of-Course Prep

Circles



Across	Down
1. A theorem that states that the sum of the square of the legs of a right triangle equals the square of its hypotenuse. 4. A line segment that passes through the center with endpoints on the circle. 7. A line segment with endpoints on the circle. 8. A line that intersects the curve at exactly two points. 10. A line segment that connects the center of a circle and a point on the circle.	2. A figure that is composed of two nonlinear rays with a common endpoint. 3. A part of the circumference of a circle with endpoints. 5. An angle with a vertex on the center of a circle. 6. A line that intersects the curve at exactly one point. 9. An angle with a vertex at the center of a circle.

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fun and exciting math for all

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Unit 7

Trigonometric Functions *Extend the domain of trigonometric functions using the unit circle.*	
NC.M3.F-TF.2 **NC.M3.F-TF.2a** **NC.M3.F-TF.2b**	Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions. a. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its y coordinate. b. Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its x coordinate.

Interpreting Functions *Understand the concept of a function and use function notation.*	
NC.M3.F-IF.1	Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.

Interpreting Functions *Analyze functions using different representations.*	
NC.M3.F-IF.7	Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Trigonometric Functions *Extend the domain of trigonometric functions using the unit circle.*	
NC.M3.F-TF.1	Understand radian measure of an angle as: • The ratio of the length of an arc on a circle subtended by the angle to its radius. • A dimensionless measure of length defined by the quotient of arc length and radius that is a real number. • The domain for trigonometric functions.

Trigonometric Functions *Model periodic phenomena with trigonometric functions.*	
NC.M3.F-TF.5	Use technology to investigate the parameters, a, b, and h of a sine function, $f(x)=a\cdot sin(b\cdot x)+h$, to represent periodic phenomena and interpret key features in terms of a context.

7.1 The Radian and Degree Measures

Warm-Up

Find the circumference of the circles.

Radius = 5 in	Diameter = 12 cm	Radius = 1 unit

Main Topic The Radian Measure

String Activity

Directions:

- Draw a circle using a string.
- Use the string to measure the circumference of the circle.
  Example responses would be 5.1 strings, 4.6 strings, 7.5 strings, etc.

Radian Measure

The **radian measure** of an angle is the length of the arc subtended by that angle divided by the radius of the circle. 1 radian = π.

Convert radian measure to degree measure.

Radian	Degree
2π	

Radian	Degree
π	

Radian	Degree
$\frac{\pi}{2}$	

Radian	Degree
$\frac{3\pi}{4}$	

Convert to angle measure.

3π	$-\pi$	$-\frac{\pi}{2}$
$-\frac{3\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{4}$
$\frac{\pi}{2}$	$\frac{7\pi}{3}$	$-\frac{5\pi}{6}$

Need Help?

Radian-Degree Matching

Match column A with equivalent measures in Column B by connecting the shapes.

Column A	Column B
$\frac{5\pi}{2}$	$210°$
-4π	$-240°$
$\frac{7\pi}{6}$	$-360°$
$-\frac{4\pi}{3}$	$450°$
-2π	$-720°$

Degree Measure

A unit used to determine the measurement of an angle around a fixed point is called a **degree measure**. 1 complete rotation = 360°

Convert degree measure to radian measure.

Radian	Degree
	$30°$

Radian	Degree
	$60°$

Radian	Degree
	$-110°$

Radian	Degree
	$-270°$

Convert to radian measure.

120^0	-45^0	-125^0
300^0	-180^0	-225^0
-400^0	720^0	-495^0

Need Help?

Degree-Radian Matching

Match column A with a correct answer in Column B by connecting the boxes.

Column A	Column B
$330°$	$\frac{5}{4}\pi$
$-405°$	$-\frac{2}{3}\pi$
$225°$	$-\frac{9}{4}\pi$
$270°$	$\frac{3}{2}\pi$
$-120°$	$\frac{11}{6}\pi$

End-of-Course Prep

7.2 Trigonometric Ratios

Warm-Up

Solve for x.

$\frac{x}{5} = \frac{8}{10}$ $\frac{6}{x} = \frac{18}{15}$ $\frac{24}{48} = \frac{x}{6}$

Main Topic Trigonometric Ratios

TASK 1

- Construct a right triangle.
- Label the legs with 3 units and 4 units.
- Calculate the length of the hypotenuse.
- Label the base angle with theta, θ.
- Label the opposite side of θ with O.
- Label the adjacent side of θ with A.
- Label the hypotenuse with H.

Trigonometric Ratios

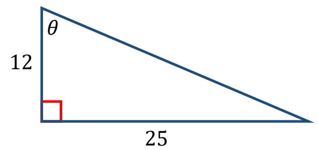
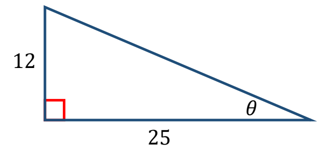
Sine	Cosine	Tangent
$\sin\theta =$ ______	$\cos\theta =$ ______	$\tan\theta =$ ______

Using the triangle above, find the trigonometric ratios.

$\sin\theta =$	$\cos\theta =$	$\tan\theta =$

Need Help?

Find the trigonometric ratios.

TASK 2

- Draw a right triangle.
- Label the base angle of the triangle with theta, θ.
- If $\sin\theta = \frac{12}{17}$, find the length of all the sides.

Find the lengths of all the sides of a right triangle.

$\sin\theta = \dfrac{13}{19}$	$\cos\theta = \dfrac{26}{31}$	$\tan\theta = \dfrac{75}{43}$

Need Help?

Angle of Elevation

The captain of a boat spotted an airplane that was 430 ft above the water. The angle of elevation between the boat and an airplane is 32°. What is the distance between the boat and the airplane?	
Illustration	Work

Angle of Depression

The angle of depression between the top of a tower and a car on the ground is 25°. The car is 350 ft away from the tower. How tall is the tower?	
Illustration	Work

Need Help?

True or False

Mark the box beside TRUE if the statement is correct, otherwise mark the box beside FALSE.
Show the complete work to justify your response.

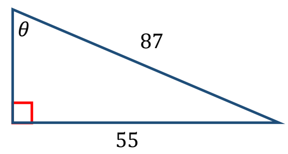
<table>
<tr><td rowspan="2">☐ TRUE
OR
☐ FALSE</td><td colspan="2">A 34-ft ladder is leaning against a wall. The angle made by the ladder with the ground is 27°. The base of the ladder is approximately 30 ft way from the wall.</td></tr>
<tr><td>Illustration</td><td>Work</td></tr>
</table>

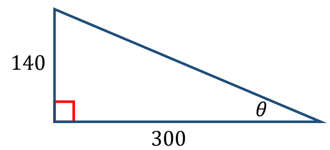
<table>
<tr><td rowspan="2">☐ TRUE
OR
☐ FALSE</td><td colspan="2">A tree casts a 53-ft shadow. The angle of depression from the tip of the tree with the tip the of shadow is 34°. The height of the tree is about 36 ft.</td></tr>
<tr><td>Illustration</td><td>Work</td></tr>
</table>

<table>
<tr><td rowspan="2">☐ TRUE
OR
☐ FALSE</td><td colspan="2">The 130-ft string of the kite is attached to the ground. The kite is 67 ft above the ground. The angle made by the string with the ground is approximately 40°.</td></tr>
<tr><td>Illustration</td><td>Work</td></tr>
</table>

Trigonometric Ratios	Cosecant	Secant	Cotangent
	$\csc\theta =$ ______	$\sec\theta =$ ______	$\cot\theta =$ ______

Find all six trigonometric ratios.





Find the lengths of all the sides of a right triangle.

$\sec\theta = \dfrac{13}{19}$	$\cot\theta = \dfrac{26}{31}$	$\csc\theta = \dfrac{75}{43}$

Need Help?

Fill in the Blank

Read the problems below and show the necessary work on a separate sheet to justify your responses.

The opposite side of a right triangle is 18 in while its hypotenuse is 32 in. The length of the adjacent side is __________ in.

The ratio of the lengths of a right triangle for $\sec\theta = \underline{\qquad}$ while $\tan\theta = \frac{23}{18}$.

The length of hypotenuse is 100 cm and the angle formed by the adjacent side and the hypotenuse is 35°. The length of the opposite side is __________ cm.

A 44-ft ladder is leaning against a wall. The angle made by the ladder with the ground is 29°. The base of the ladder is approximately _________ft way from the wall.

A tree casts a 67-ft shadow. The angle of depression from the tip of the tree with the tip the of shadow is 28°. The height of the tree is about _________ ft.

If $\sin\theta = \frac{13}{15}$ then $\cot\theta = \underline{\qquad}$.

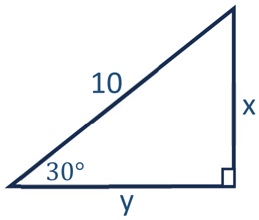
The 130-ft string of the kite is attached to the ground. The kite is _____ ft above the ground. The angle made by the string with the ground is approximately 40°.

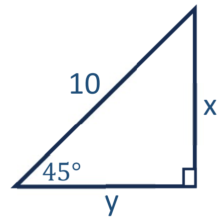
End-of-Course Prep

7.3 The Unit Circle

Warm-Up

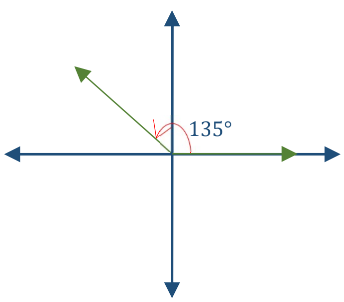
Solve for x and y.

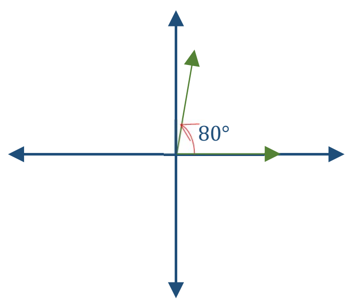


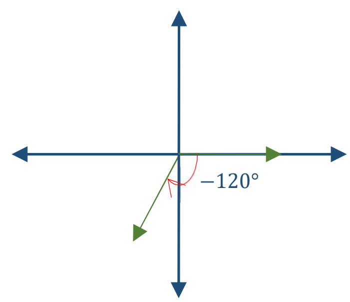


Main Topic The Unit Circle

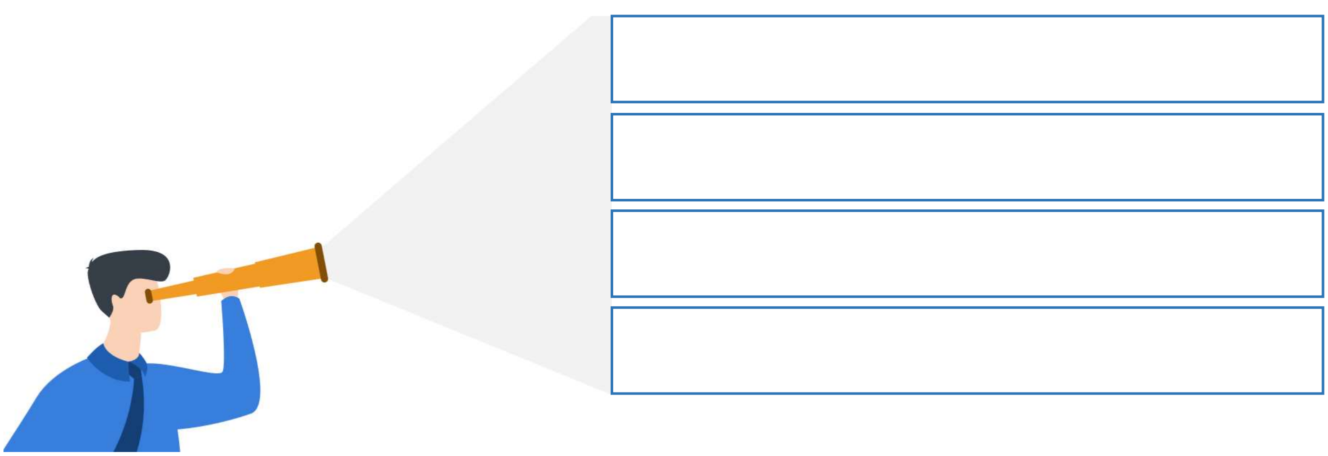
Pay attention to the sketched **angles in standard position**.







Write your observations.

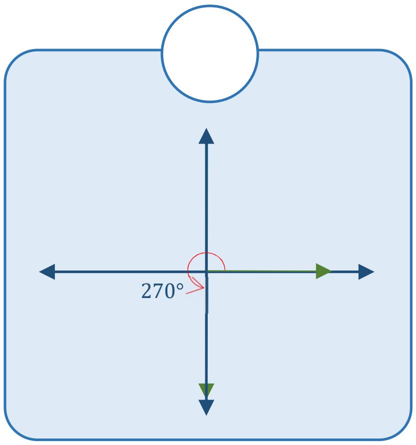


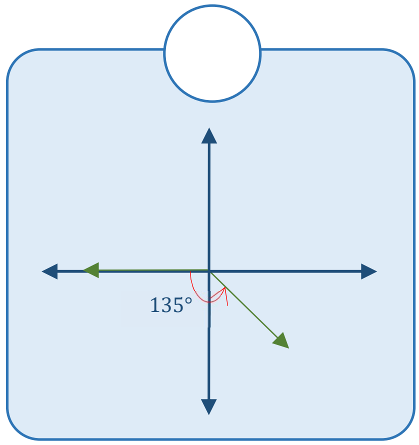
Need Help?

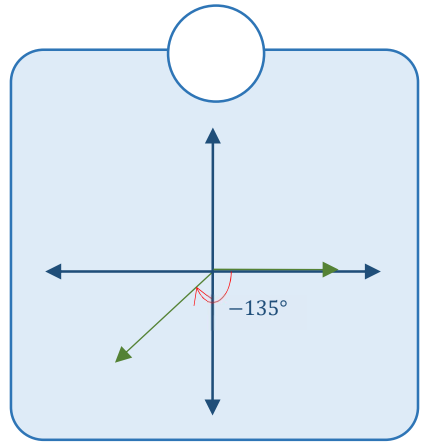
Standard Position

An angle in **standard position** is when its vertex is at the origin and one of its rays is on the positive x-axis.

Write **“Yes”** in the circle if the angle is in standard position. Otherwise, write **“No”**.





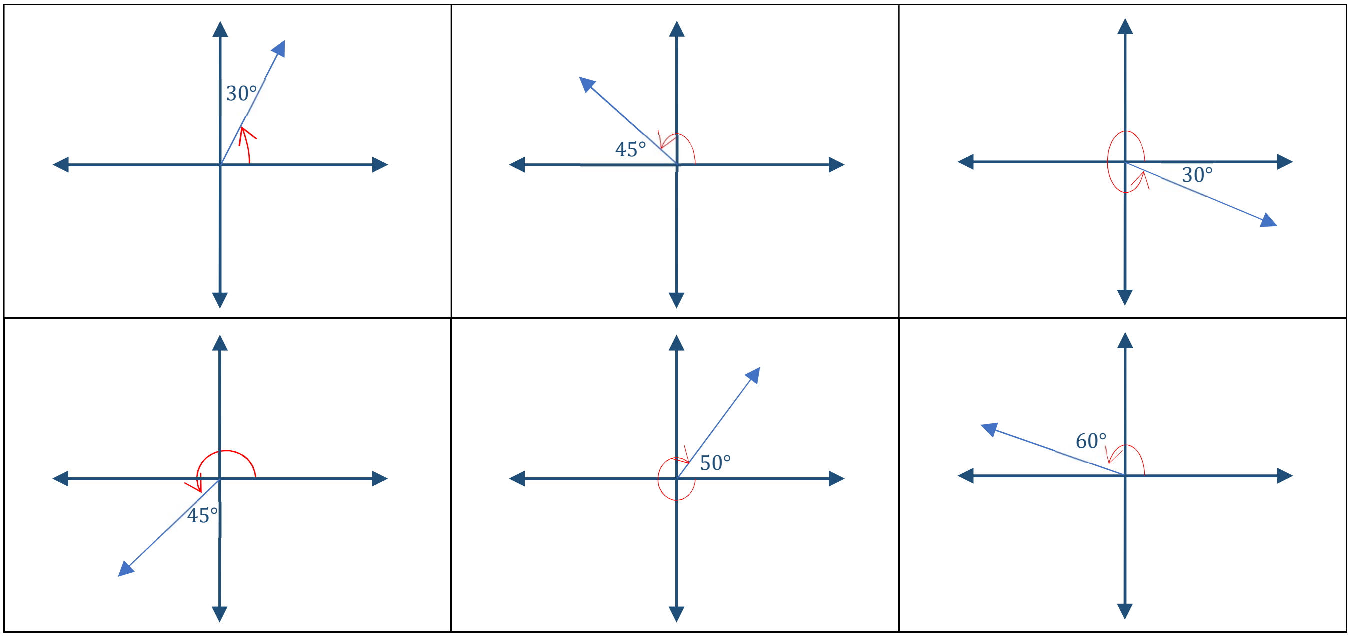


Sketch the given angles.

300°	-30°	125°
405°	-135°	225°

Need Help?

Find the measure of the sketched angles.



Unit Circle Coordinates $(30°)$

- Draw a circle with 1 unit radius.
- Sketch a 30° angle in standard position.
- Label the intersection between the rotating side of the angle and the circle with P.
- From P, draw a line perpendicular to the x-axis.
- Pay attention to the features of the constructed right triangle.
- Calculate the lengths of the right triangle.

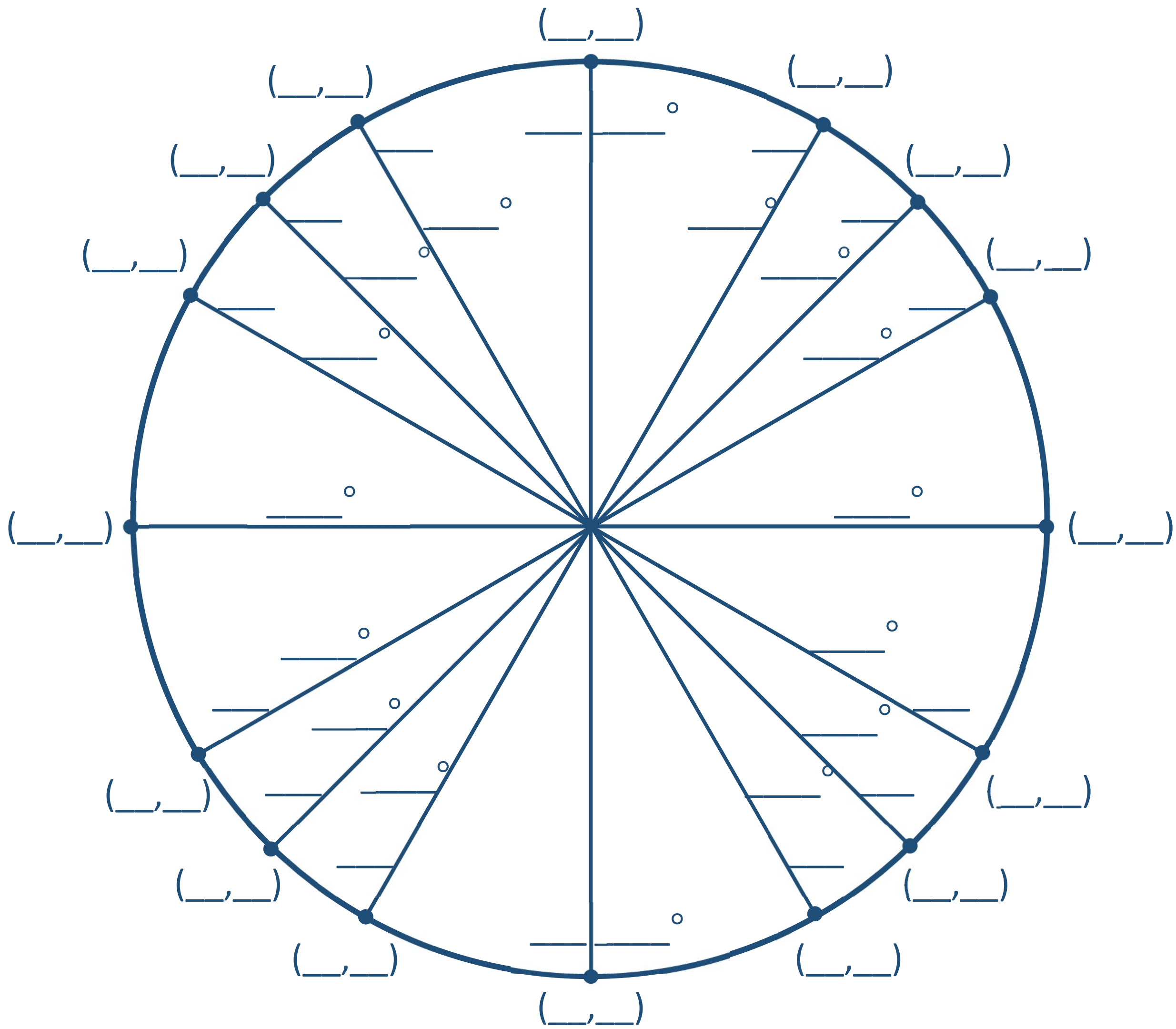
Unit Circle Coordinates ($60°$)

- Draw a circle with 1 unit radius.
- Sketch a $60°$ angle in standard position.
- Label the intersection between the rotating side of the angle and the circle with P.
- From P, draw a line perpendicular to the x-axis.
- Pay attention to the features of the constructed right triangle.
- Calculate the lengths of the right triangle.

Unit Circle Coordinates ($45°$)

- Draw a circle with 1 unit radius.
- Sketch a $45°$ angle in standard position.
- Label the intersection between the rotating side of the angle and the circle with P.
- From P, draw a line perpendicular to the x-axis.
- Pay attention to the features of the constructed right triangle.
- Calculate the lengths of the right triangle.

Coordinates of the Unit Circle



Find the value.

$\sin 60°$	$\cos -\frac{\pi}{4}$	$\tan 225°$	$\sec -30°$
$\sec 330°$	$\cot -360°$	$\sin \frac{5\pi}{6}$	$\cos -270°$

Need Help?

Unit Circle Stickies Gallery

Copy each question on a sticky note, show complete work, and circle the final answer.
Post your work next to the questions across the classroom.

$\sec 135°$

$\cot -270°$

$\cos \frac{11}{6}\pi$

$\sin -\frac{4}{3}\pi$

$\sin 360°$

$\csc \frac{3}{4}\pi$

End-of-Course Prep

7.4 The Sine Function

Warm-Up

Write the equation of the transformed equation, $g(x)$.

$f(x) = x^2 - 5$

Rule: Translate 3 units left and 2 units up

$f(x) = x^2 - 5$

Rule: Reflect over the x-axis.

Main Topic The Sine Function

Spaghetti Activity

Materials

Flat spaghetti noodles　Cardboard　Glue　Ruler　Marker　Calculator

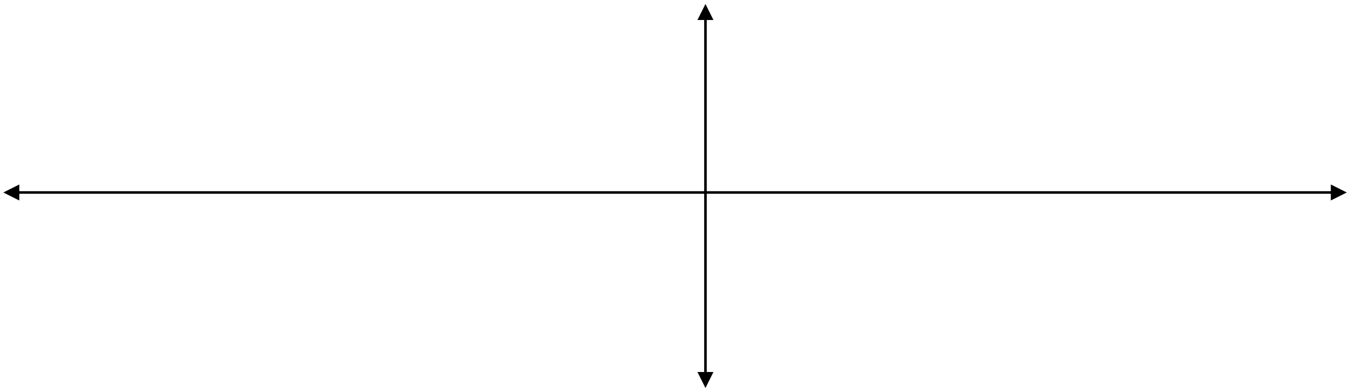
Directions

- Complete the table below.
    - Reset the calculator, for TI calculator, press 2^{nd}, +, 7, 1, 2.
    - Press the MODE button, select DEGREE, then press the CLEAR button.
    - Press the SIN button then enter the angle, no need to add the degree symbol.

Angle	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
Sine																	

- Draw a Cartesian plane.
- Label the x-axis from $0°$ to $360°$, space out the labels accordingly.
- Label the y-axis from -1 to 1, leave a large gap between 0 and -1, and 0 and 1.
- Plot the pairs of coordinates from the table.
- Connect the plotted point with the x-axis vertically using the spaghetti noodle, one end of the spaghetti is at the point and the other end is on the x-axis.

$f(x) = \sin x$



Properties of Sine Graph

Domain:

Minimum:

Amplitude:

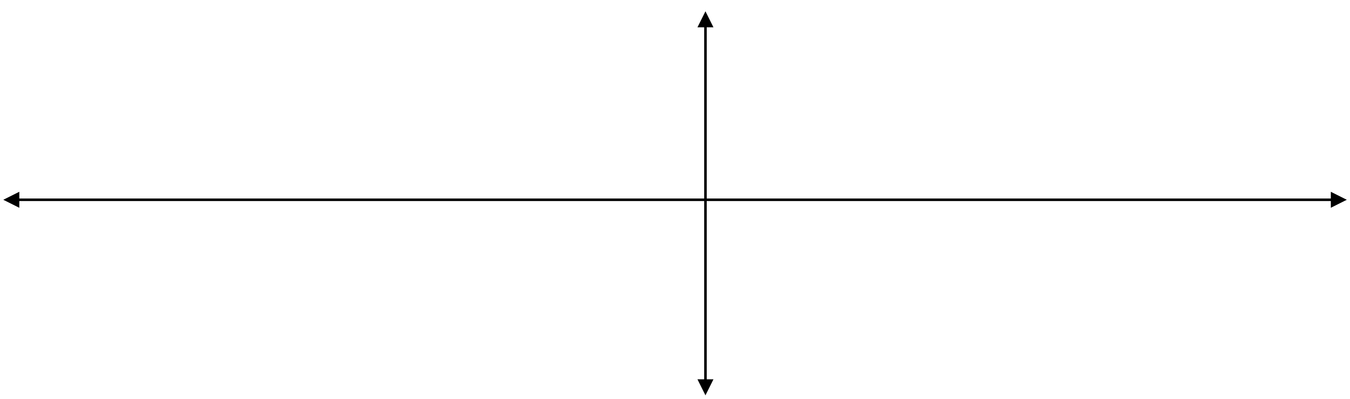
Period:

Range:

Maximum:

Zeros:

$f(x) = 3\sin x$



Properties of Sine Graph

Domain:

Minimum:

Amplitude:

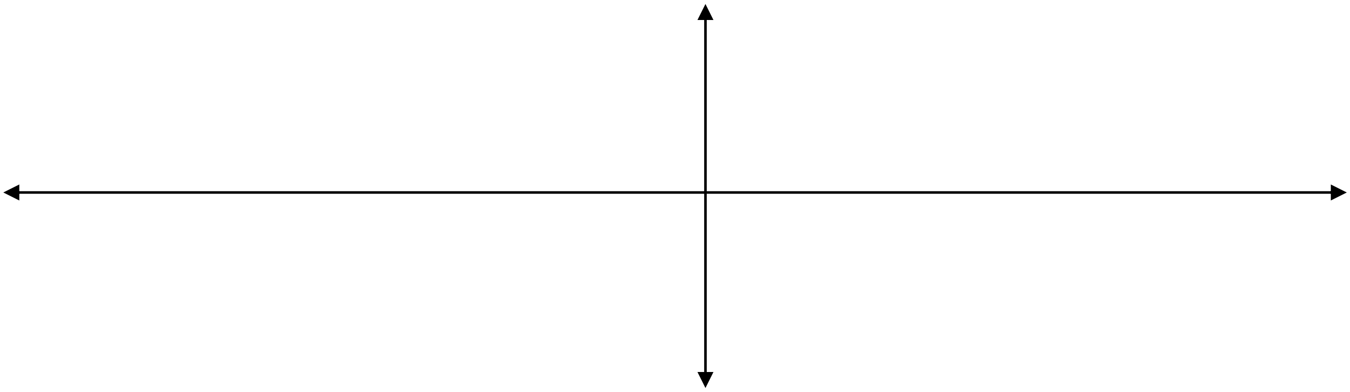
Period:

Range:

Maximum:

Zeros:

$f(x) = 3\sin(2x)$

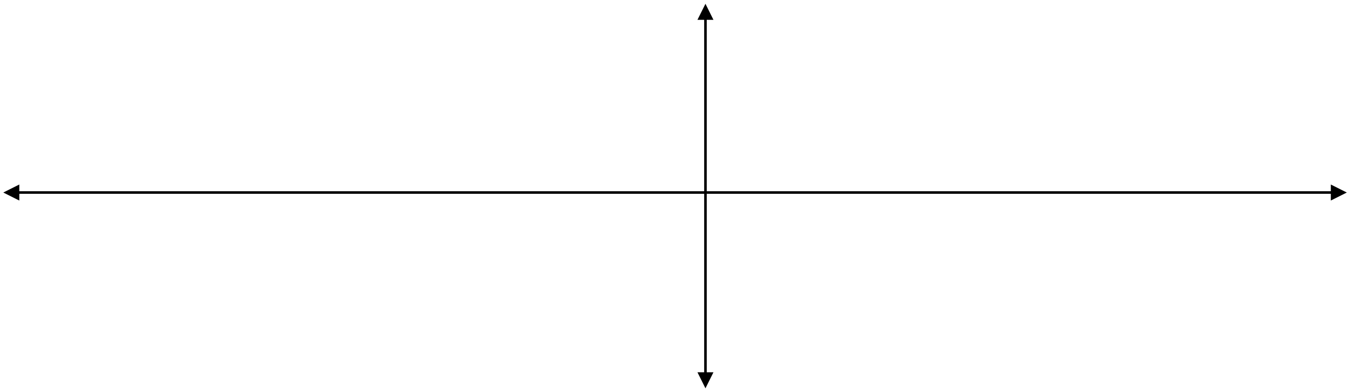


Properties of Sine Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = -3\sin(5x)$

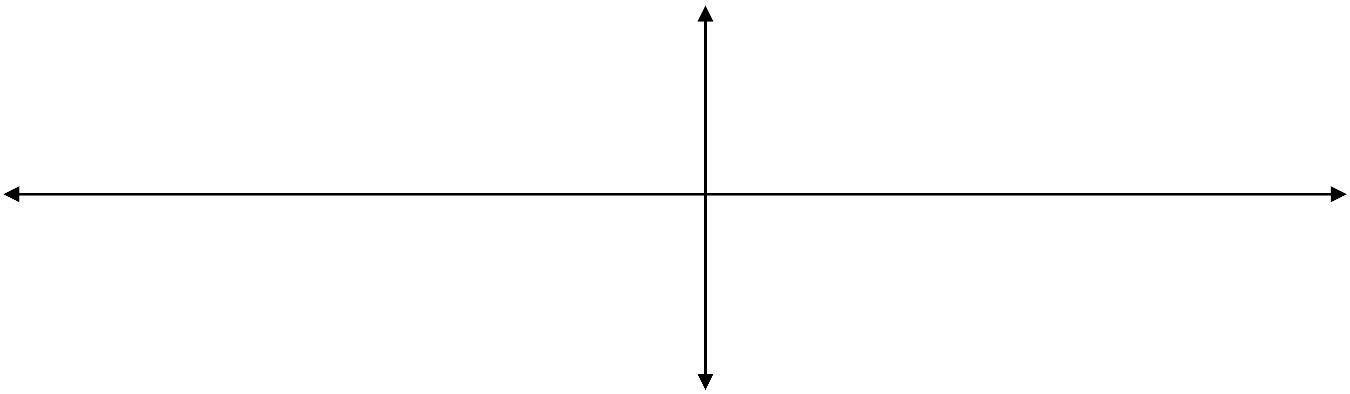


Properties of Sine Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = 2\sin(x - 30)$



Properties of Sine Graph

Domain:

Minimum:

Amplitude:

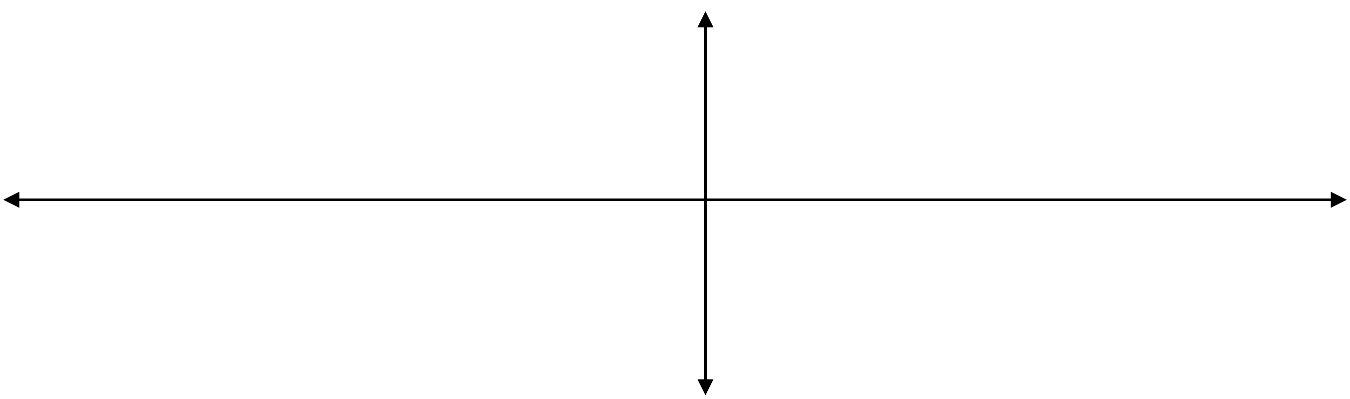
Period:

Range:

Maximum:

Zeros:

$f(x) = -3\sin(x + 60)$



Properties of Sine Graph

Domain:

Minimum:

Amplitude:

Period:

Range:

Maximum:

Zeros:

End-of-Course Prep

7.5 The Cosine Function

Warm-Up

Find the zeros of the functions.

$f(x) = x^2 + 3x - 10$

$g(x) = -x^2 + 3x - 10$

Main Topic The Cosine Function

Spaghetti Activity

Materials

Flat spaghetti noodles
Cardboard
Glue
Ruler
Marker
Calculator

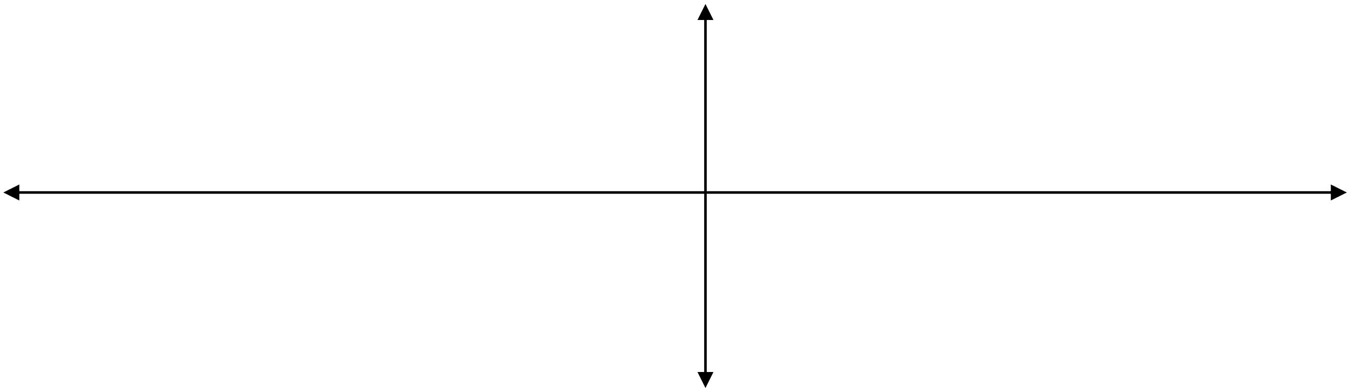
Directions

- Complete the table below.
    - Reset the calculator, for TI calculator press 2nd, +, 7, 1, 2.
    - Press the MODE button, select DEGREE, then press the CLEAR button.
    - Press the COS button then enter the angle, no need to add the degree symbol.

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Cosine																	

- Draw a Cartesian plane.
- Label the x-axis from 0° to 360°, space out the labels accordingly.
- Label the y-axis from -1 to 1, leave a large gap between 0 and -1, and 0 and 1.
- Plot the pairs of coordinates from the table.
- Connect the plotted point with the x-axis vertically using the spaghetti noodle, one end of the spaghetti is at the point and the other end is on the x-axis.

$f(x) = \cos x$

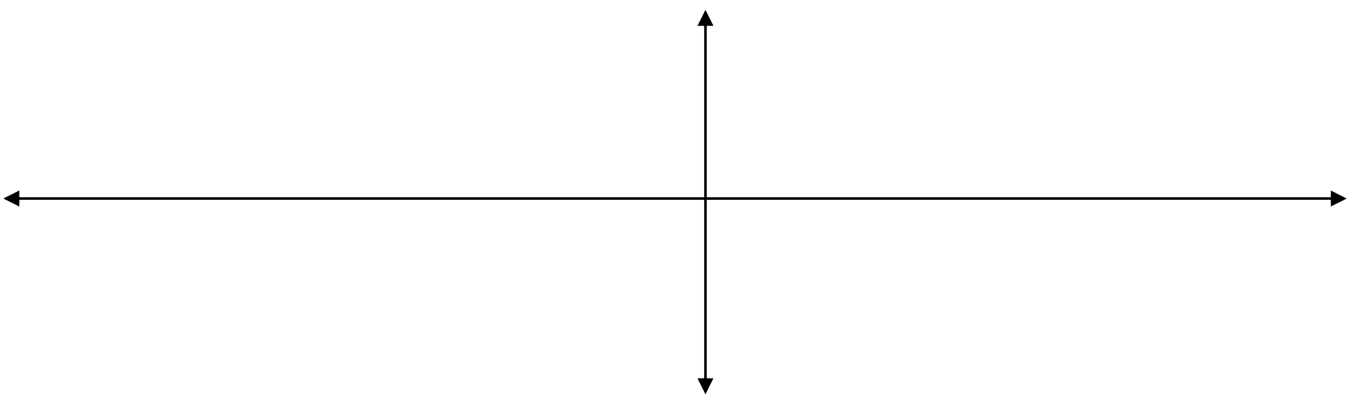


Properties of Cosine Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = 3\cos x$

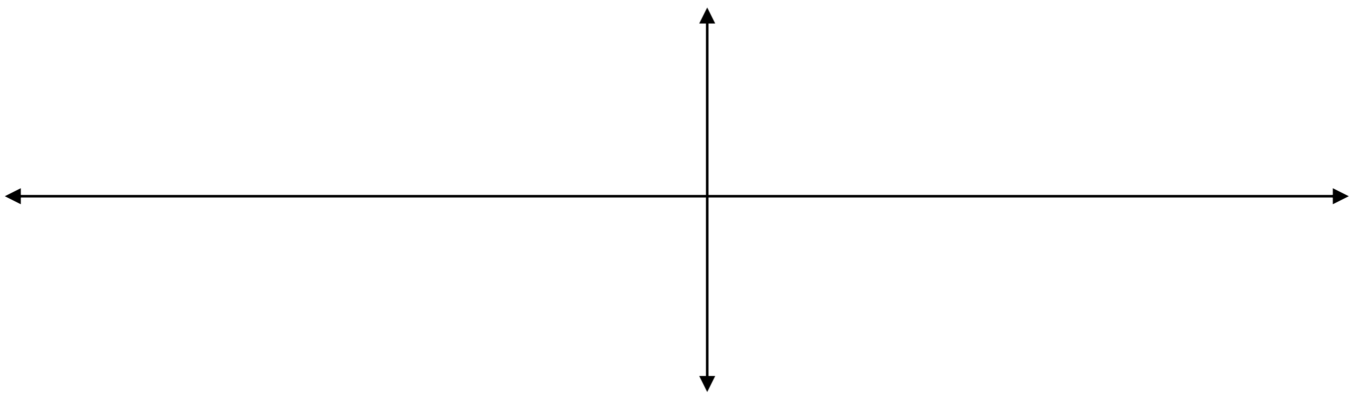


Properties of Cosine Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

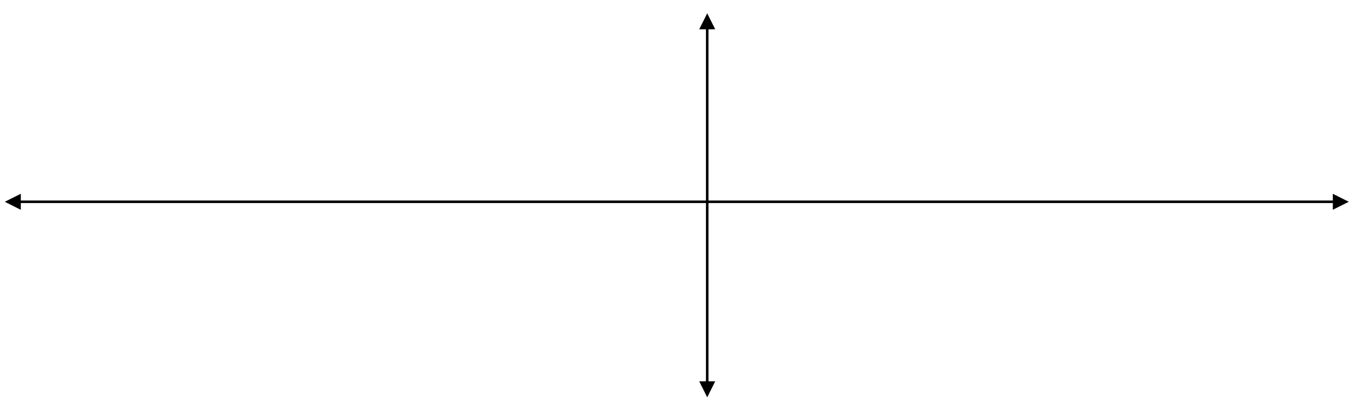
$f(x) = 3\cos(2x)$



Properties of Cosine Graph

Domain:	Minimum:	Amplitude:	Period:
Range:	Maximum:	Zeros:	

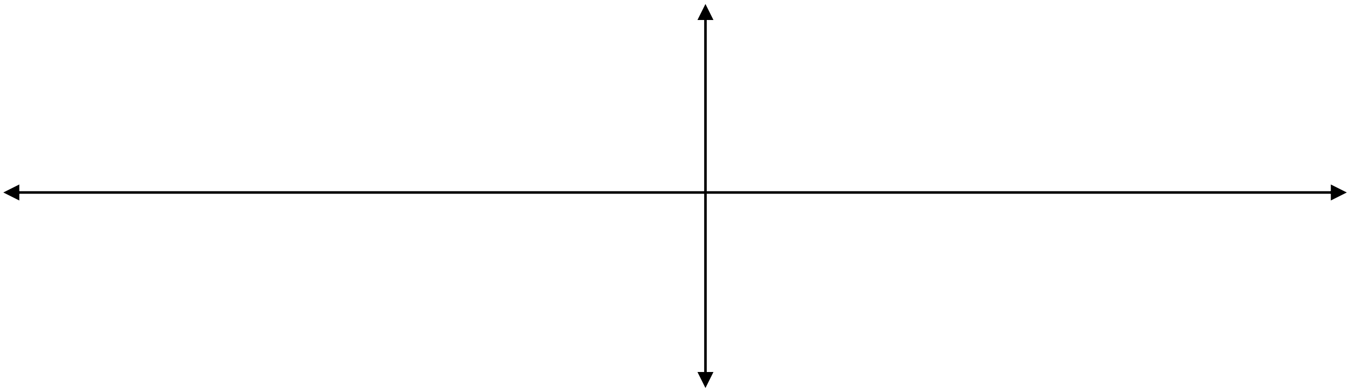
$f(x) = -3\cos(5x)$



Properties of Cosine Graph

Domain:	Minimum:	Amplitude:	Period:
Range:	Maximum:	Zeros:	

$f(x) = 2\cos(x - 30)$

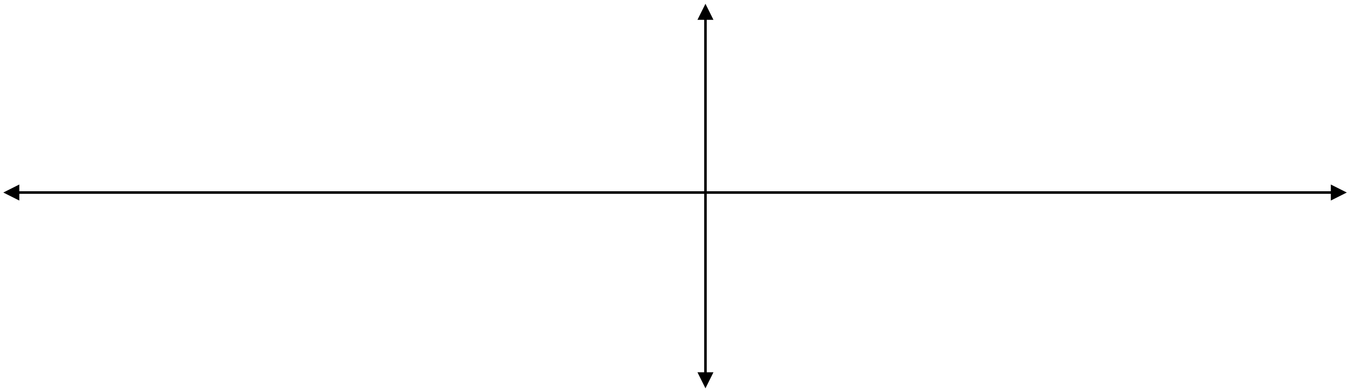


Properties of Cosine Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = -3\cos(x + 60)$



Properties of Cosine Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

End-of-Course Prep

7.6 The Tangent Function (Honors Level)

Warm-Up

Find the minimum and maximum of the functions.

$f(x) = x^2 - 5$

$f(x) = -x^2 - 5$

Main Topic The Tangent Function

Tangent Function $f(x) = \tan x$

Fill out the table.

Angle	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
Tangent																	

Properties of Tangent Graph

Domain:

Minimum:

Amplitude:

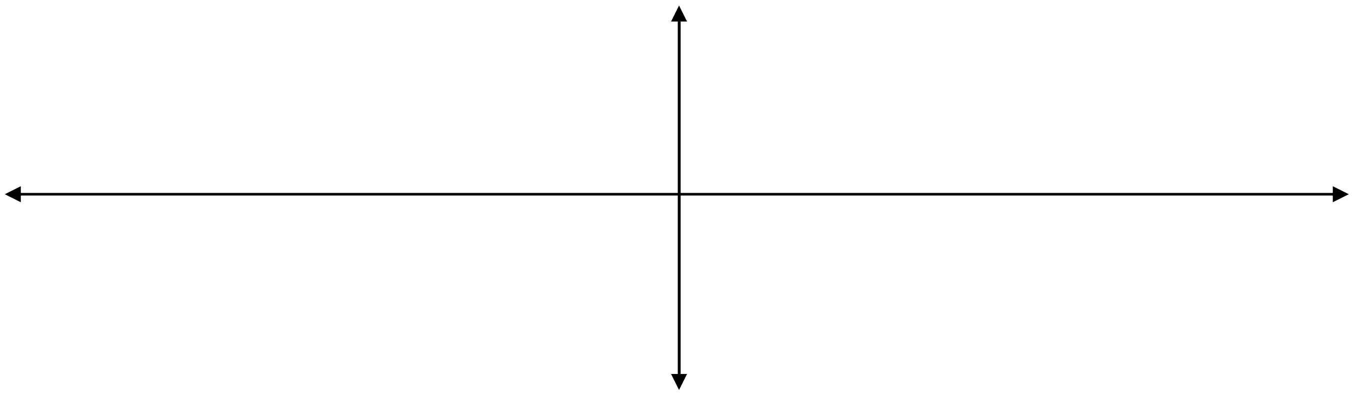
Period:

Range:

Maximum:

Zeros:

$f(x) = 3\tan x$



Properties of Tangent Graph

Domain:

Minimum:

Amplitude:

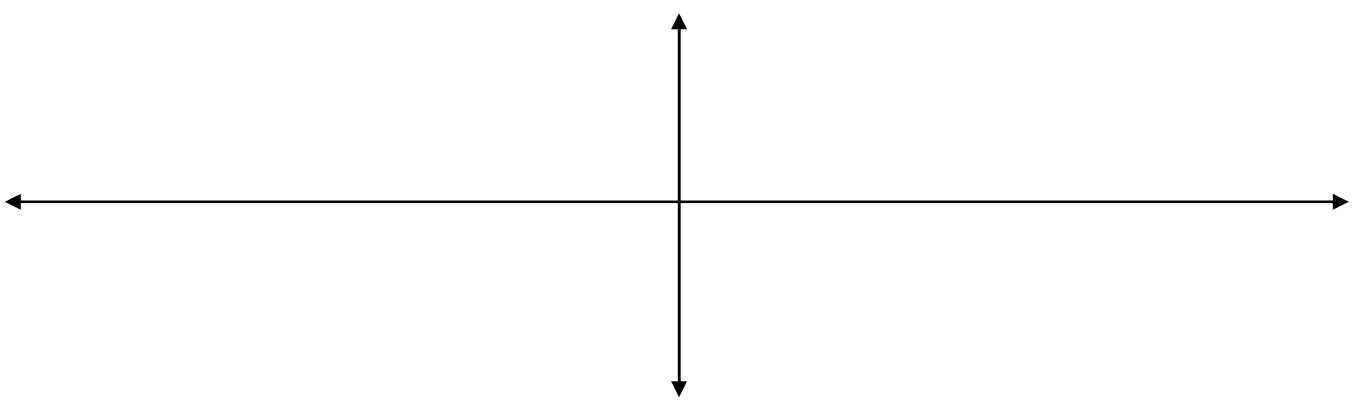
Period:

Range:

Maximum:

Zeros:

$f(x) = -5\tan x$



Properties of Tangent Graph

Domain:

Minimum:

Amplitude:

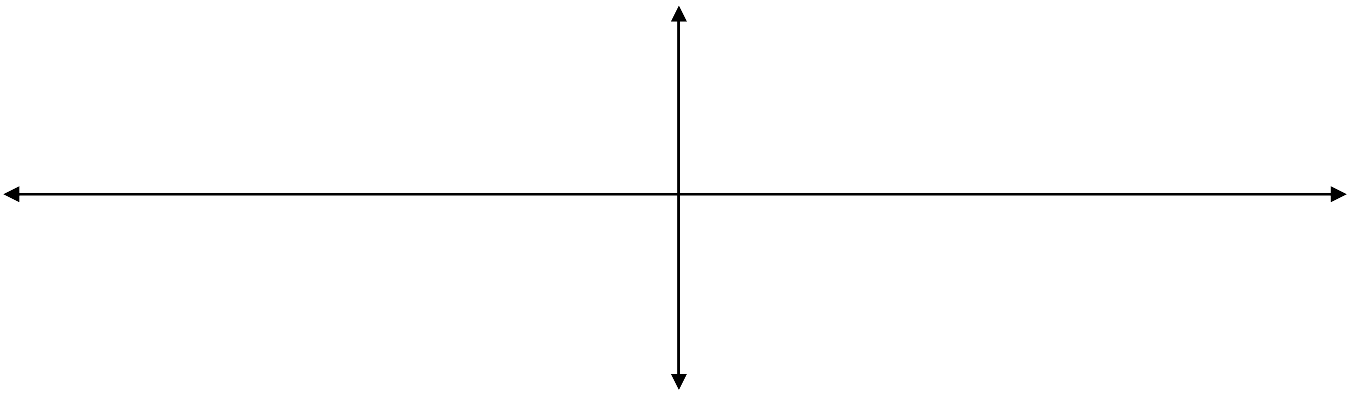
Period:

Range:

Maximum:

Zeros:

$f(x) = 6\tan(3x)$

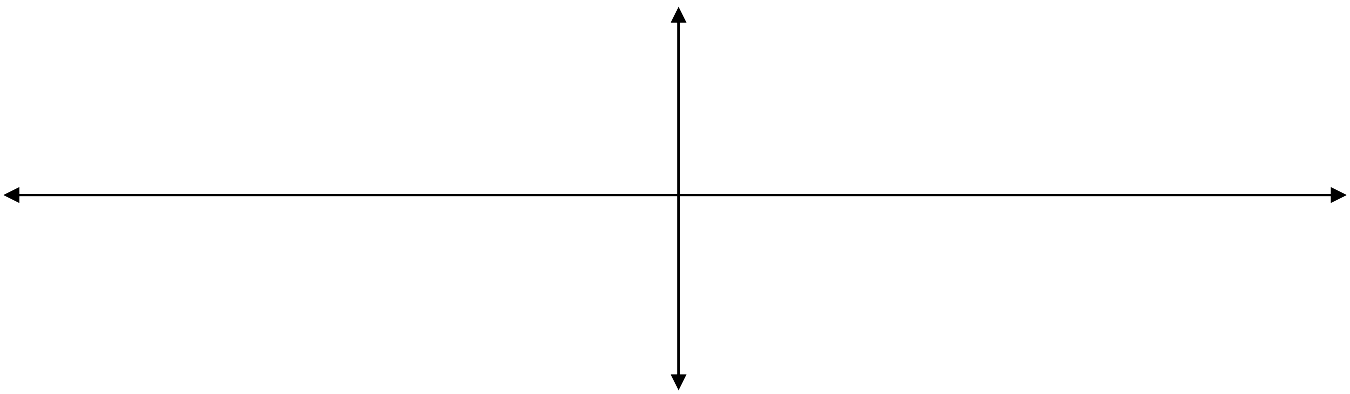


Properties of Tangent Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = -3\tan(5x)$

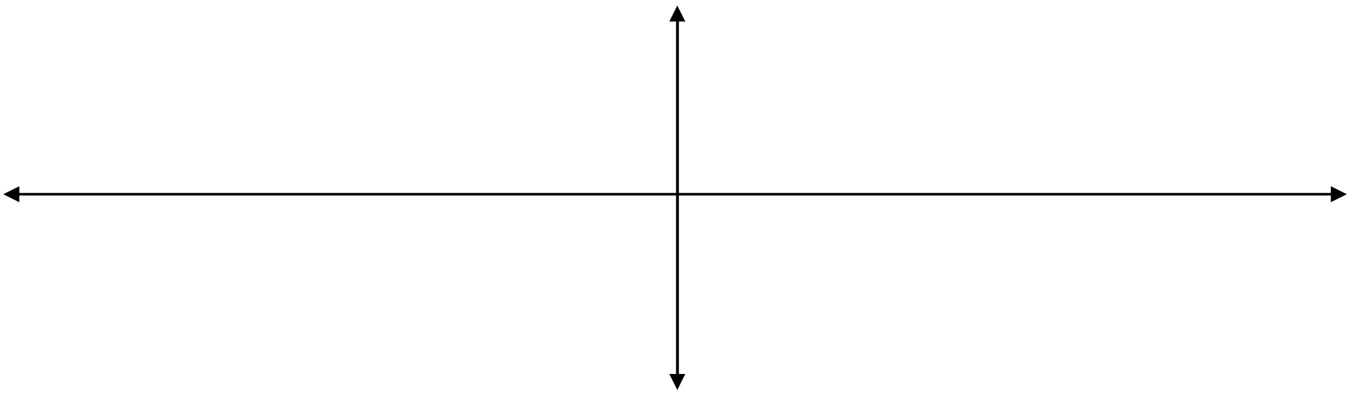


Properties of Tangent Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = 2\tan(x + 180)$

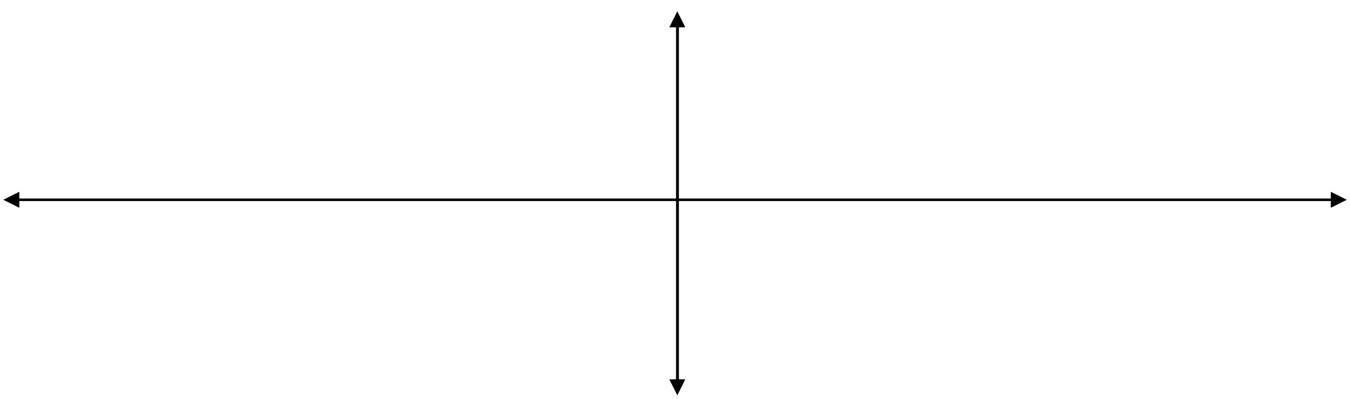


Properties of Tangent Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

$f(x) = -\tan(3x - 45)$



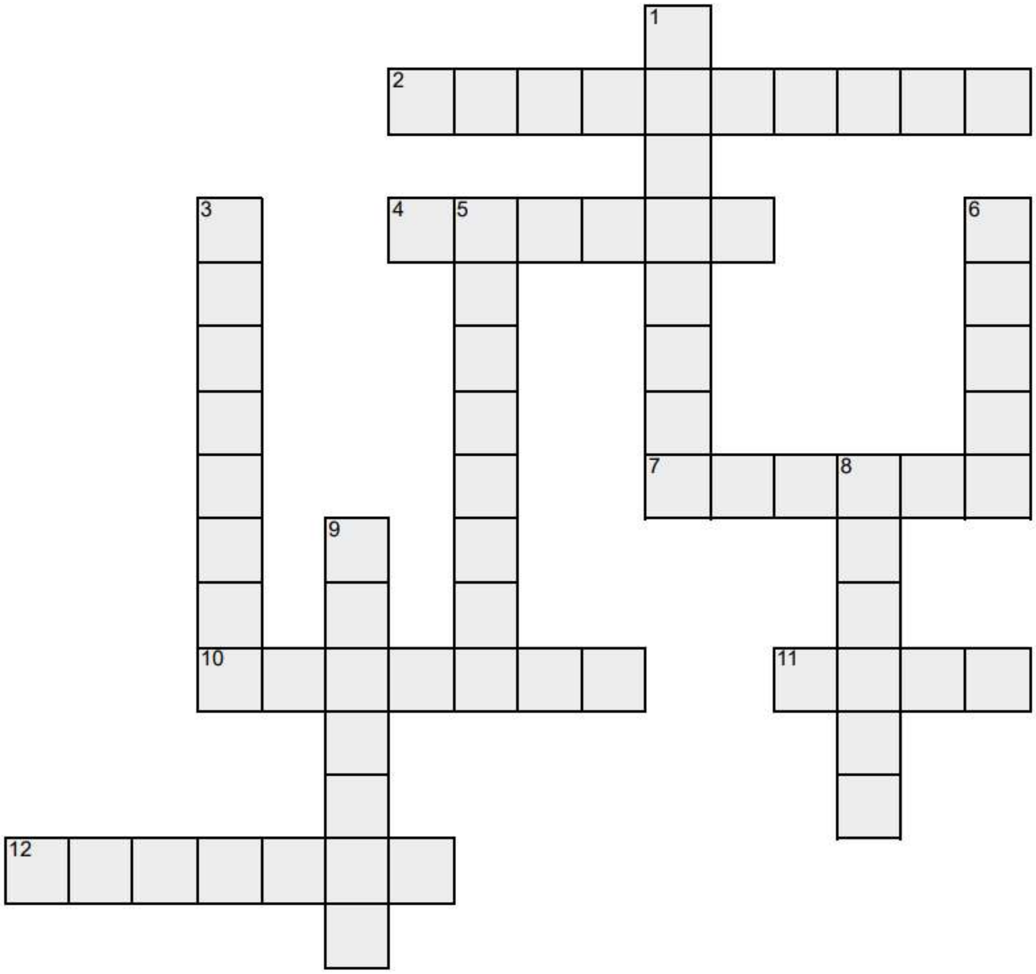
Properties of Tangent Graph

Domain: Minimum: Amplitude: Period:

Range: Maximum: Zeros:

End-of-Course Prep

Trigonometry



Across	Down
2. The longest side of the right triangle.	1. An angle in ____________ position is when its vertex is at the origin and one of its rays is on the positive x-axis.
4. A trigonometric ratio of the adjacent side and hypotenuse side.	3. The side of a right triangle next to the given angle.
7. The measurement of an angle around a fixed point.	5. The side of a right triangle located across the given angle.
10. A trigonometric ratio of the opposite side and adjacent side.	6. A geometric figure that is composed of two rays sharing the same endpoint.
11. A trigonometric ratio of the opposite side and hypotenuse side.	8. The length of the arc subtended by that angle divided by the radius of the circle.
12. The highest point of the graph.	9. The lowest point of the graph.

fun and engaging math for all

Unit 8 | Statistics

High School Mathematics 3

North Carolina Math 3 Standards

Unit 8

Making Inference and Justifying Conclusions *Understand and evaluate random processes underlying statistical experiments.*	
NC.M3.S-IC1	Understand the process of making inferences about a population based on a random sample from that population.

Making Inference and Justifying Conclusions *Make inferences and justify conclusions from sample surveys, experiments, and observational studies.*	
NC.M3.S-IC.3	Recognize the purposes of and differences between sample surveys, experiments, and observational studies and understand how randomization should be used in each.
NC.M3.S-IC.4	Use simulation to understand how samples can be used to estimate a population mean or proportion and how to determine a margin of error for the estimate.
NC.M3.S-IC.5	Use simulation to determine whether observed differences between samples from two distinct populations indicate that the two populations are actually different in terms of a parameter of interest.
NC.M3.S-IC.6	Evaluate articles and websites that report data by identifying the source of the data, the design of the study, and the way the data are graphically displayed.

8.1 Types of Studies

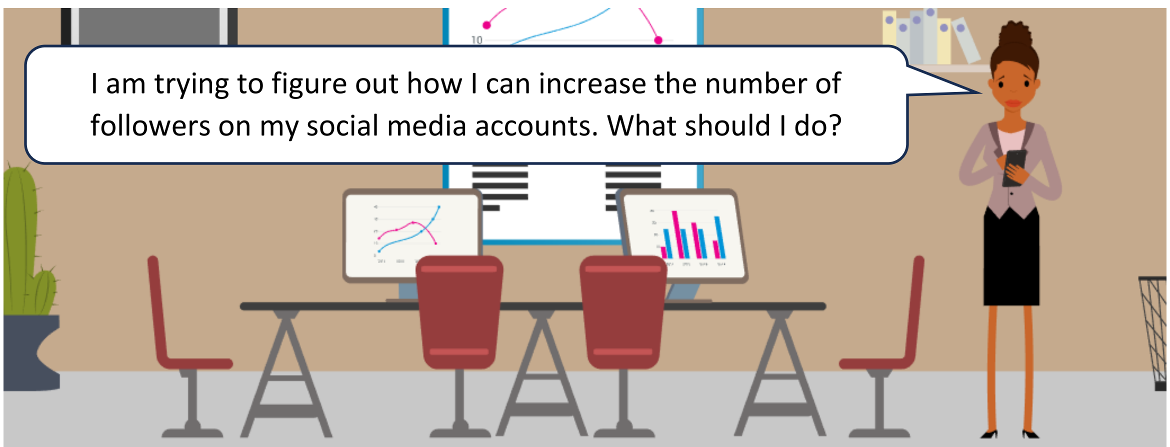
Warm-Up

Find the average of the following.

89, 94, 96, 95, 82, 68

Main Topic Data Collection

TASK 1



Pick the most effective way to increase the number of followers.

- ☐ Actively engage your audience
- ☐ Update your posts
- ☐ Collaborate with other influencers

Explain your answer.

____

____

Types of Studies		
Survey	Observational	Experimental

Which type of study is the *best* option for the questions below.

A small group of teachers wants to find out how many hours teachers work outside of regular school hours?	A company that manufactures pool pumps select 2 pumps at random. Then the pumps are tested to see how long they last.	In a group of adults, you want to see which works best on headaches…Motrin or Tylenol.
You want to start your own soap making business and you need help with a logo. You ask 20 people which logo they like the best.	You want to start your own soap making business and you need help with a logo. You invite 20 people to your conference room to watch their reactions to the different logos.	You are trying out three different stain removers to see which one gets red drink off of a white shirt the best.
A student research group wants to determine whether high school students that participated in a summer reading program tested higher than their peers on the ELA exam.	A clothing store conducts an online study that inquires about their shopping experience.	A college research group randomly selects 15 students to see which fruits spike their sugar levels.

Main Topic Populations and Samples

Definition

Statistics

____

____

Sample

____

____

Population

____

____

Sample or Population

Connect the outer shapes with the inner shapes.

The age of the passengers sitting in the aisle seats on the plane.	Sample	Scores of all students that took the math EOC.
The number of points scored by all basketball players.	Population	Salary of every 10^{th} employee that enters the building in the morning.

The age of all graduates in the 2025 graduating class at NC State.

Sample

Scores of only the sophomores that took the math EOC.

The number of points scored by all football players.

Population

The number of female employees that work the night shift at Taco Bell.

The age of all children hat went to the state fair.

Sample

The mayor wanted to see if very person in the town of Hope Mills thought she was a good mayor.

The voters of ages 25-40 that voted in the presidential election.

Population

The number of all students that attend Cape Fear High School.

Create your own shapes (like the ones above) with different scenarios and explain why you think a population or sample best fits that situation.

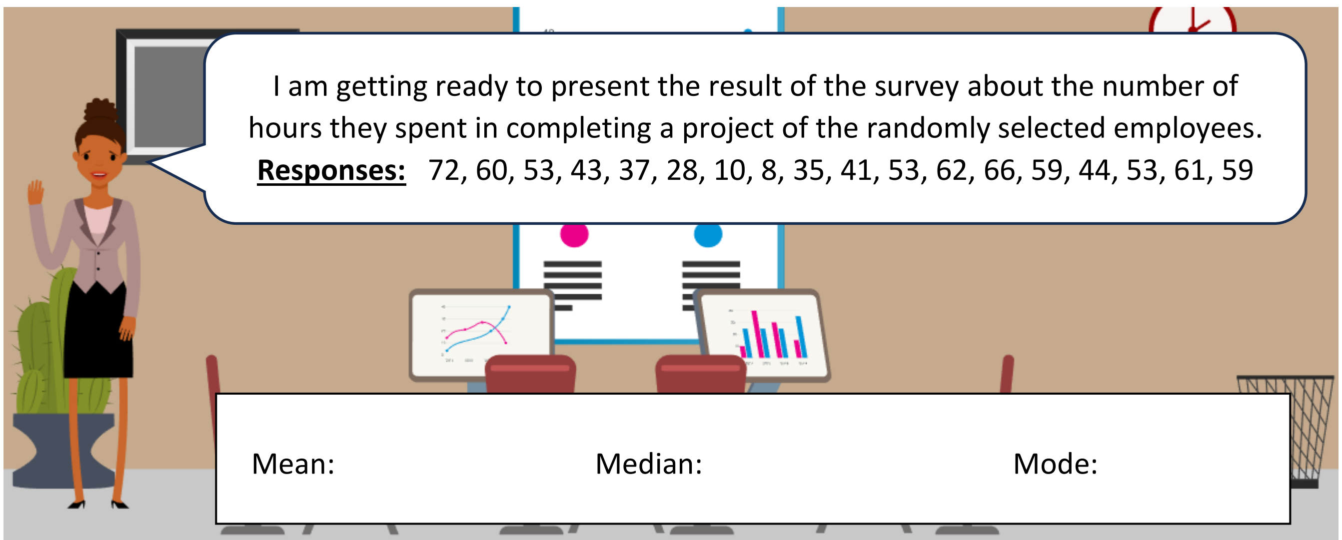
Sample

Population

Main Topic Measures of Central Tendencies

Fill in the blank

_______________ is the sum of all the data divided by the number of data. A more commonly used term is ________________. _____________ is the middle value of the dataset arranged in order from lowest to highest or highest to lowest. If there are two values in the middle, find the _____________. _____________ is the most frequent value in the dataset. The lowest value in the dataset is called ______________ while the highest is called ________________.



TASK 2

Dataset: 70, 60, 52, 43, 27, 28, 11, 6, 34, 41, 52, 62, 64, 64, 64, 52, 60, 58

Mean: Median: Mode:

Find the measures of central tendency for each data set.

12, 15, 18, 21, 25, 27, 30, 33, 36, 39	45, 52, 60, 60, 65, 70, 72, 75, 80, 80, 85, 90	5, 8, 9, 12, 12, 14, 16, 18, 20, 20, 21, 23, 25
100, 95, 85, 100, 92, 88, 97, 90, 100, 94, 98, 93	32, 36, 38, 40, 40, 42, 44, 44, 45, 46, 48, 50, 52, 54, 54, 56	2, 3, 3, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 10, 10, 11, 11, 12
75, 80, 82, 85, 87, 87, 88, 89, 90, 91, 93, 95, 95, 96, 98, 100	120, 130, 130, 135, 140, 140, 145, 145, 150, 150, 155, 160, 165, 170, 175	5, 10, 10, 15, 20, 20, 25, 25, 30, 35, 35, 40, 45, 45, 50, 55, 60, 65, 70

End-of-Course Prep

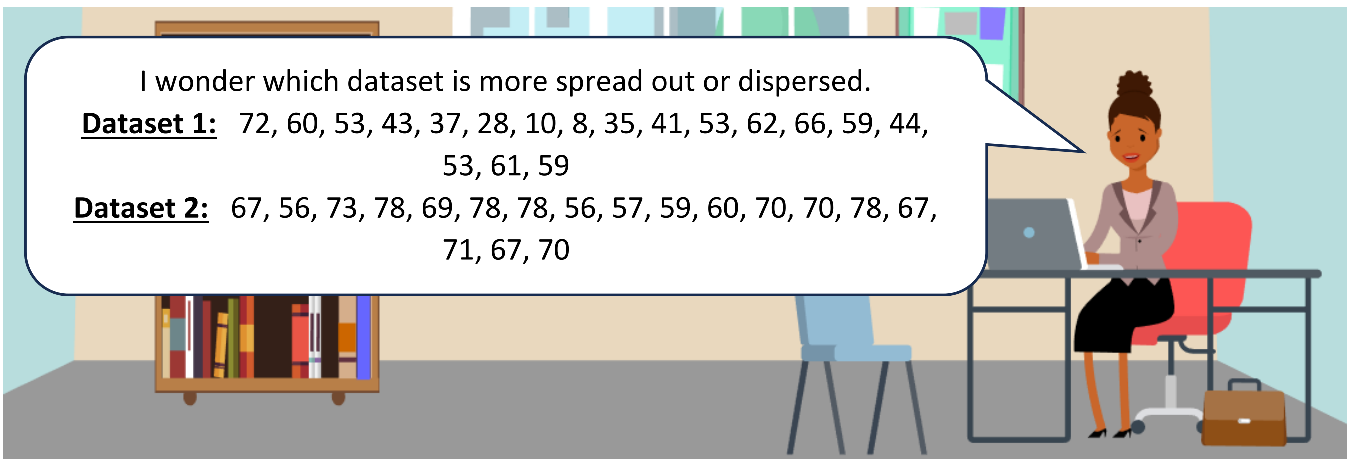
8.2 Measures of Variability

Warm-Up

Find the average distance of the numbers below.

-5, 2, 6, 12, 18, -1

Main Topic Range



Range	The difference between the maximum and minimum values.

TASK 1

- What is the range of the dataset 1?
- What is the range of the dataset 2?
- Which dataset is more spread out or dispersed?
- What are the advantages and disadvantages of calculating the range in determining the spread of the data?

Main Topic Standard Deviation

Standard Deviation

Standard Deviation measures the average distance of the dataset about the mean.

Calculator Steps for finding the Standard Deviation

- Press STAT, EDIT, Enter data list into L_1.
- Press STAT, CALC,#1(1-Var Stats), ENTER
- Sx represents the Sample Standard Deviation
  σx is the Population Standard Deviation

$$S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}}$$

Use a calculator to calculate the Standard Deviation of a dataset.

Dataset 1: 72, 60, 53, 43, 37, 28, 10, 8, 35, 41, 53, 62, 66, 59, 44, 53, 61, 59

Dataset 2: 67, 56, 73, 78, 69, 78, 78, 56, 57, 59, 60, 70, 70, 78, 67, 71, 67, 70

Find the standard deviation for each data set.

45, 47, 49, 50, 51, 53, 55, 58, 60

10, 15, 20, 25, 30, 35, 40, 45, 50, 55

90, 85, 92, 88, 84, 86, 91, 89, 87

22, 24, 24, 25, 26, 27, 28, 30, 30, 31, 32, 33

100, 102, 104, 106, 108, 110, 112, 114, 116, 118, 120

75, 80, 82, 85, 87, 90, 92, 95, 98, 100, 102, 105, 108, 110, 112

End-of-Course Prep

8.3 Margin of Error (M.O.E.)

Warm-Up

When does Margin of Error mean to you?

Margin of Error

Describes how far the sample statistic is expected to be from the true population parameter, within a certain level of confidence.

$$M.E. = \underbrace{2}_{\text{Critical Value}} \frac{\overbrace{s}^{\text{Standard Deviation}}}{\underbrace{\sqrt{n}}_{\text{Sample Size}}}$$

Find the Margin of Error for the following data set.	Find the Margin of Error for the following data set.
2.3, 4.1, 4.2, 6.5, 6.3, 6.5, 3.6, 5.2, 4.5, 2.6	A sample data set of 218 people had an average weight of 112.5 lbs with a margin of error for a 95% confidence interval is 0.134.

Margin of Error

A new software company offers a test to their 125 employees that test their skills in their new roles. The standard deviation is 500. Find the margin of error.	A sample of baby cubs at the new zoo has a mean weight of 634 pounds. The standard deviation of sample is 322 pounds and the margin of error is 88.4. How many cubs were in the sample?
A college researcher is working on a study that has a sample size of 64 and a standard deviation of 0.123. Find the margin of error.	A class of 25 students had a test average of 85 with a margin of error of 6.2. What is the standard deviation of the class?
A survey of 275 people shows that 185 have been to the movies in the last month. The standard deviation is 0.186. Find the margin of error.	When you are working with sampling distribution of a sample statistic, which of the following would increase the margin of error? A. Increase sample size B. Decrease sample size C. Increase the standard error D. Increase the standard deviation

End-of-Course Prep

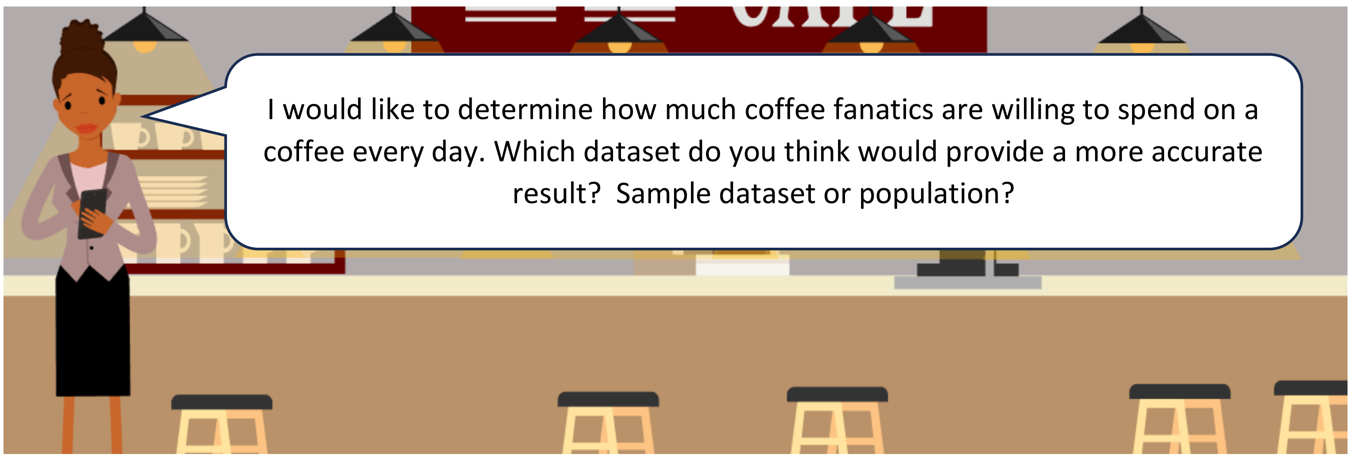
8.4 Normal Distribution & the Bell Curve

Warm-Up

Write all you know about the terms below.

Sample Data	Population Data

Main Topic **The Normal Distribution**



Explain the advantages and disadvantages of using a sample data vs. population data.

	Sample Data	**Population Data**
Advantages		
Disadvantages		

Parameter

Numerical value of a population.

$\boldsymbol{\mu}$: *Population Mean*

$\boldsymbol{\sigma}$: *Population Standard Deviation*

$\boldsymbol{p}$: *Population Proportion*

Statistic

Numerical value of a sample.

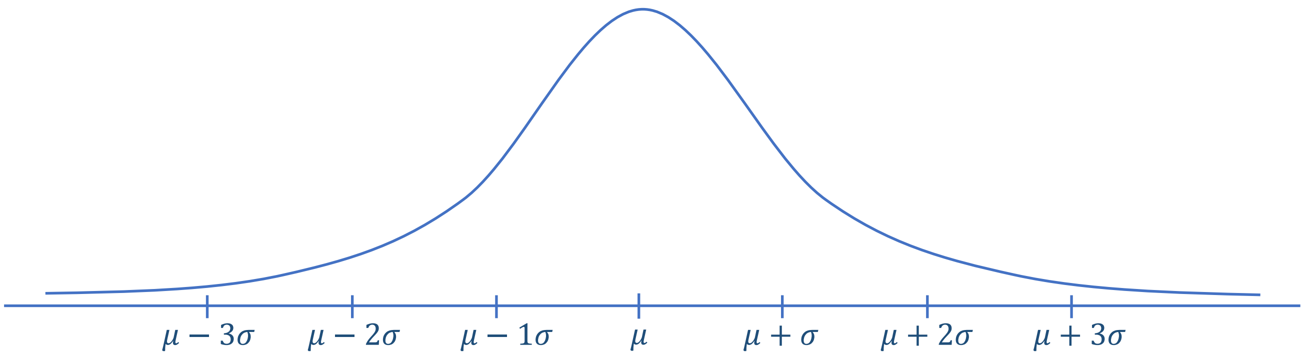
$\bar{\boldsymbol{x}}$: *Sample Mean*

$\boldsymbol{s}$: *Sample Standard Deviation*

$\hat{\boldsymbol{p}}$: *Sample Proportion*

Need Help?

Normal Curve / Bell Curve / Normal Distribution



Describe a normal curve.

____

____

Draw a normal curve with Population Mean (μ) of 30 and a Standard Deviation (σ) of 6

Draw a normal curve with Population Mean (μ) of 50 and a Standard Deviation (σ) of 4.

Draw a normal curve with Population Mean (μ) of 70 and a Standard Deviation (σ) of 10.

Need Help?

Population Standard Deviation

Draw a normal curve given the following conditions.

Population Mean (μ) = 50 Population Standard Deviation (σ) = 10.

Draw a normal curve given the following conditions.

Population Mean (μ) = 50 Population Standard Deviation (σ) = 20.

How do you describe Population Mean?

____

____

How do you describe Population Standard Deviation?

____

____

Main Topic	Empirical Rule

Draw the normal curve of the results.

Empirical Rule

The **68-95-99.7 Rule** states that almost all data in a normally distributed data fall within three standard deviations of the mean.

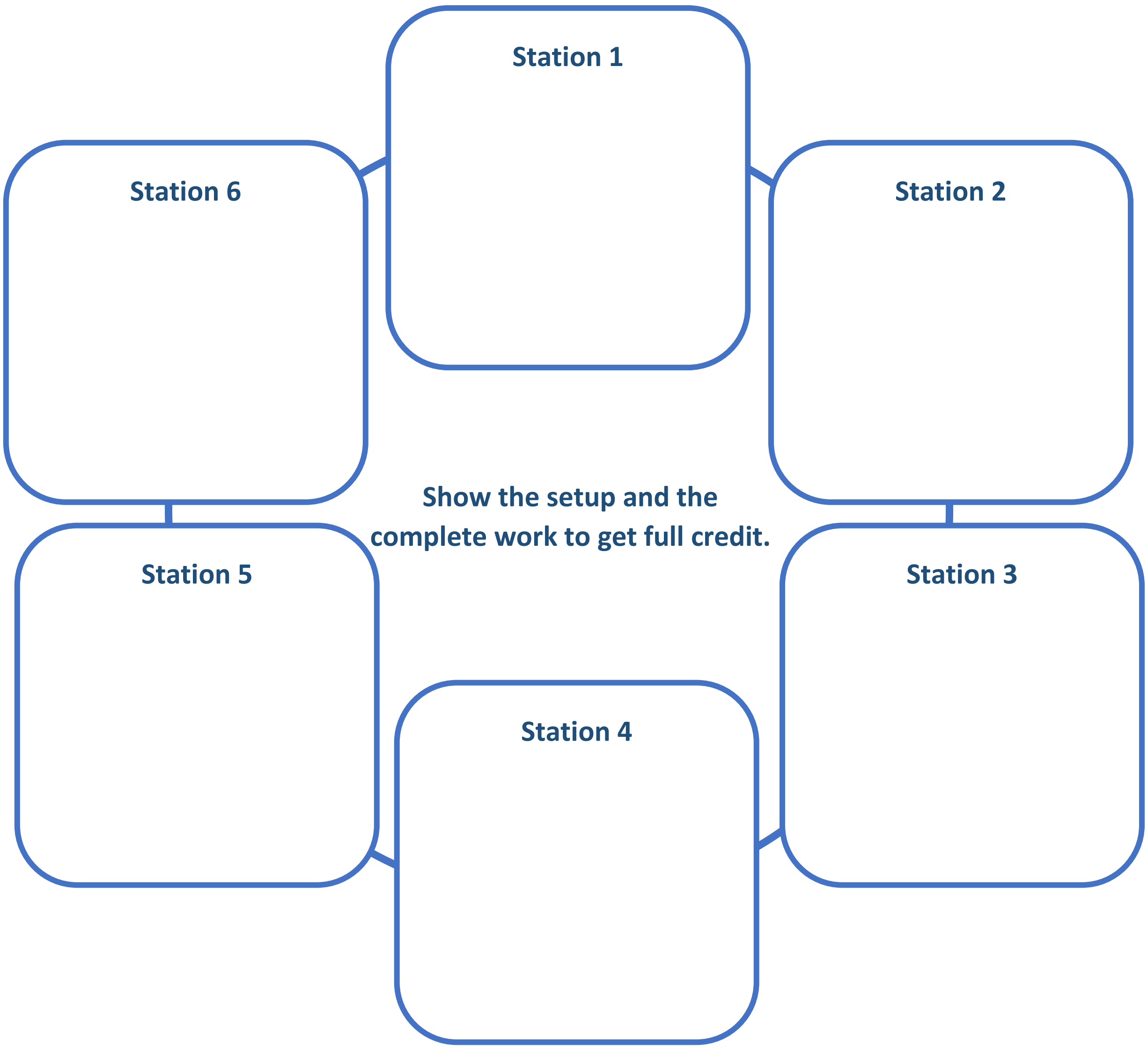
68% Approximately 68% of the observed data are within <u>one</u> standard deviation of the mean

95% Approximately 95% of the observed data are within <u>two</u> standard deviations of the mean

99.7% Approximately 99.7% of the observed data are within <u>three</u> standard deviations of the mean

Standard Deviation Stations

Answer the question(s) in each station with a partner.



End-of-Course Prep

8.5 Z-scores

Warm-Up

Use a calculator to find the standard deviation of the sample data below.

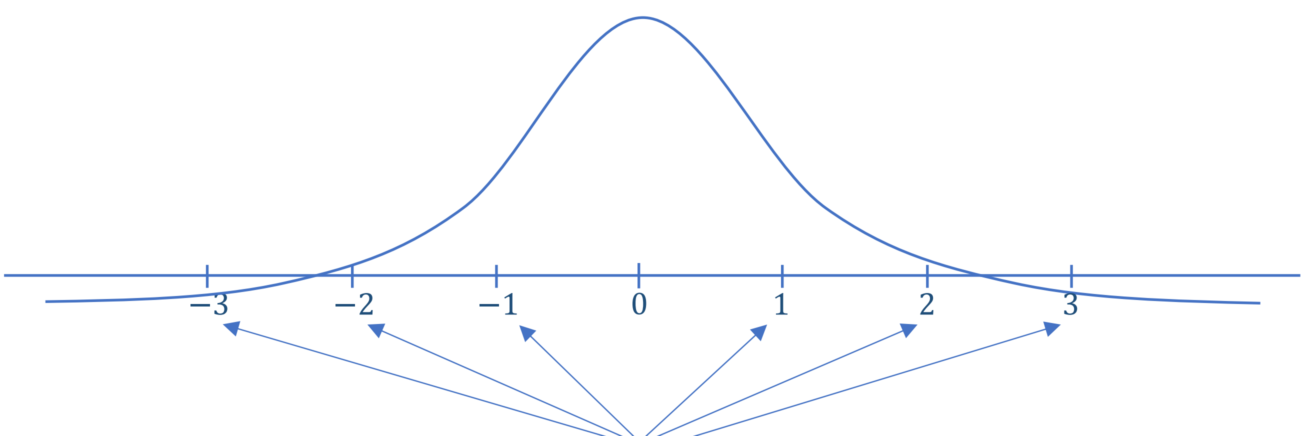
34, 37, 28, 40, 33

Main Topic Standard Normal Distribution

Standard Normal Distribution

It is a normal distribution that has a mean of 0 and a standard deviation of 1.

Standard Normal Distribution

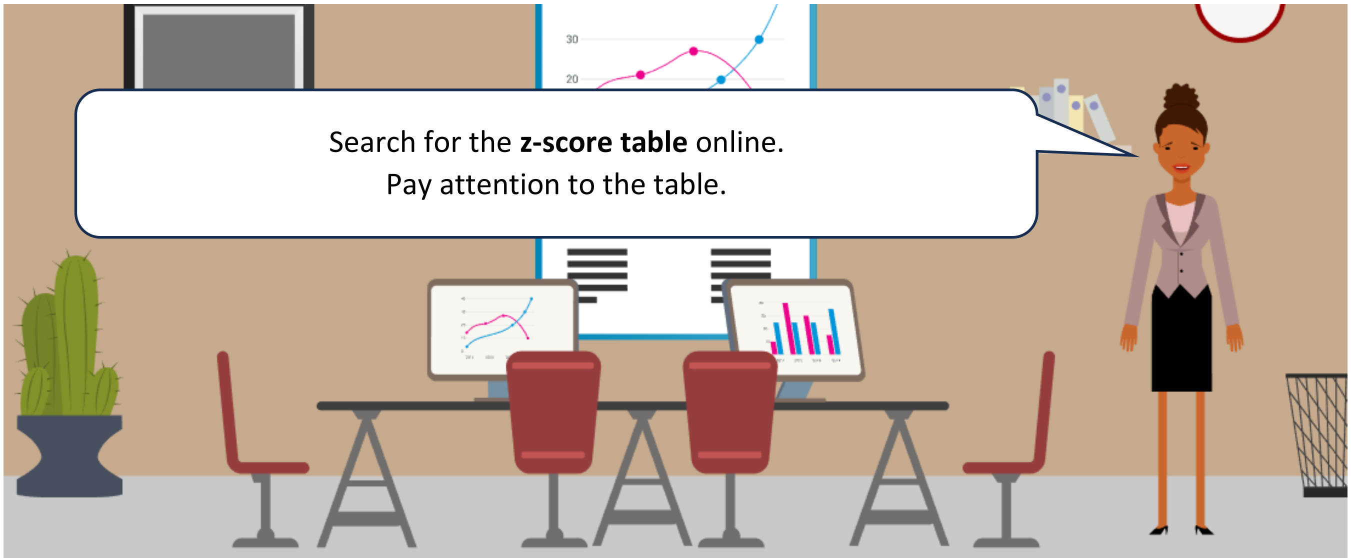


z-score is a measurement of the distance of a specific data from the mean.
z-score is also known as the **standard score**.

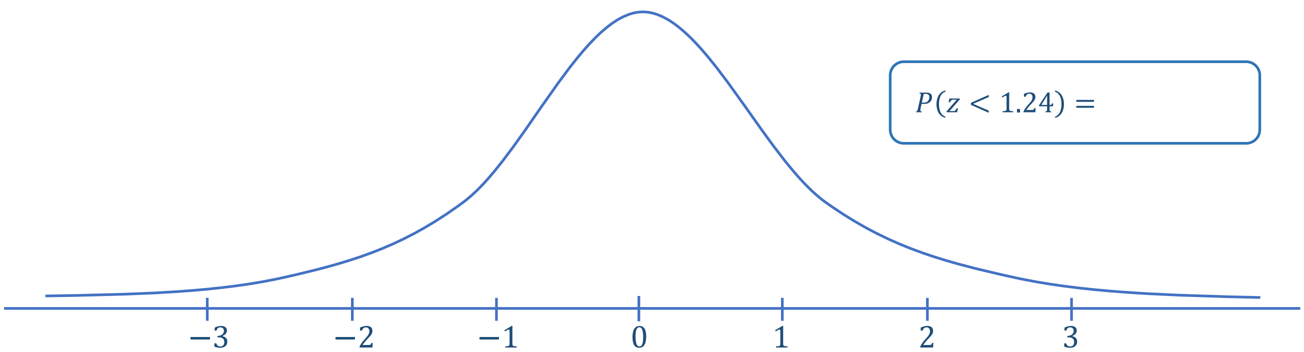
- What does it mean when the z-score is 2.4?
- What does it mean when the z-score is -1.2?

Note: The total area under the normal curve is equal to 1.

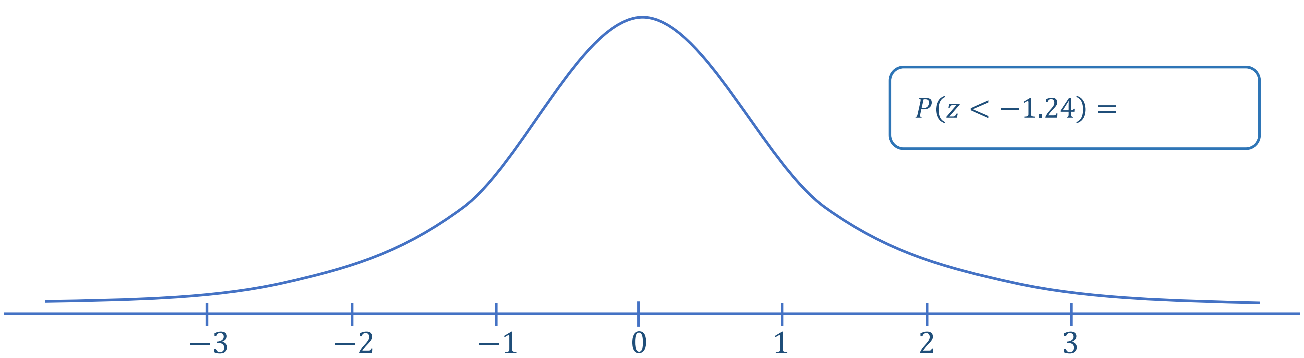
Need Help?



- Sketch z = 1.24 and find the area of the shaded region.

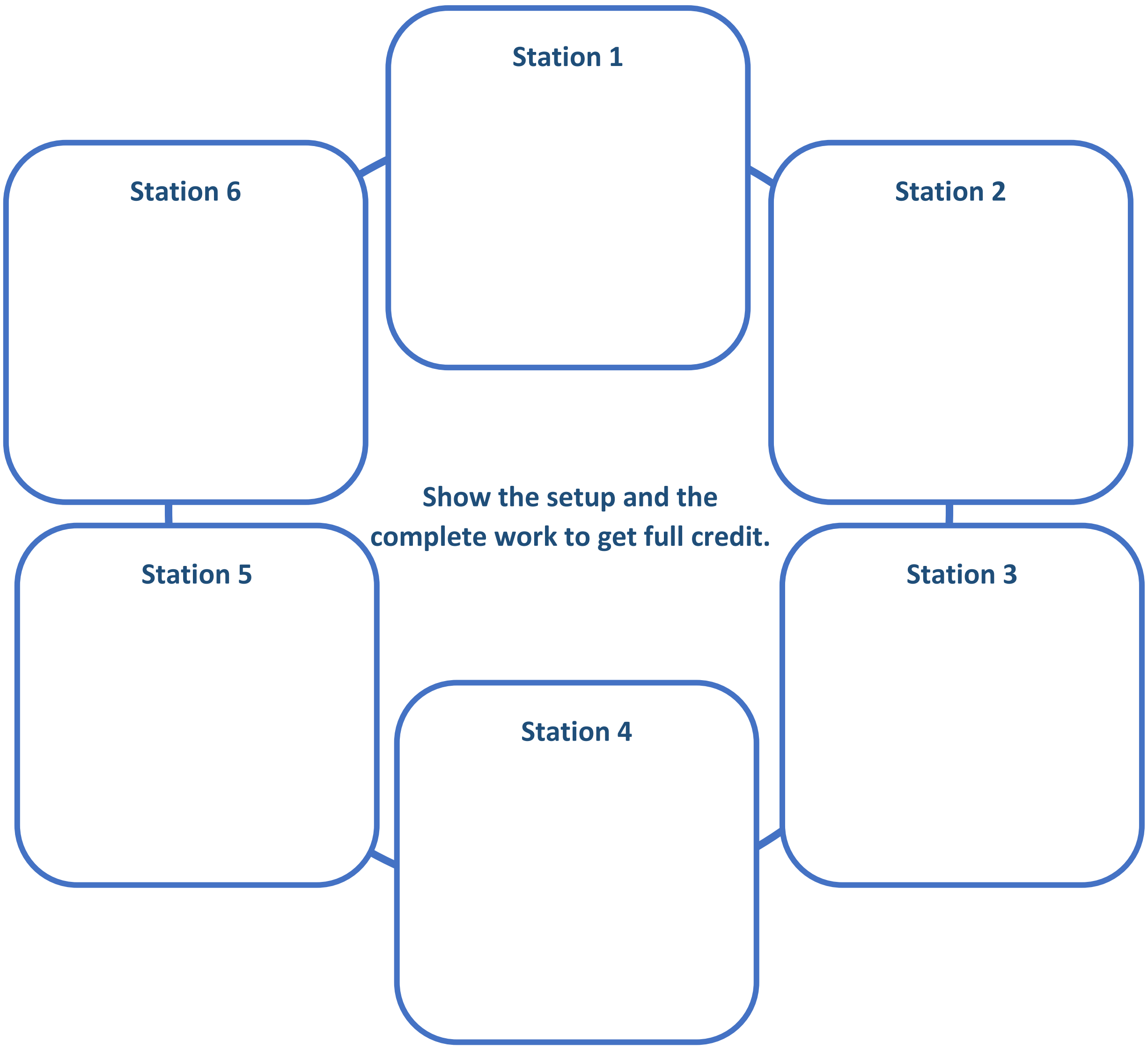


- Sketch z = -1.24 and find the area of the shaded region.



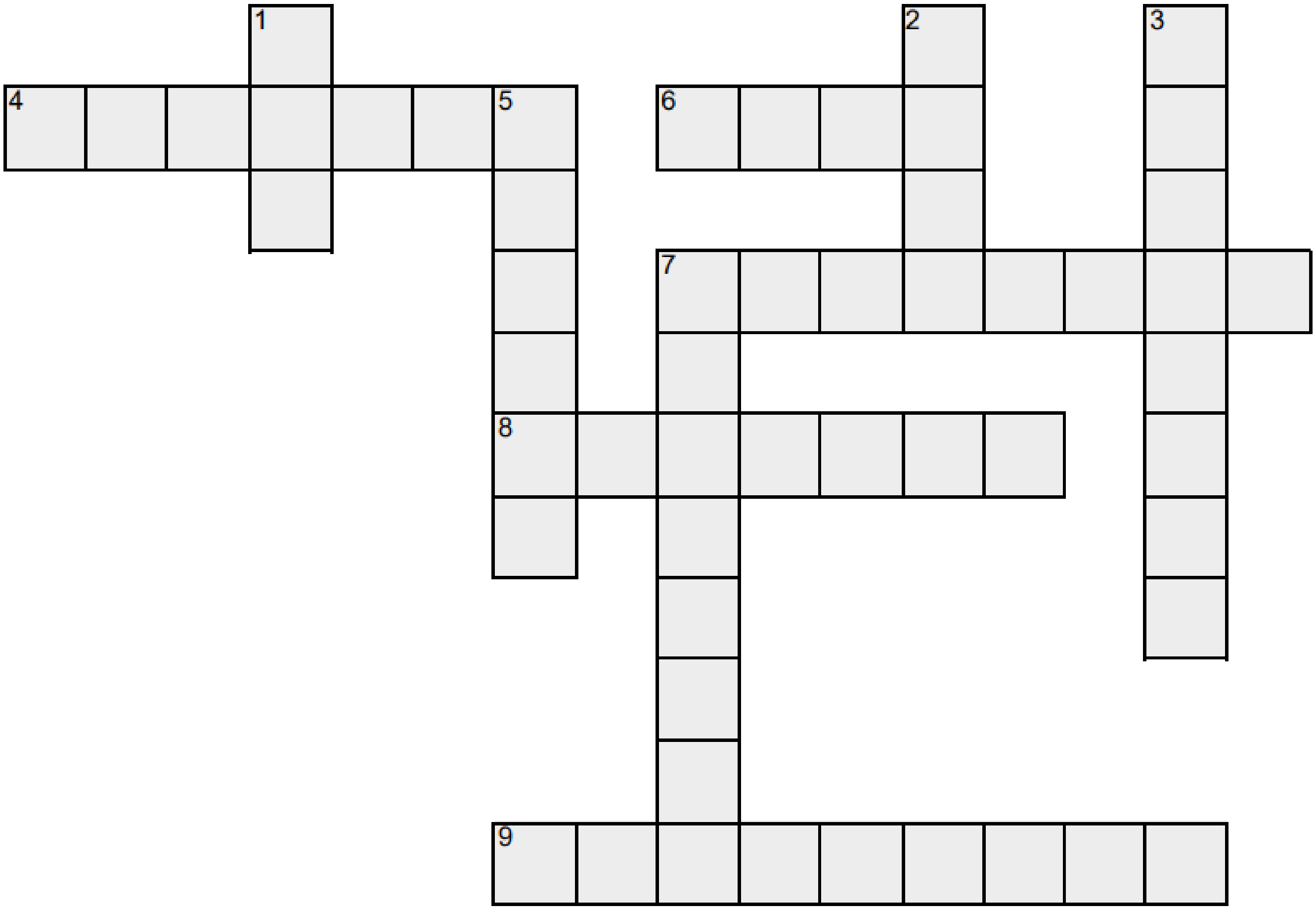
z-scores Stations

Answer the questions in each station with a partner.



End-of-Course Prep

Statistics



Across	Down
4. The lowest value in the dataset.	1. A visual representation of the dataset represented by the slices in a circle.
6. The most frequent value in the dataset.	2. The average value of the dataset.
7. A _____________ deviation measures how individual values deviate from the mean of the dataset.	3. Divides the dataset into four equal parts.
8. A more common term for mean.	5. The middle value of the dataset arranged in ascending or descending order.
9. A visual representation of the distribution that uses bars.	7. The ___________ of a curve refers to the asymmetry of the distribution.

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