MathBlast2

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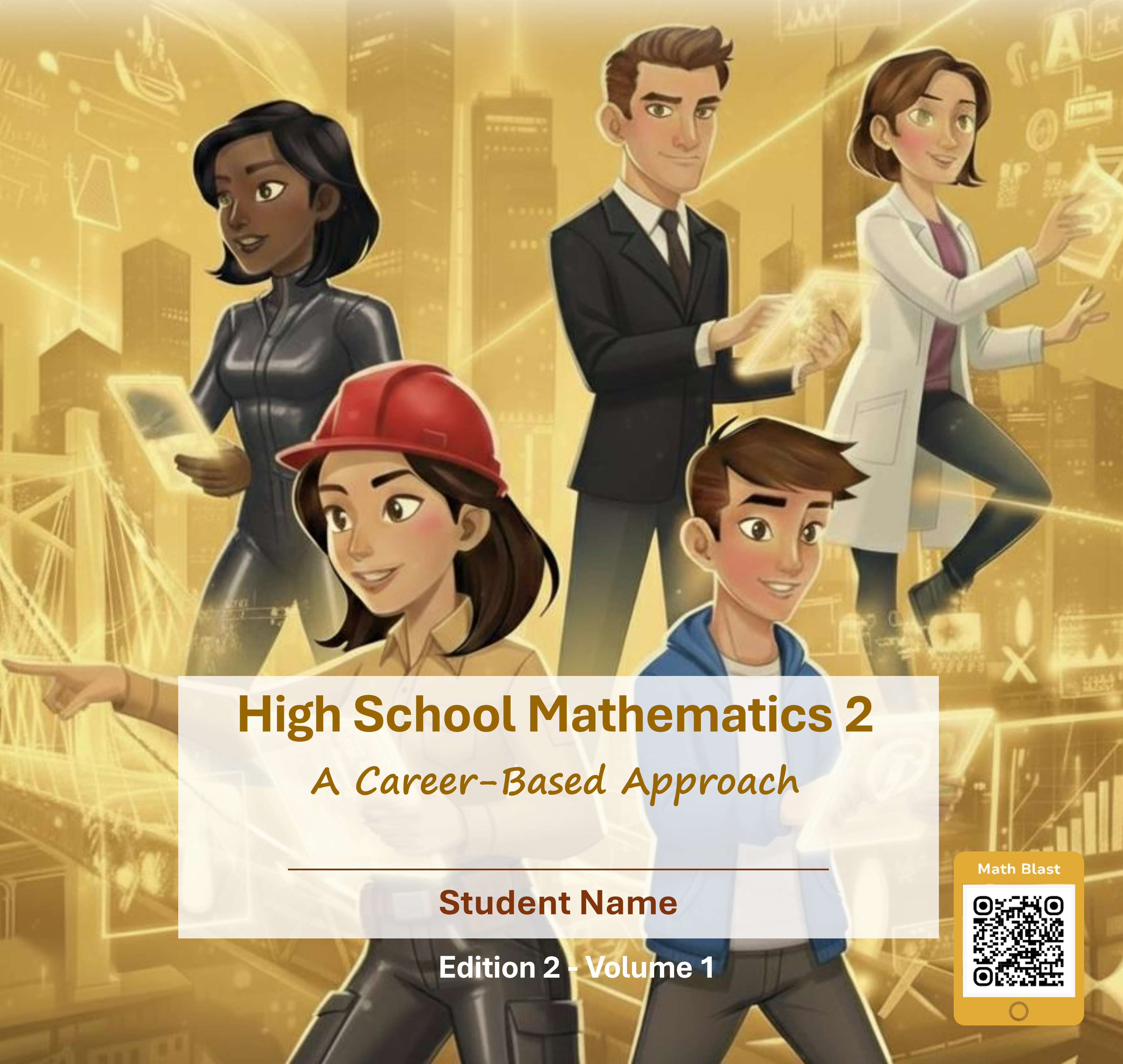

High School Mathematics 2

A Career-Based Approach

Student Name

Edition 2 - Volume 1

Math Blast

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fun and interactive math for all

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Volume 1

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Interior Design

High School Mathematics 2

North Carolina Math 2 Standards

Unit 1

Congruence *Experiment with transformations in the plane.*	
NC.M2.G-CO.2	Experiment with transformations in the plane. • Represent transformations in the plane. • Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations). Understand that rigid motions produce congruent figures while dilations produce similar figures.
NC.M2.G-CO.3	Given a triangle, quadrilateral, or regular polygon, describe any reflection or rotation symmetry i.e., actions that carry the figure onto itself. Identify center and angle(s) of rotation symmetry. Identify line(s) of reflection symmetry.
NC.M2.G-CO.4	Verify experimentally properties of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
NC.M2.G-CO.5	Given a geometric figure and a rigid motion, find the image of the figure. Given a geometric figure and its image, specify a rigid motion or sequence of rigid motions that will transform the pre-image to its image.

Congruence *Understand congruence in terms of rigid motions.*	
NC.M2.G-CO.6	Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.

1.1 Translations of Geometric Figures

Warm-Up

Write all you know about translations of figures.

Main Topic | **Translations of Geometric Figures**

TASK 1

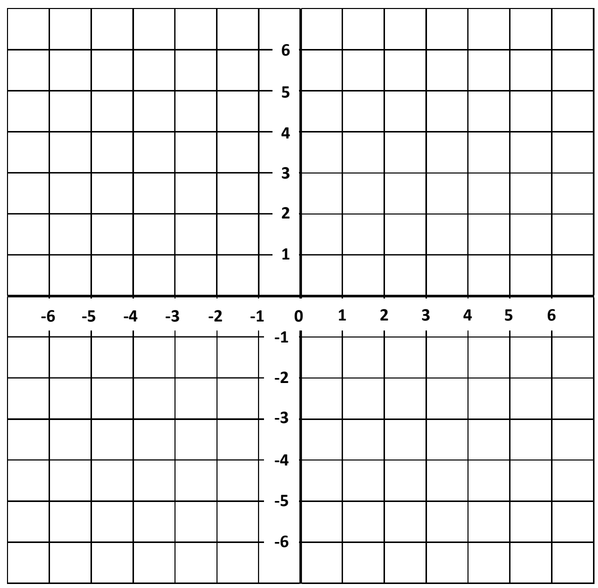
- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)
- Translate the figure 4 units left and 2 units up.

A’(,)

B’(,)

C’(,)

D’(,)



Create a rule.

____

____

Write the translation rule and the coordinates of the translated point.

A(1, -5) 3 units up 1 unit left	B(-6, 3) 1 unit down 5 units right	C(4, 0) 9 units down 4 units left	D(-3, -8) 27 units up 8 units right

Need Help?

The Flexible Living Room Layout

Client: Sarah and Mark, who live in a small, open-concept apartment. They often host friends and need their living room to be adaptable for different social situations – from quiet evenings to lively game nights.

The Challenge: Maximize flexibility in their living space. They have a modular sofa, a movable coffee table, and several accent chairs. Sarah and Mark want to easily visualize and adjust their furniture layout without physically moving heavy items.

The Interior Designer's Approach (Using Coordinate Plane Translation):
The interior designer decides to model the room and furniture on a coordinate plane.

1. **Establishing the Coordinate Plane:**
    - The living room floor is represented as a 2D coordinate plane.
    - The bottom-left corner of the room is designated as the origin (0,0).
    - Each grid unit represents one foot.

2. **Initial Furniture Placement (Baseline Layout):**
    - **Sofa Segment A (chaise):** Its primary corner is at (2, 2) and it occupies a 3x6 foot area.
    - **Sofa Segment B (2-seater):** Its primary corner is at (5, 2) and it occupies a 3x4 foot area. (Initially forming an L-shape with Segment A).
    - **Coffee Table:** Its center is at (4, 7) and it's a 2x3 foot rectangle.
    - **Armchair 1:** Its primary corner is at (8, 2) and it's a 2x2 foot square.

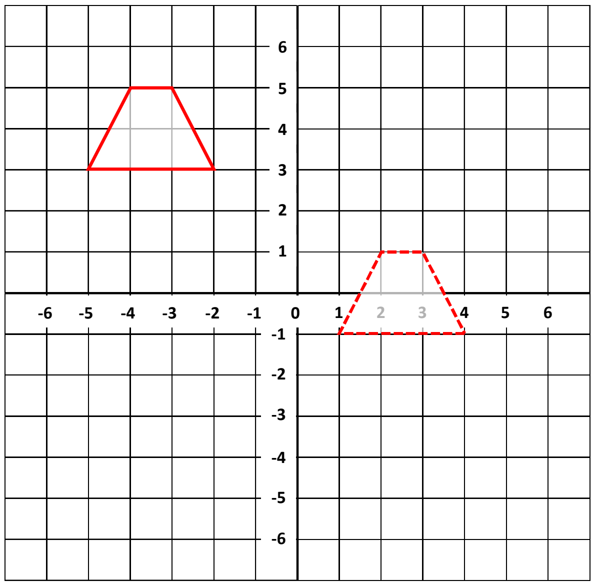
3. **Applying Translation for a "Game Night" Layout:** Sarah and Mark want to shift the furniture to create a more open central space for games. The designer proposes the following **translations**:
    - **Sofa Segment A:** Translate (x+0, y+3). This moves it 3 feet *up* along the y-axis, separating it from Segment B.
        - New corner for Sofa Segment A: (2+0, 2+3) = (2, 5).
    - **Sofa Segment B:** Translate (x-1, y+7). This moves it 1 foot *left* and 7 feet *up*, positioning it opposite Segment A.
        - New corner for Sofa Segment B: (5-1, 2+7) = (4, 9).
    - **Coffee Table:** Translate (x+0, y-3). This moves it 3 feet *down*, closer to the original sofa position.
        - New center for Coffee Table: (4+0, 7-3) = (4, 4).
    - **Armchair 1:** Translate (x-2, y+0). This moves it 2 feet *left* to be closer to the center.
        - New corner for Armchair 1: (8-2, 2+0) = (6, 2).

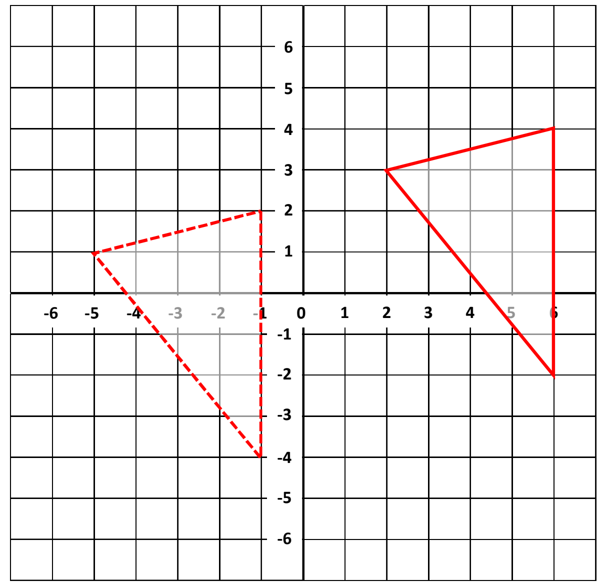
The Outcome:
By applying simple translations to the coordinates of each furniture piece, the interior designer can quickly show Sarah and Mark how their living room can transform. This saves time, prevents physical exertion, and allows them to explore multiple layouts digitally before committing to a physical arrangement. The use of translations on a coordinate plane provides a precise and efficient method for manipulating and visualizing spatial arrangements in interior design.

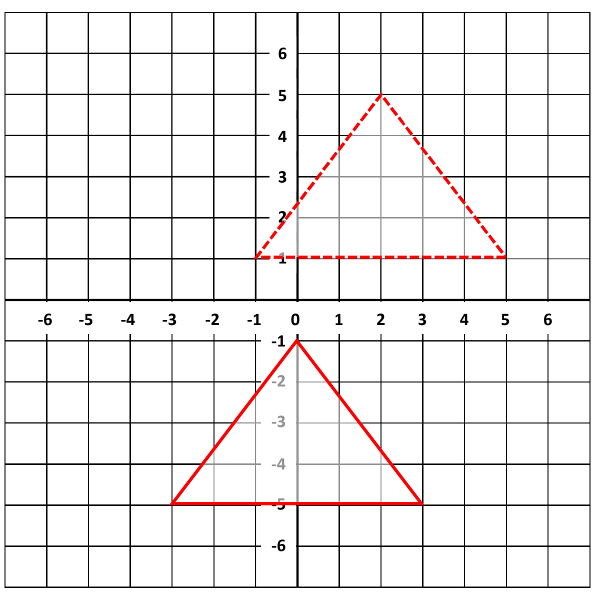
The Flexible Living Room Layout
(Pre-Image)

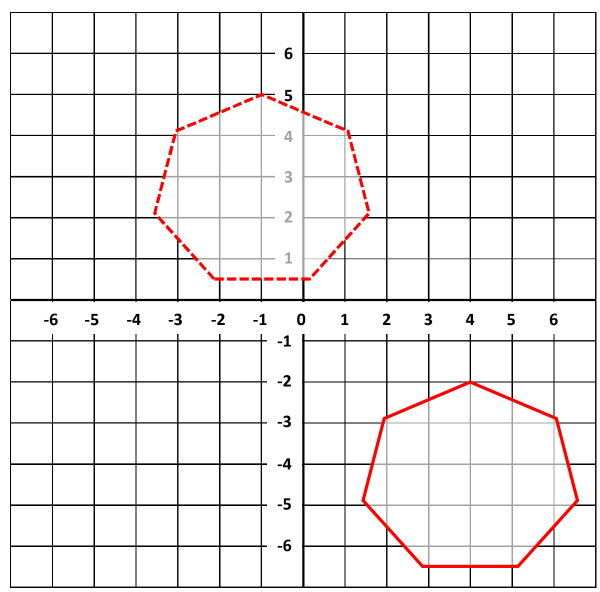
The Flexible Living Room Layout
(Image)

Label the vertices of the pre-image (solid line). Write the rule and the coordinates of the vertices of the image (dashed line).









Honors Level

What is the fundamental difference between a rigid transformation (like translation) and a non-rigid transformation (like dilation) on the coordinate plane?

Conceptually, what does the translation rule $(x,y) \rightarrow (x+a,y+b)$ guarantee about the relationship between the pre-image and the image?

A figure is translated by the rule $(x,y) \rightarrow (x-3,y+2)$. How does the length of any segment in the image compare to the length of the corresponding segment in the pre-image, and why?

If a polygon is translated, how does the slope of a side in the pre-image compare to the slope of the corresponding side in the image?

Why does the translation of a circle only require tracking the coordinates of a single point?

End-of-Course Prep

1.2 Reflections of Geometric Figures

Warm-Up

Write all you know about reflections of figures.

Main Topic	**Reflections of Geometric Figures**

TASK 1

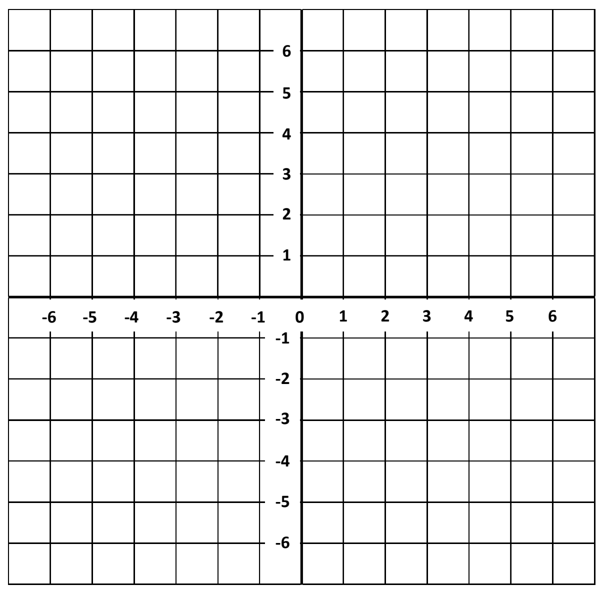
- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

- Reflect the figure over the x-axis.
  A'(,)

  B'(,)

  C'(,)

  D'(,)



Create a rule.

____

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The Symmetrical Primary Bathroom Design

Client: Mr. and Mrs. Henderson, who are renovating their primary bathroom. They desire a classic, symmetrical, and harmonious look, particularly around the vanity area. They have chosen specific fixtures and storage units and want to see how a mirrored layout will appear.

The Challenge: Design a vanity area that perfectly mirrors itself across a central axis, creating a strong sense of balance and luxury. They want to ensure that if a design element is on one side, its exact counterpart is on the other, reflected as if looking into a mirror.

The Interior Designer's Approach (Using Coordinate Plane Reflection):
The interior designer decides to model the bathroom wall and fixtures on a coordinate plane.

1. **Establishing the Coordinate Plane and Axis of Reflection:**
    - The main vanity wall is represented as a 2D coordinate plane.
    - The bottom-left corner of the wall is (0,0).
    - Each grid unit represents 6 inches (or 0.5 feet).
    - A **central vertical line** on the wall will serve as the **axis of reflection (the mirror line)**. Let's say the wall is 10 feet wide (20 units), so the axis of reflection is the line x = 10 (or x=5 if units are 1 foot). For simplicity, let's use 1 unit = 1 foot and the wall is 20 feet wide. So the axis of reflection is x = 10.

2. **Placing Initial Elements (One Side of the Design):** The designer starts by placing elements on the left side of the reflection axis x = 10.
    - **Vanity Cabinet (Left):** A 4-foot wide cabinet. Its top-left corner is at (1, 2) and it occupies a 4x3 foot area.
        - Key points: (1,2), (5,2), (5,5), (1,5)
    - **Sink Basin (Left):** Centered within the left vanity. Its center is at (3, 4).
    - **Wall Sconce (Left):** A decorative light fixture. Its mounting point is at (7, 8).
    - **Small Storage Tower (Left):** A slim vertical cabinet. Its primary corner is at (8.5, 2) and it's 1.5x3 feet.

3. **Applying Reflection Across the Line x = 10:** To create the perfectly symmetrical design, the designer applies a reflection transformation for each element across the line x = 10.

    - **Rule for Reflection across x = k:** If a point (x, y) is reflected across the line x = k, its new coordinates are (2k - x, y).
    - Here, k = 10. So, the rule is (2*10 - x, y) or (20 - x, y).

Let's reflect the key points:

- **Vanity Cabinet (Right - reflected from Left):**
    - Original key points: (1,2), (5,2), (5,5), (1,5)
    - Reflected key points:
        - (20 - 1, 2) = (19, 2)
        - (20 - 5, 2) = (15, 2)
        - (20 - 5, 5) = (15, 5)
        - (20 - 1, 5) = (19, 5)
    - This creates a cabinet starting at (15, 2) and ending at (19, 5).

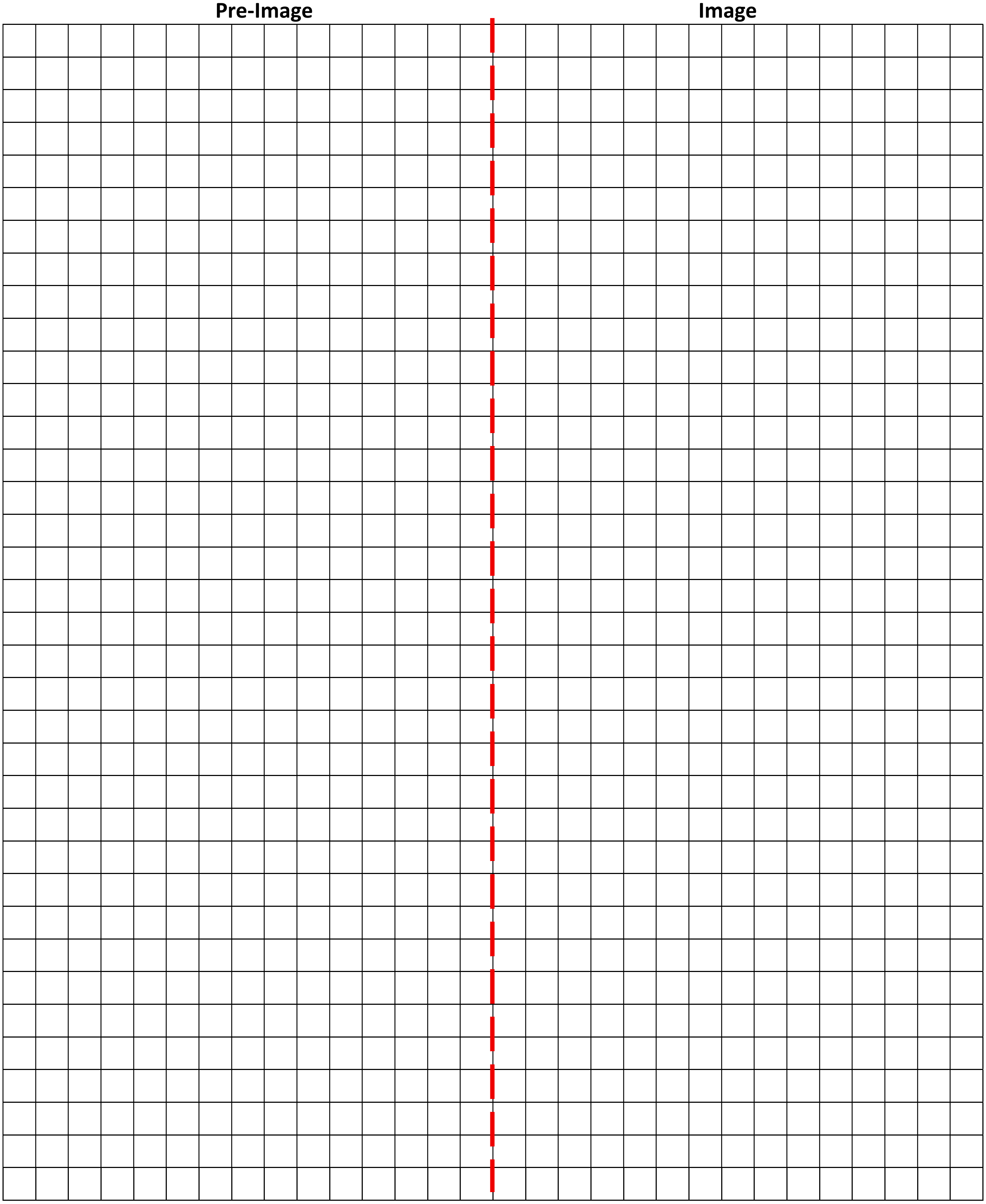
- **Sink Basin (Right - reflected from Left):**
    - Original center: (3, 4)
    - Reflected center: (20 - 3, 4) = (17, 4).

- **Wall Sconce (Right - reflected from Left):**
    - Original mounting point: (7, 8)
    - Reflected mounting point: (20 - 7, 8) = (13, 8).

- **Small Storage Tower (Right - reflected from Left):**
    - Original primary corner: (8.5, 2)
    - Reflected primary corner: (20 - 8.5, 2) = (11.5, 2).

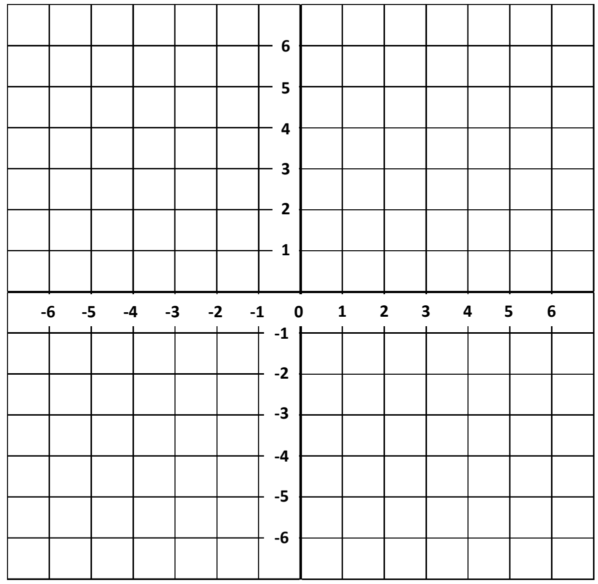
The Symmetrical Master Bathroom Design

Pre-Image **Image**



TASK 2

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)



- Reflect the figure over the y-axis.

  A'(,)

  B'(,)

  C'(,)

  D'(,)

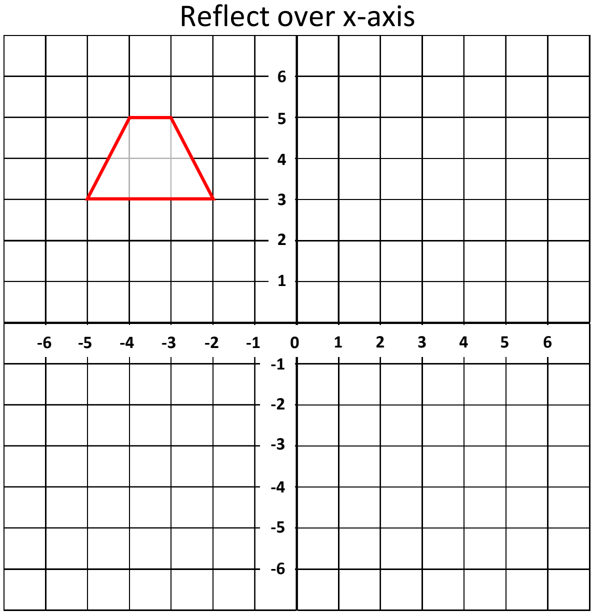
Create a rule.

____

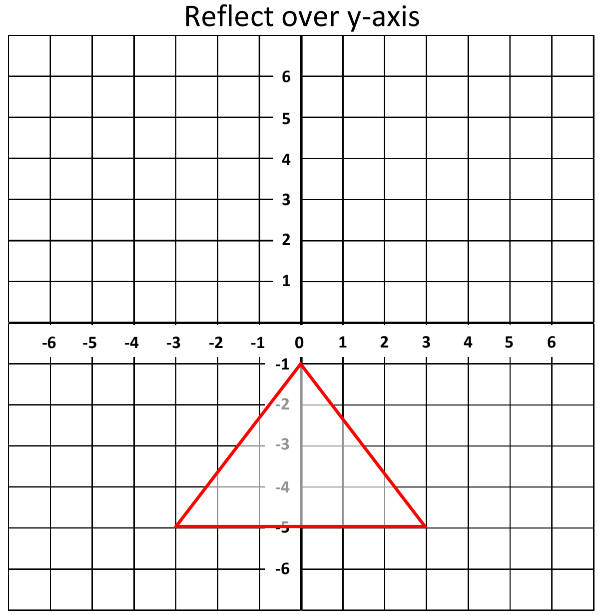
____

Label the vertices of the pre-image. Draw the image (reflected figure) using a dashed line. Write the rule and the coordinates of the vertices.

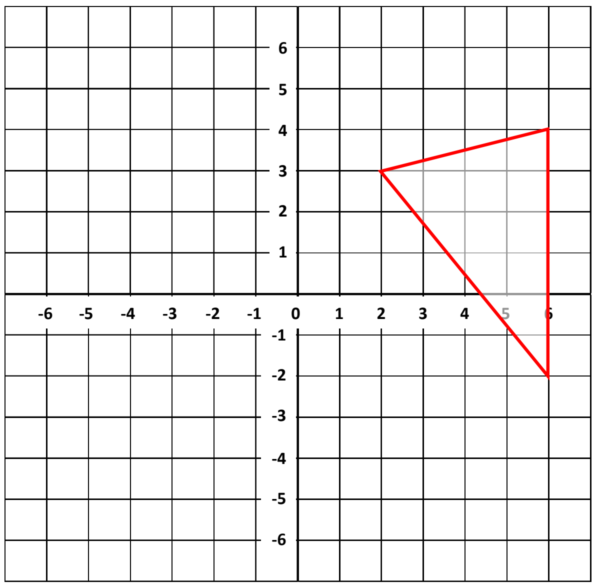
Reflect over x-axis



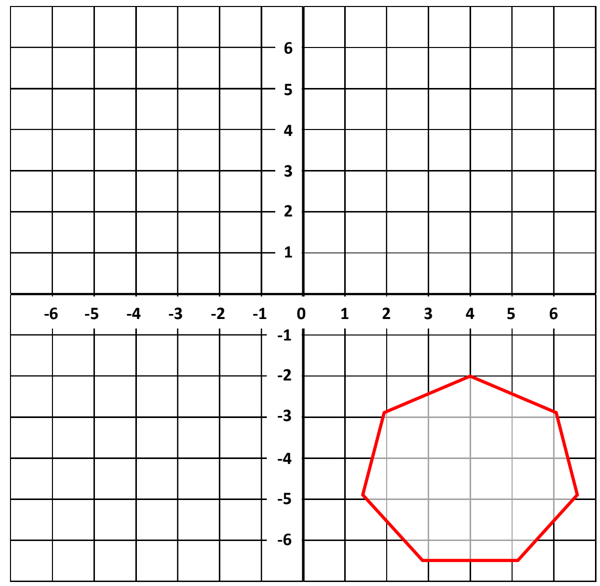
Reflect over y-axis



Reflect over x-axis



Reflect over y-axis



TASK 3

- Plot, connect, and label the points.

  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

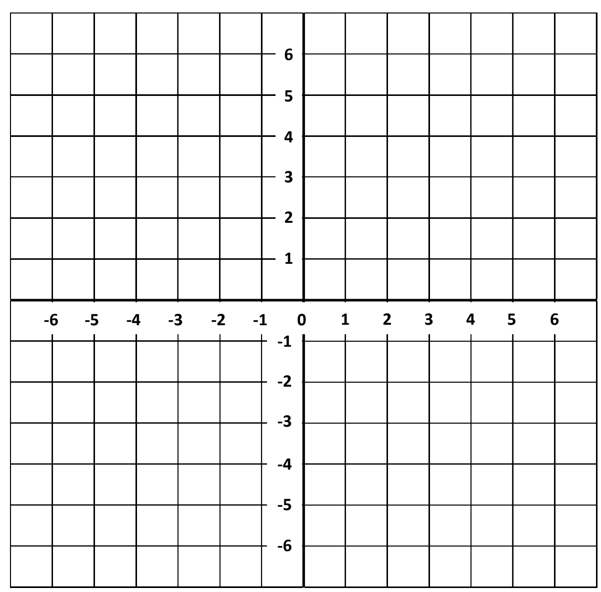
- Reflect the figure over x = -1.

  A’(,)

  B’(,)

  C’(,)

  D’(,)



Create a rule.

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TASK 4

- Plot, connect, and label the points.

  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

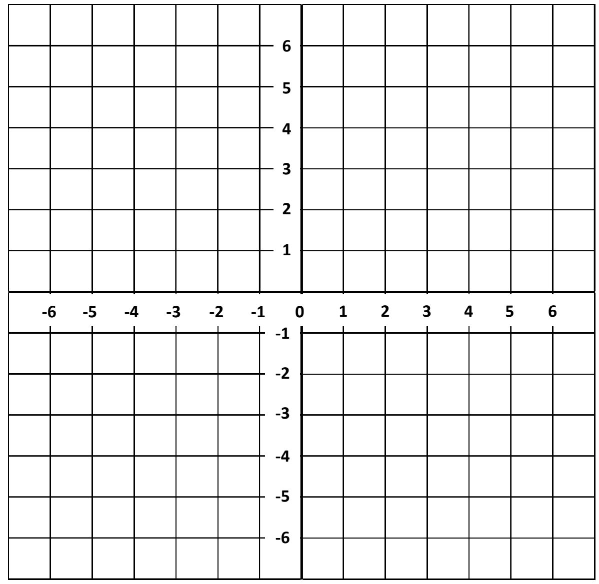
- Reflect the figure over y = 2.

  A’(,)

  B’(,)

  C’(,)

  D’(,)



Create a rule.

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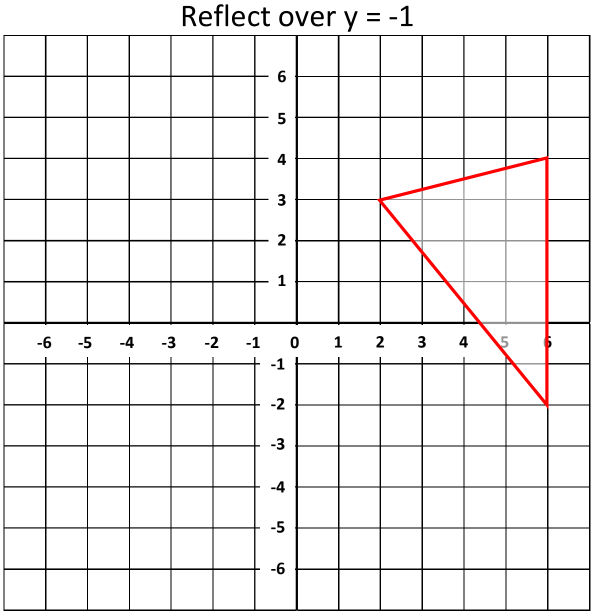
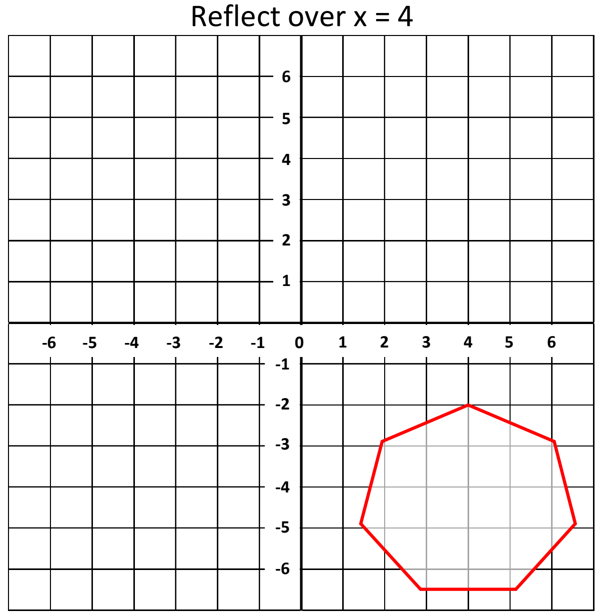
____

Label the vertices of the pre-image. Draw the image (reflected figure) using a dashed line.
Write the rule and the coordinates of the vertices.

Reflect over y = -1



Reflect over x = 4

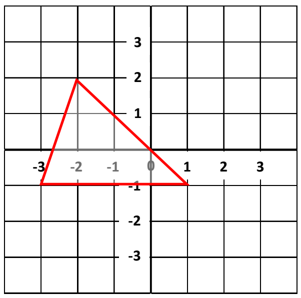


Matching Activity

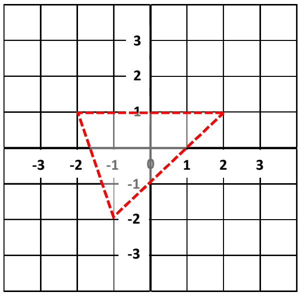
Match column A with Column B by connecting them with the rules in the middle column.

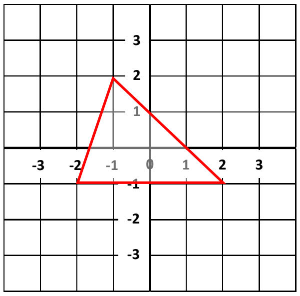
Column A

Column B

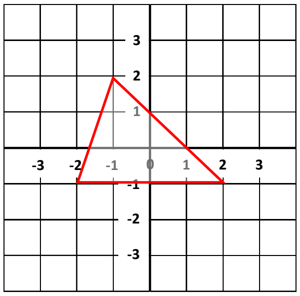


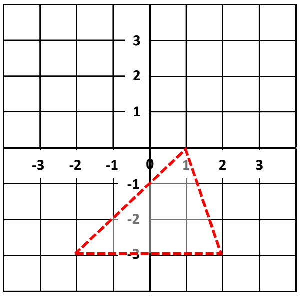
Reflection
over y-axis
Translation
2 units down



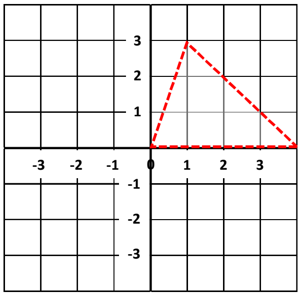


Translation
3 units right
1 unit up





Reflection
over y = 1
Translation
2 units down
1 unit right



Honors Level

What is the fundamental difference between a reflection and a translation in terms of their effect on a figure's orientation?

What geometric concept always exists and is equidistant from any point on the pre-image and its corresponding point on the image after a reflection?

Is reflection a rigid transformation (isometry)? Justify your answer.

How does the algebraic rule for reflecting across the line $y=-x$ differ from reflecting across $y=x$?

Why does reflecting a general line $y = mx + b$ across the x-axis result in a new line with the same magnitude of slope but opposite sign?

End-of-Course Prep

1.3 Rotations of Geometric Figures

Warm-Up

Describe clockwise rotation.

Describe counterclockwise rotation.

Main Topic | **Rotations of Geometric Figures**

TASK 1

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

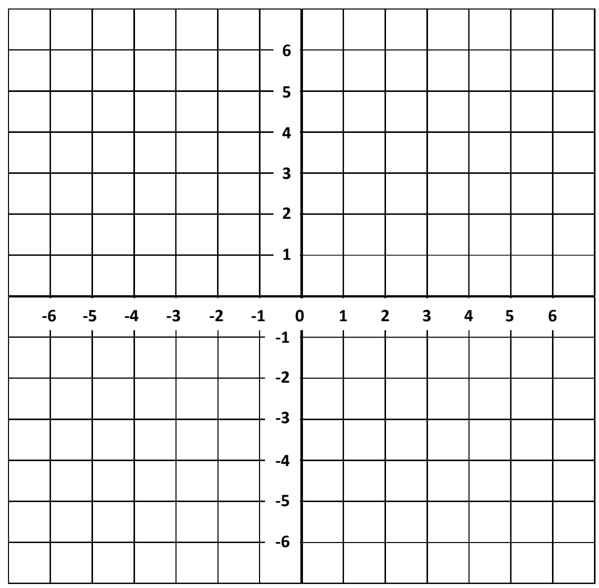
- Rotate the figure 90° clockwise about the origin.

  A′(,)

  B′(,)

  C′(,)

  D′(,)



Create a rule.

____

____

Rotate each point 90° clockwise about the origin. Write the rule and the coordinates of the rotated point.

A(1, -5)	B(-6, 3)	C(4, 0)	D(-3, -8)

Need Help?

The Dynamic Penthouse Living Room

Client: Ms. Evelyn Reed, a modern art collector and socialite, lives in a penthouse with stunning panoramic city views. She frequently rearranges her living room to highlight different art pieces, optimize conversation areas, or showcase the changing city skyline. Her key furniture pieces are often unique and asymmetrical.

The Challenge: Ms. Reed wants to explore how her large, custom-designed L-shaped sofa and a sculptural chaise lounge can be rotated to create various focal points and flow patterns in her open-plan living room. She wants to visualize these changes precisely without the heavy lifting.

The Interior Designer's Approach (Using Coordinate Plane Rotation):
The interior designer models the living room and furniture on a coordinate plane, with a specific pivot point for rotation.

1. **Establishing the Coordinate Plane and Center of Rotation:**
    - The living room floor is represented as a 2D coordinate plane.
    - The bottom-left corner of the room is (0,0).
    - Each grid unit represents one foot.
    - The designer identifies a strategic point in the room, perhaps the center of a large rug or an architectural feature, as the **Center of Rotation**. Let's say this point is (10, 8).

2. **Initial Furniture Placement (Baseline Layout):** The designer places the key furniture pieces in their current (initial) configuration.
    - **L-Shaped Sofa:** This is a complex figure, so we'll define it by its main vertices. Let's assume its "pivot" corner (closest to the center of rotation) is at (7, 6).
        - Vertices (relative to the corner (7,6)): (0,0), (6,0), (6,3), (3,3), (3,7), (0,7)
        - Absolute coordinates: (7,6), (13,6), (13,9), (10,9), (10,13), (7,13)
    - **Sculptural Chaise Lounge:** A unique, curved piece. For simplicity, let's represent its main rectangular bounding box corner at (15, 5) and it's 2x5 feet.
        - Key points: (15,5), (17,5), (17,10), (15,10)

3. **Applying Rotation for a "Conversation Area" Layout:** Ms. Reed wants to rotate the sofa to face a different view and bring the chaise lounge into a closer conversational grouping. The designer proposes the following **rotations** around the center of rotation (10, 8).

   - **Rule for Rotation around a point (a, b):**
     1. Translate the figure so the center of rotation is at the origin: (x - a, y - b).
     2. Apply the rotation rule around the origin (e.g., for 90° clockwise: (y, -x); for 90° counter-clockwise: (-y, x); for 180°: (-x, -y)).
     3. Translate back: (x' + a, y' + b).

Let's apply a **90° counter-clockwise rotation** for the L-shaped sofa and a **180° rotation** for the chaise lounge, both around (10, 8).

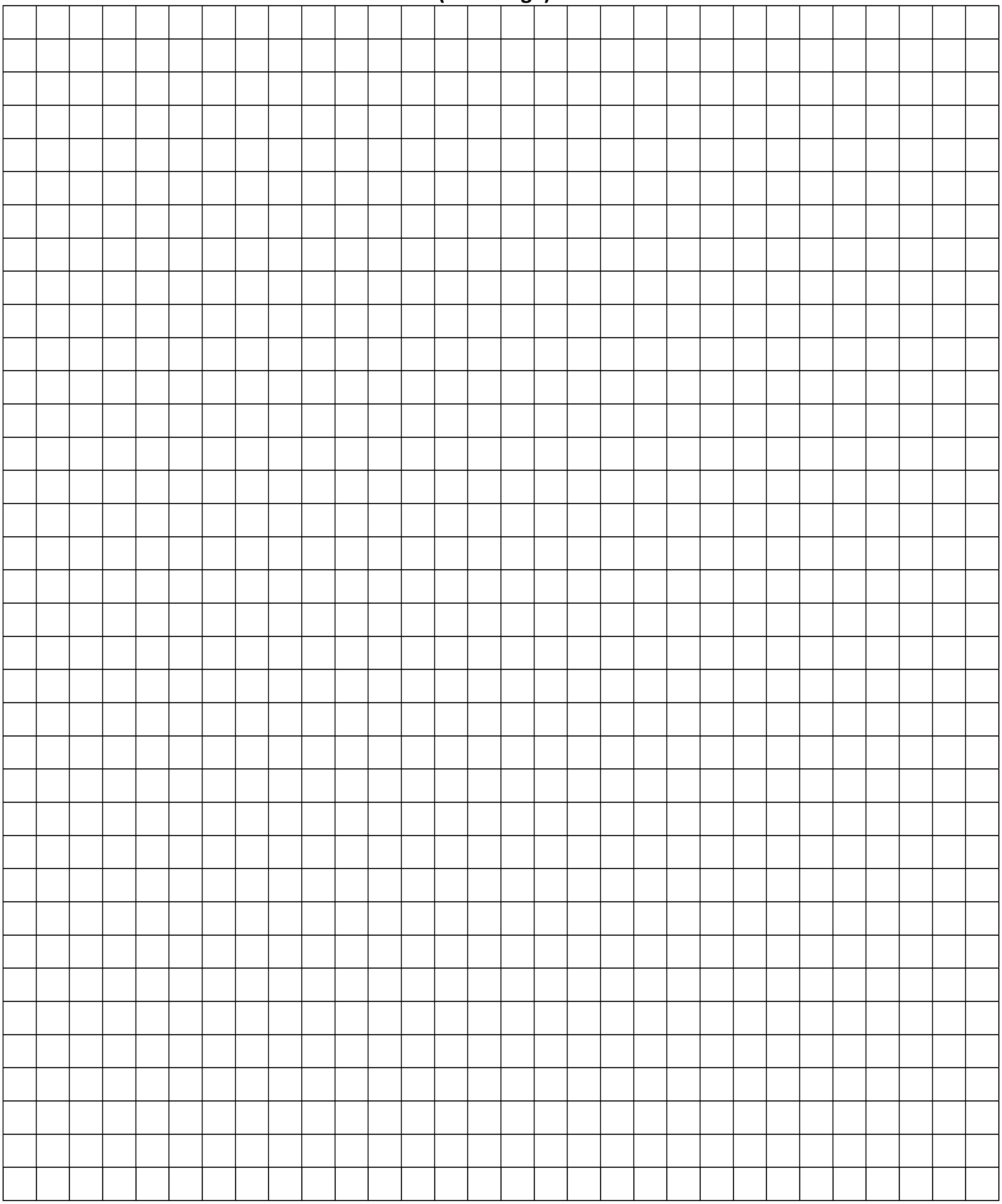
A. L-Shaped Sofa (90° Counter-Clockwise Rotation around (10, 8)):

- Original vertex: (7, 6)
  1. Translate to origin: (7-10, 6-8) = (-3, -2)
  2. Rotate 90° CCW: (-(-2), -3) = (2, -3)
  3. Translate back: (2+10, -3+8) = (12, 5)
- (Repeat for all vertices to get the full rotated shape.)
  - New key vertices for Sofa (e.g., pivot corner now at (12, 5)), oriented differently.

B. Sculptural Chaise Lounge (180° Rotation around (10, 8)):

- Original corner: (15, 5)
  1. Translate to origin: (15-10, 5-8) = (5, -3)
  2. Rotate 180°: (-5, -(-3)) = (-5, 3)
  3. Translate back: (-5+10, 3+8) = (5, 11)
- (Repeat for all corners.)
  - New key corners for Chaise (e.g., (5, 11)), now facing the opposite direction.

The Dynamic Penthouse Living Room
(Pre-image)



The Dynamic Penthouse Living Room
(Image)

TASK 2

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

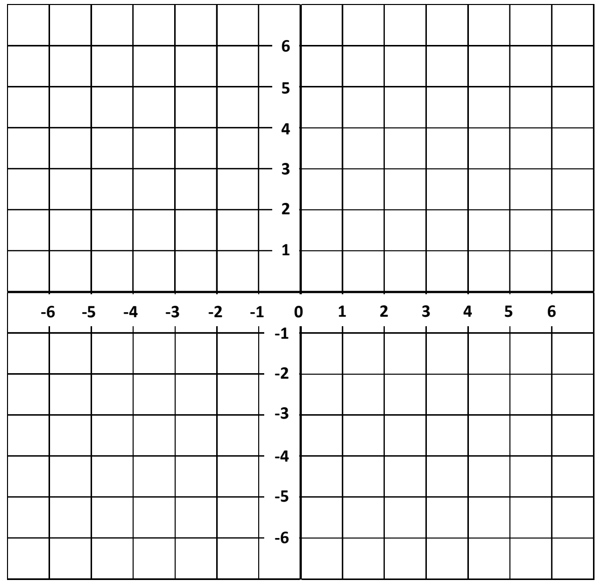
- Rotate the figure 90° counterclockwise about the origin.

  A’(,)

  B’(,)

  C’(,)

  D’(,)



Create a rule.

____

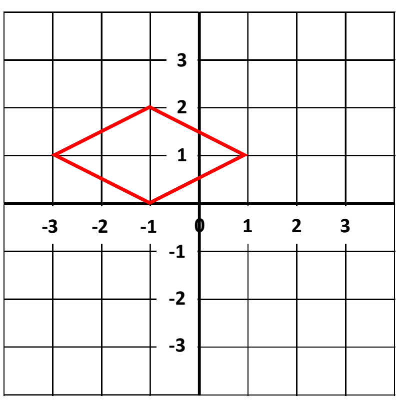
____

Rotate each point 90° counterclockwise about the origin. Write the rule and the coordinates of the rotated point.

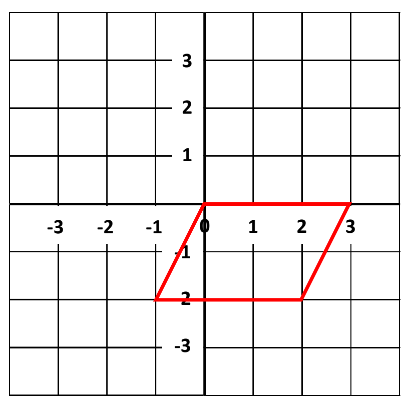
A(1, -5)	B(-6, 3)	C(4, 0)	D(-3, -8)

Label the vertices of the pre-image. Draw the image (rotated figure) using a dashed line. Write the rule and the coordinates of the vertices.

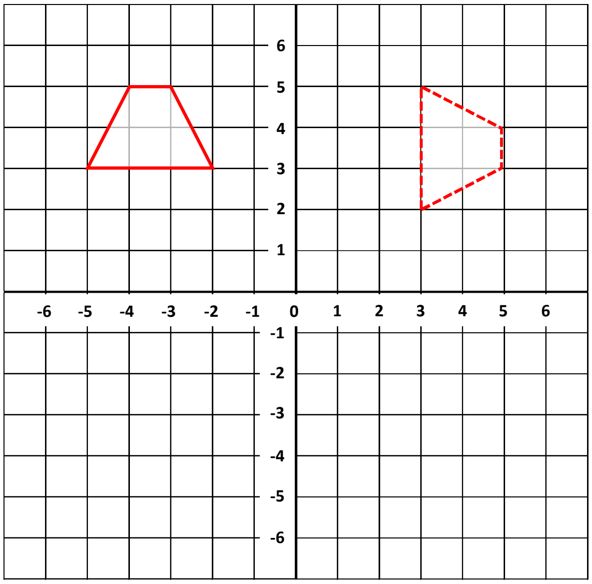
90° counterclockwise

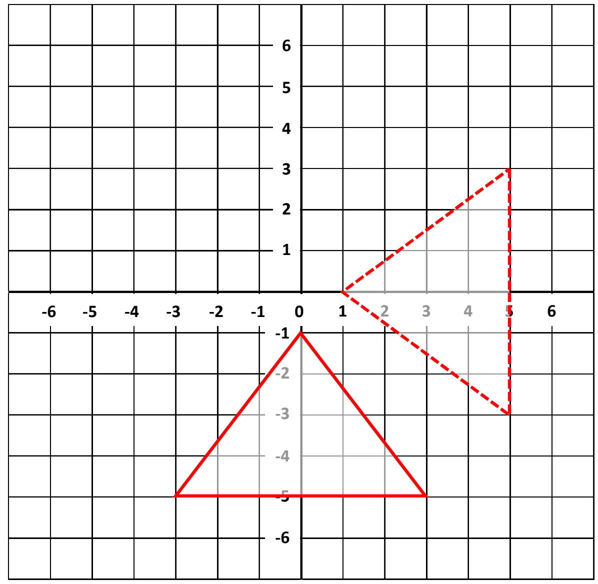


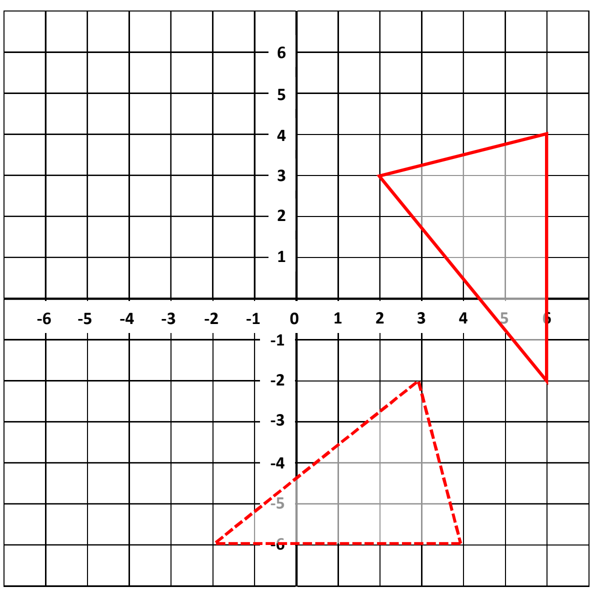
90° clockwise

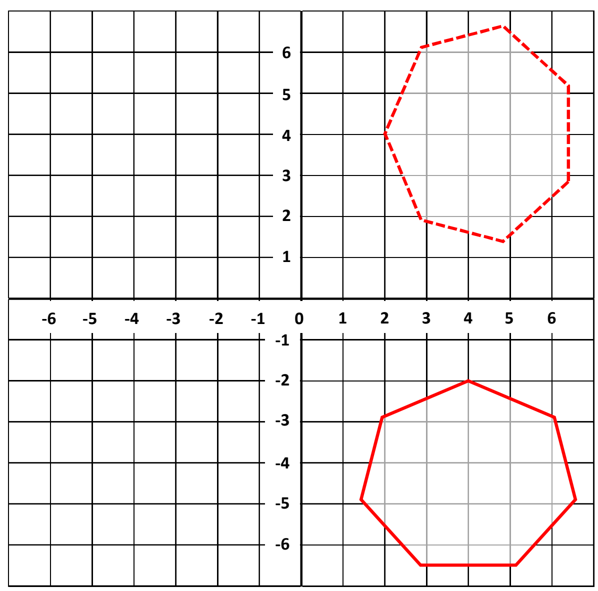


Label the vertices of the pre-image and image. Write the rule and the coordinates of the vertices.









TASK 3

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

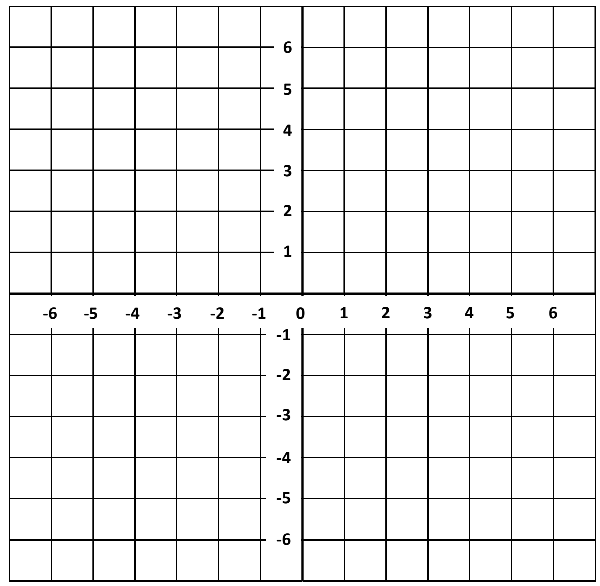
- Rotate the figure 180° clockwise about the origin.

  A′(,)

  B′(,)

  C′(,)

  D′(,)



Create a rule.

____

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TASK 4

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

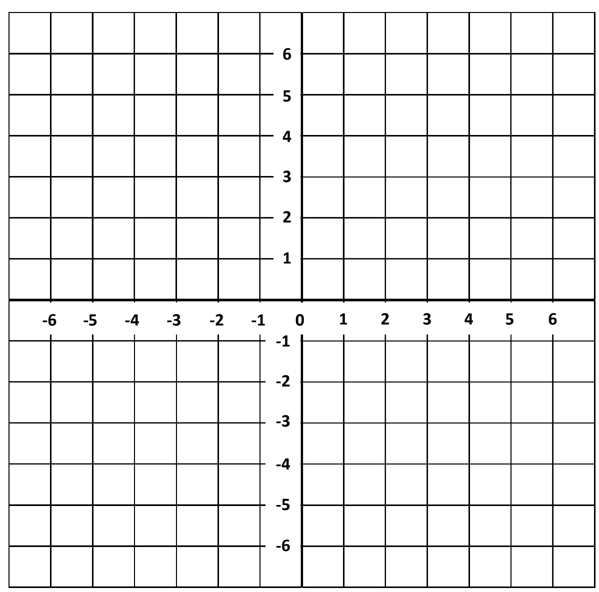
- Rotate the figure 180° counterclockwise about the origin.

  A′(,)

  B′(,)

  C′(,)

  D′(,)



Create a rule.

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TASK 5

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

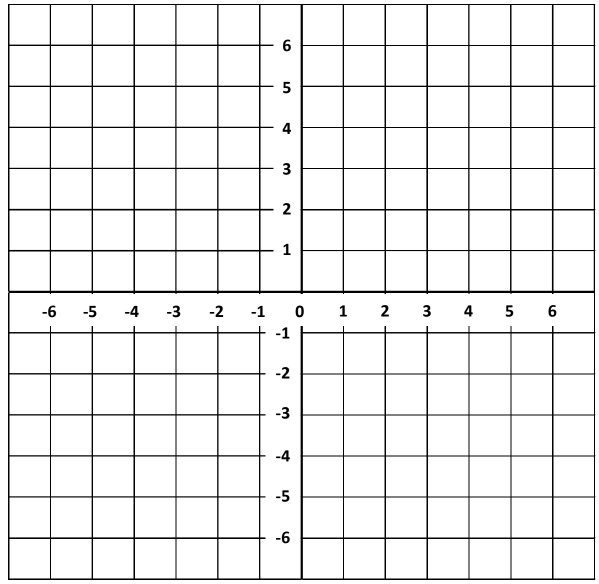
- Rotate the figure 90° clockwise about (2, 1).

  A’(,)

  B’(,)

  C’(,)

  D’(,)



Create a rule.

____

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TASK 6

- Plot, connect, and label the points.
  A(1, 2) B(2, 5) C(5, 3) D(5, 2)

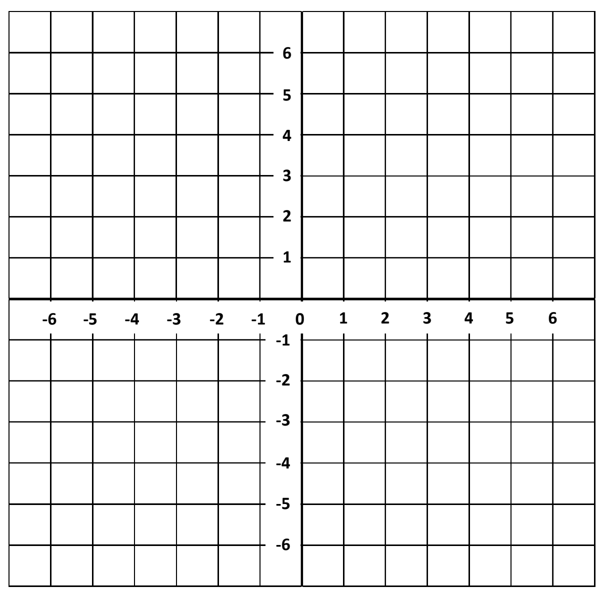
- Rotate the figure 180° counterclockwise about (2, 0).

  A’(,)

  B’(,)

  C’(,)

  D’(,)



Create a rule.

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Honors Level

What is the defining characteristic of a rigid transformation,
and why is a rotation considered one?

Explain the difference between the center of rotation and the angle of rotation.

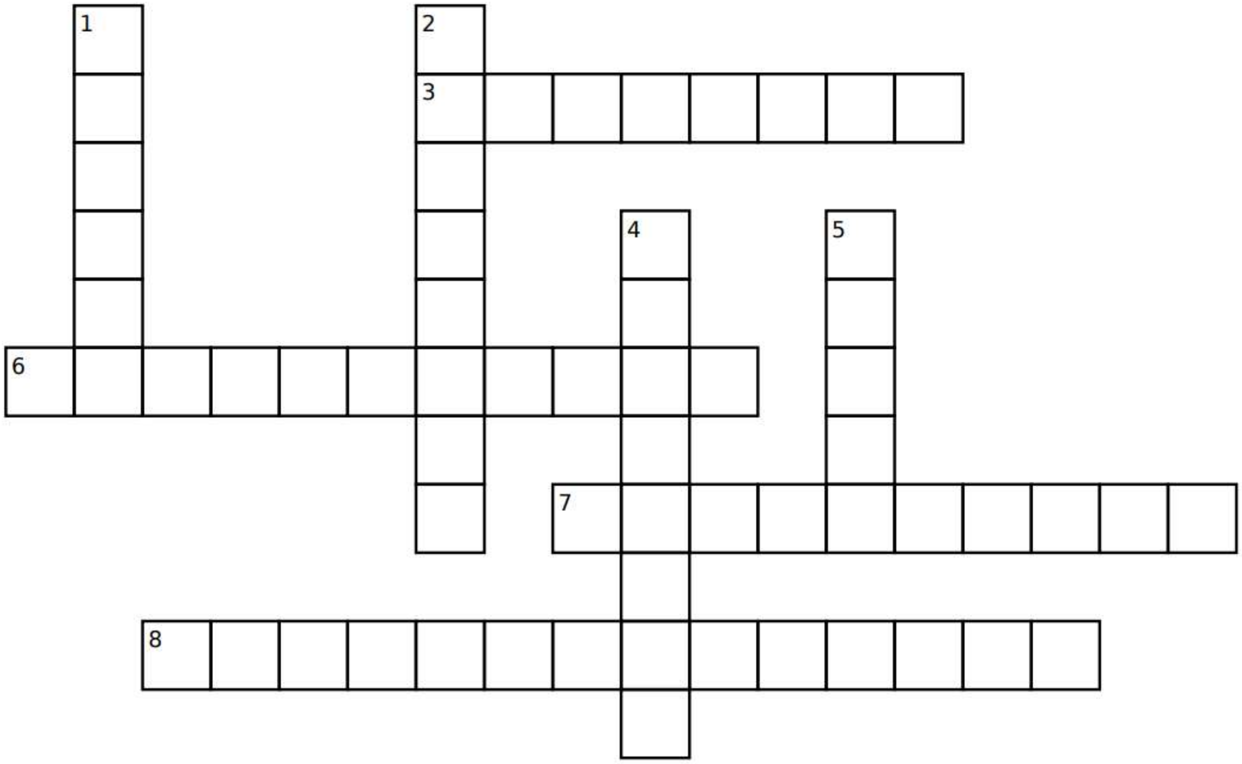
How is a 270◦ counterclockwise rotation related to a 90° clockwise rotation in terms of coordinate rules?

If a line segment AB is rotated 180° about its midpoint, M,
what is the relationship between the pre-image AB and the image A'B'?

How can you determine the center of rotation if you are given a figure (pre-image) and its rotated image?

End-of-Course Prep

Geometric Figure Transformations



Across	Down
3. A transformation that turns a figure about a fixed point.	1. A _____ of rotation is the fixed point around which a figure is turned.
6. A transformation that moves every point of a figure the same distance in the same direction.	2. The original figure before any transformation has been applied.
7. A transformation that creates a mirror image of a figure across a specific line.	4. A transformation that preserves distance and angle measures. (This means the size and shape stay exactly the same; only the position changes).
8. A general term for four specific ways to manipulate the shape and/or position of a point, a line, or a geometric figure.	5. The new figure that results after a transformation.

Unit 2 | Triangles and Construction

Construction Engineering

High School Mathematics 2

North Carolina Math 2 Standards

Unit 2

Congruence Understand congruence in terms of rigid motions.	
NC.M2.G-CO.5	Given a geometric figure and a rigid motion, find the image of the figure. Given a geometric figure and its image, specify a rigid motion or sequence of rigid motions that will transform the pre-image to its image.
NC.M2.G-CO.6	Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.
NC.M2.G-CO.7	Use the properties of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
NC.M2.G-CO.8	Use congruence in terms of rigid motion.

Congruence *Prove geometric theorems.*	
NC.M2.G-CO.9	Prove theorems about lines and angles and use them to prove relationships in geometric figures including: • Vertical angles are congruent. • When a transversal crosses parallel lines, alternate interior angles are congruent. • When a transversal crosses parallel lines, corresponding angles are congruent. • Points are on a perpendicular bisector of a line segment if and only if they are equidistant from the endpoints of the segment. Use congruent triangles to justify why the bisector of an angle is equidistant from the sides of the angle.
NC.M2.G-CO.10	Prove theorems about triangles and use them to prove relationships in geometric figures including: • The sum of the measures of the interior angles of a triangle is 180º. • An exterior angle of a triangle is equal to the sum of its remote interior angles. • The base angles of an isosceles triangle are congruent. The segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length.

2.1 Interior Angles of Triangles

Warm-Up

Solve for x.

$2x - 3 = 25$ $\qquad$ $8x + 2x + 7 = 3x - 1$

Main Topic | **Interior Angles of Triangles**

A team of **surveyors** is performing a **boundary survey** for a new commercial development site. The site plan indicates a triangular-shaped lot (Lot 3) at a critical junction near an existing utility easement. The surveyors need to precisely determine all three **interior angles** of this triangular lot to ensure the placement of **property markers** (monuments) and the accuracy of the final plat map.

The surveyors have already measured two of the interior angles of Lot 3 using a **total station** instrument:

- **Angle A** (at the intersection with Main Street): $65°$
- **Angle B** (at the intersection with the Utility Easement): $48°$

TASK 1

The team needs to calculate the third interior angle, **Angle C** (the final property corner), to verify their field measurements and complete the survey data.

The measure of the third interior angle at the final property corner is ________. This angle is critical for plotting the lot accurately, calculating the bearing (direction) of the third boundary line, and laying out the final boundary monument (marker). The slight difference from a perfect ________ angle demonstrates the need for precise angular measurements in land surveying.

Main Topic | **Triangle Construction**

TASK 2

- Construct ΔABC using a ruler and a compass.
- $\angle A, \angle B$, and $\angle C$ are acute angles.
- Use a protractor to measure each interior angle.
- What is the sum of the interior angles?
- What type of triangle is it?
- Write your observations about the triangle.

____

____

TASK 3

- Construct ΔABC using a ruler and a compass.
- $\angle A$ and $\angle B$ are acute angles.
- $\angle C$ is a right angle.
- Use a protractor to measure each interior angle.
- What is the sum of the interior angles?
- What type of triangle is it?
- Write your observations about the triangle.

____

____

Need Help?

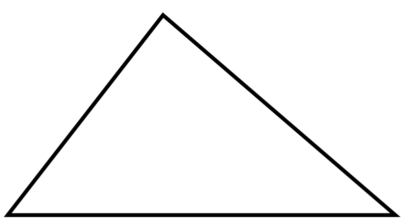
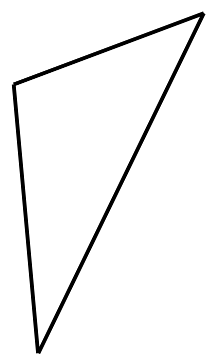
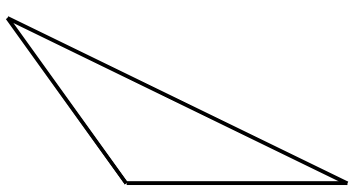
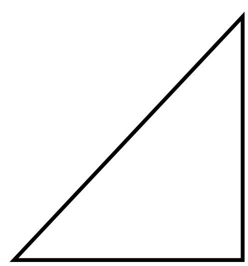
TASK 4

- Construct ΔABC using a ruler and a compass.
- $\angle A$ and $\angle B$ are acute angles.
- $\angle C$ is an obtuse angle.
- Use a protractor to measure each interior angle.
- What is the sum of the interior angles?
- What type of triangle is it?
- Write your observations about the triangle.

  ____

  ____

Use a protractor to measure each interior angle of the triangles. State the sum of the interior angles.

Interior Angles Cutouts

Directions:

- Cut a triangle.
- Trace the triangle on this page.
- Label each interior angle with 1, 2, and 3.
- Tear the corners off.
- Put the corners together.

Write your general observations.

____

____

TASK 5

- Construct ΔABC.
- $\angle A$, $\angle B$, and $\angle C$ are congruent.
- Use a protractor to measure each interior angle.
- What is the sum of the interior angles?
- What type of triangle is it?
- Write your observations about the triangle.

____

____

Show complete work.

$\angle A = 3x - 18, \angle B = 2x + 20, \angle C = x - 2$ Find the measure of each interior angle.	$\angle L = 3x + 4, \angle M = 12x, \angle N = 3x - 4$ Find the measure of each interior angle.
$\angle X = 6x, \angle Y = 4x + 10, \angle Z = 21x + 15.$ Find the measure of each interior angle.	$\angle H = 2x + 5, \angle I = 8x - 5, \angle J\ is\ a\ right\ angle.$ Find the measure of each interior angle.

Need Help?

Honors Level

Is it possible for a triangle to have three obtuse interior angles?
Justify your answer using the Angle Sum Theorem.

If the measures of the interior angles of a triangle are in the ratio 1:2:3, what type of triangle is it, and what are the measures of its angles?

If one interior angle of a triangle measures 120°, what is the largest possible measure for the smallest of the remaining two interior angles?

In an acute triangle, what is the maximum possible integer measure for any single interior angle?

What condition is required for a point *P* inside a triangle $\triangle ABC$ such that $\angle APB = \angle BPC = \angle CPA$?

End-of-Course Prep

2.2 Sides of Triangles

Warm-Up

Write all you know about the following.

Isosceles Triangle

Equilateral Triangle

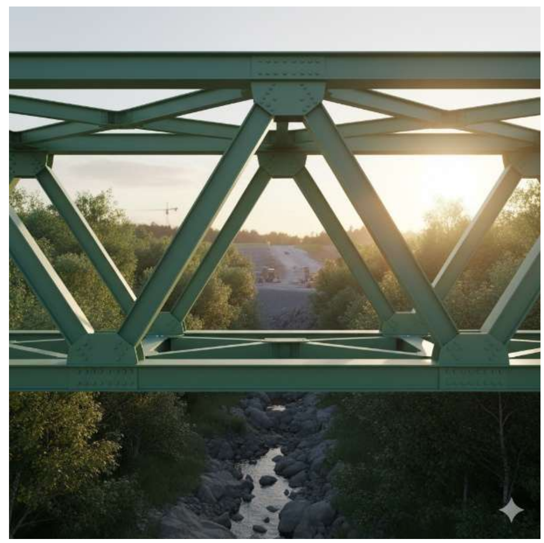
Scalene Triangle

Congruent

Main Topic | **Sides of Triangles**

A structural engineer is designing a **Warren Truss** bridge to span a small ravine. The truss structure relies entirely on triangular units for strength and rigidity. The engineer has fixed the lengths of two adjacent structural members (sides) in one critical triangular section, ΔABC.

- **Member** (Top Chord Segment): 15 meters
- **Member** (Diagonal Member): 8 meters



The engineer now needs to determine the viable range of lengths for the third member, **Member** (Bottom Chord Segment), which will complete the triangular unit.

Any length outside this range would result in a structural impossibility and a non-rigid connection.

TASK 1

The engineer must apply the **Triangle Inequality Theorem**, which states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

The length of the bottom chord segment (Member AC) must be strictly between ____ meters and ___ meters. If the third member were exactly ___ meters or ___ meters, the three members would form a degenerate triangle (a straight line), resulting in a point of collapse rather than a rigid, load-bearing joint. The Triangle Inequality Theorem is a fundamental check for the viability of any triangulated structure in civil engineering.

TASK 2

- Construct ΔABC using a ruler and a compass.
- $\overline{AB}, \overline{BC}$, and $\overline{AC}$ are not congruent.
- Use a ruler to measure each side.
- Label each side with its measurement.
- What type of triangle is it?
- Write your observations about the triangle.

____

____

Solve each problem and show all steps.

The longest side of a triangular panel is twice the shortest side. The middle side is 3 centimeters longer than the shortest side. If the perimeter of the panel is 31 centimeters, what are the lengths of the three sides?	The three sides of a triangular garden plot are consecutive even integers. If the total perimeter of the plot is 42 yards, what are the lengths of the sides?
The side lengths of a triangular roof support are in the ratio 3:4:5. The perimeter of the support is 72 kilometers. Find the length of each side.	An engineer designs an isosceles triangular bracket where the two equal sides are 4 centimeters longer than half the length of the unequal base. If the perimeter of the bracket is 40 centimeters, what are the side lengths?

Need Help?

TASK 3

- Construct ΔABC using a ruler and a compass.
- $\overline{AB} \cong \overline{BC}$
- Use a ruler to measure each side.
- Label each side with its measurement.
- What type of triangle is it?
- Write your observations about the triangle.

____

____

TASK 4

- Construct ΔABC using a ruler and a compass.
- All sides are congruent.
- Use a ruler to measure each side.
- Label each side with its measurement.
- What type of triangle is it?
- Write your observations about the triangle.

____

____

I Do - We Do - You Do

Show complete work to receive full credit.

I Do

Teacher models an example

$\overline{AB} = 2x + 3$, $\overline{BC} = 21 - x$, and $\overline{AC} = 2x$

The perimeter of the triangle is 42. Find the length of each side.

We Do

Students work together

$\overline{AB} = 6x + 8$

Find the value of x if the perimeter of an equilateral triangle is 240.

You Do

Students work independently

$\overline{AB} = 4x + 2$, $\overline{BC} = 5x$, and $\overline{AC} = 6x + 1$

The perimeter of the triangle is 63. Find the length of each side.

Honors Level

The lengths of two sides of a triangle are a and b.
What is the strict mathematical range for the length of the third side, c?

What does it mean for a triangle to be degenerate in the context of its side lengths?

If a triangle has two equal sides, what can be definitively concluded about its interior angles?

In any triangle, why must the side opposite the smallest angle be the shortest side?

If two triangles are similar, $\triangle ABC \sim \triangle DEF$, and the ratio of side AB to side DE is 1:3, what is the ratio of their perimeters?

End-of-Course Prep

2.3 Exterior Angles of Triangles

Warm-Up

Write all you know about the following terms.

Linear pairs

Adjacent Angles

Complementary Angles

Supplementary Angles

Main Topic **Exterior Angles of Triangles**

Exterior Angles Cutouts

Directions:

- Draw a triangle on a separate sheet.
- Extend one endpoint on each side.
- Mark each exterior angle.
- Cut each exterior angle.
- Put together all exterior angles on the dot.

Write your general observations.

____

____

Road Alignment and Transition Curves

A team of civil engineers is designing a new segment of a highway that needs to transition smoothly around a large hill. This requires a sharp initial curve followed by a gentler curve. The key is to calculate the precise angle needed for the transition—known as the **deflection angle** in surveying and road design.

The engineers have set up a baseline and established a critical triangular layout (ΔABC) to model the geometry of the curve:

- **Point A:** The start of the sharp initial curve.
- **Point B:** The point where the initial sharp curve ends.
- **Point C:** The point where the second, gentler curve begins.

The engineers used a **total station** to measure two interior angles within this triangular setup:

- **Interior Angle $\angle CAB$** (Angle at the start of the first curve): $70°$
- **Interior Angle $\angle ACB$** (Angle at the start of the second curve): $45°$

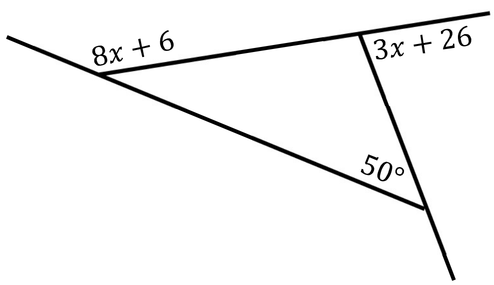
The design requires the engineers to determine the **exterior angle** at Point B, specifically the angle formed by extending the line segment AB past B. This exterior angle, $\angle CBD$, represents the **total deflection angle** required for the road to transition from the initial alignment (AB) to the new segment (BC).

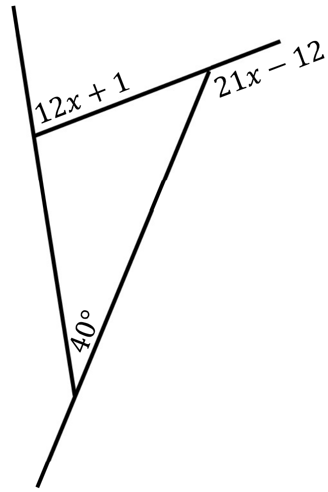
TASK 1

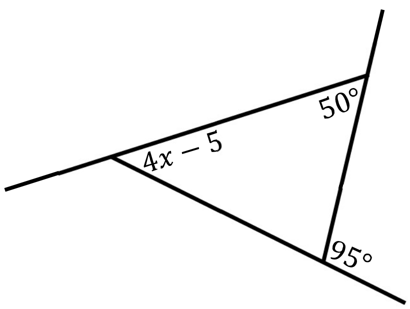
Draw an illustration of the triangle. Find the measure of the exterior angle.

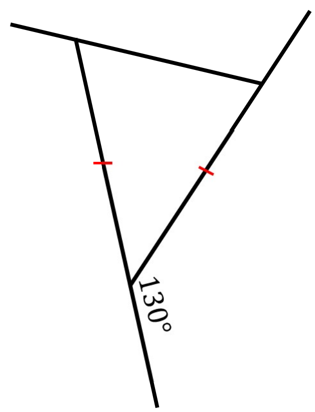
The required **total deflection angle** for the road segment transition at Point B is ______. This calculation is crucial for programming the precise curve geometry into the automated grading equipment, ensuring the road surface transitions smoothly and meets safety standards. Using the Exterior Angle Theorem provides an efficient check on the field measurements and allows for direct calculation of the needed deflection angle.

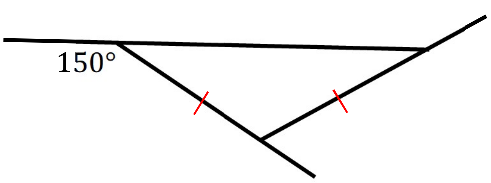
Find the measure of each exterior angle.

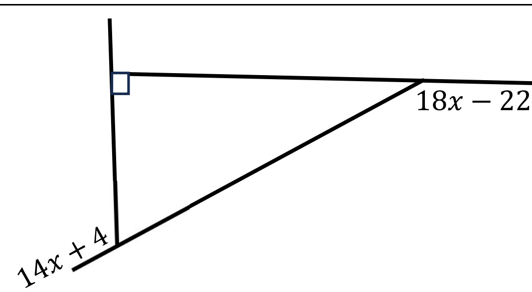








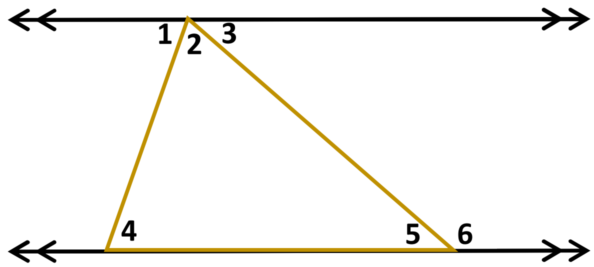




Need Help?

Main Topic | **Exterior Angle Theorem**

TASK 2

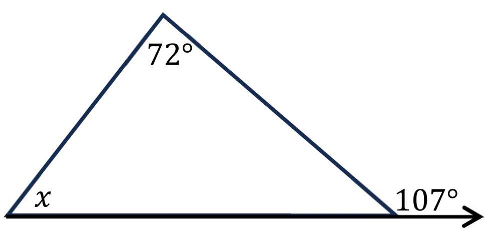


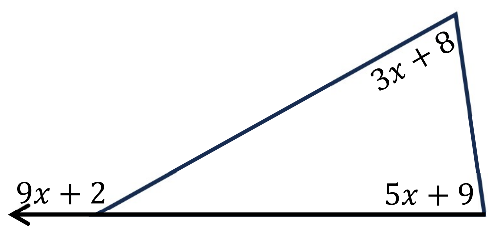
- What is the sum of the measures of $\angle 1, \angle 2, and\ \angle 3$?
- What can you say about $\angle 1\ and\ \angle 4$?
- What can you say about $\angle 3\ and\ \angle 5$?
- What can you say about $\angle 1 + \angle 2\ and\ \angle 6$?
- What can you say about $\angle 4 + \angle 2\ and\ \angle 6$?

Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

Solve for x.





Need Help?

Exterior Angle Theorem Cutouts

Directions:

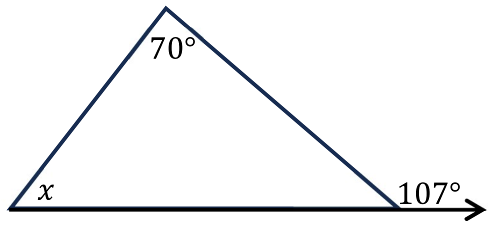
- Cut a triangle and label the interior angles with 1, 2, and 3.
- Trace the triangle on this page.
- Extend the base segment of the traced triangle to the right.
- Tear the remote interior angles off and put them together on the exterior angle.

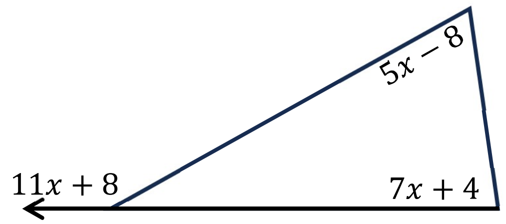
Write your general observations.

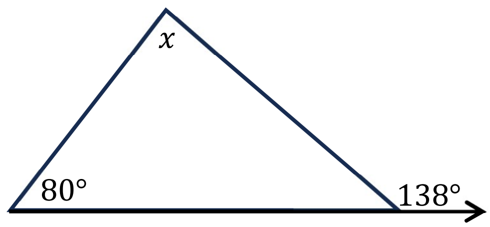
____

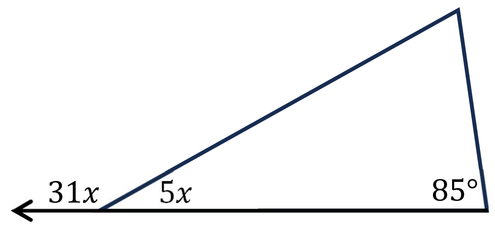
____

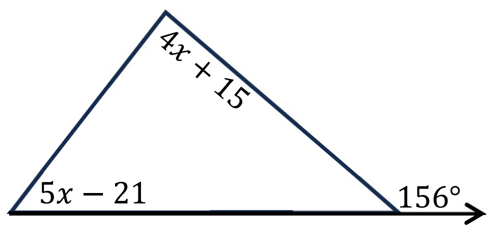
Solve for x.

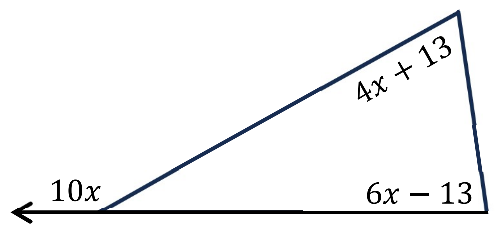












Need Help?

Honors Level

State the Exterior Angle Theorem in terms of its relationship to the interior angles of a triangle.

The exterior angle and its adjacent interior angle form what type of angle pair, and what is their sum?

In an equilateral triangle, what is the measure of every exterior angle?

If the measures of the remote interior angles of a triangle are represented by the expressions $(2x+10)^\circ$ and $(3x-5)^\circ$, and the exterior angle is 125°, how would you set up the equation to solve for x?

Why is it impossible for a triangle to have two right (90°) exterior angles?

End-of-Course Prep

2.4 Isosceles and Equilateral Triangles

Warm-Up

Find the value of x if the measure of each interior angle of an equilateral triangle is 8x + 4.

Main Topic | **Isosceles Triangle**

TASK 1

- Construct an isosceles ΔABC.
- $\overline{AB} \cong \overline{BC}$
- Draw $\overline{BD}$ as the perpendicular bisector of $\overline{AC}$.
- What can you say about $\overline{AD}$ and $\overline{CD}$?
- What can you say about $\overline{BD}$ and $\overline{DB}$?
- What can you say about ΔABD and ΔCBD?

Need Help?

Designing a Cable-Stayed Bridge Pylon

A civil engineering firm is designing a signature **cable-stayed bridge** that will span a major harbor. The bridge's central feature will be its main pylon (tower), which anchors the cables that support the bridge deck. The pylon needs to be not only incredibly strong and stable to resist enormous forces from the cables and environmental loads (wind, seismic activity) but also aesthetically iconic.
The engineer is designing the internal bracing and external form of the pylon.

The key challenges are:

- **Anchoring Cables:** The cables exert massive tension, and the pylon's internal structure must transfer these forces efficiently to the foundation.
- **Wind and Seismic Resistance:** The tall pylon must withstand significant lateral forces.
- **Visual Impact:** The pylon's shape should be elegant and distinctive.

The engineer decides to heavily incorporate **isosceles triangles** into the pylon's design, both for its internal cross-bracing and for shaping its external facade.

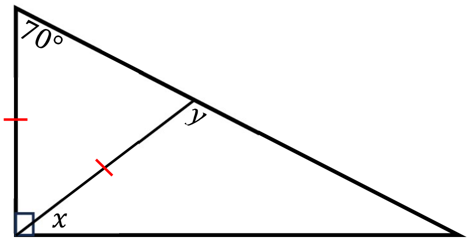
TASK 2

Draw an illustration of a Cable-Stayed Bridge Pylon

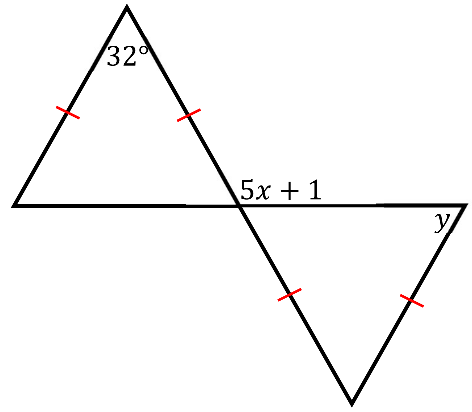
(must utilize a ruler and a compass)

By strategically integrating **isosceles triangles** into both the hidden internal bracing and the visible external form of the cable-stayed bridge pylon, the engineer achieves a design that is:
Structurally robust: Capable of resisting immense forces and ensuring bridge stability.
Highly stable: Providing excellent resistance against lateral loads.
Economically efficient: Through standardized member fabrication.
Iconically aesthetic: Creating a landmark structure that combines strength with visual elegance.

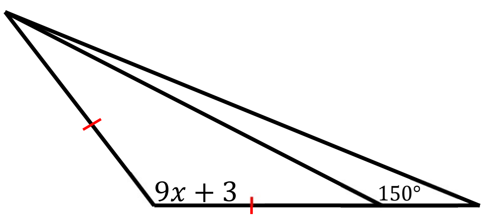
Solve for x and y.



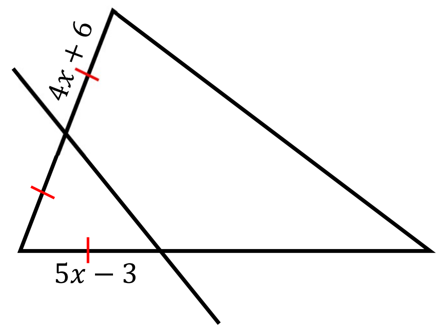
Solve for x and y.



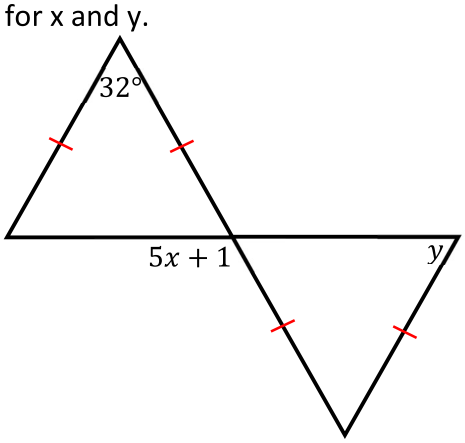
Solve for x.



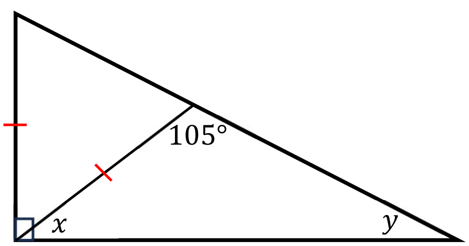
Solve for x.



Sol ve for x and y.



Solve for x and y.



Need Help?

Main Topic | **Equilateral Triangle**

TASK 3

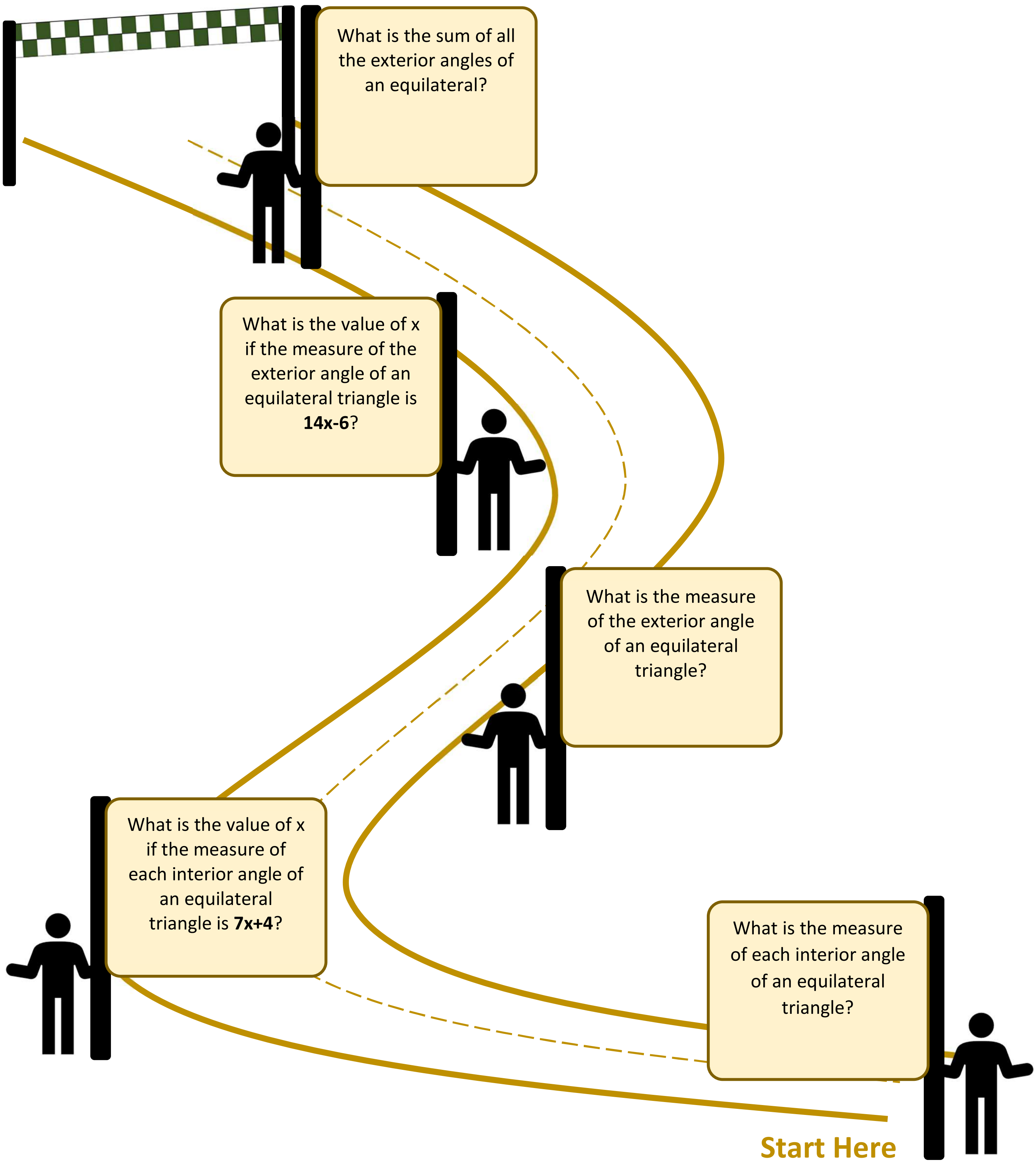
- Construct an equilateral ΔABC.
- Use a protractor to measure each interior angle.
- Compare the interior angles of your constructed triangle with other students.
- What did you discover?

Equilateral Triangle Cutouts

Directions:

- Construct an equilateral triangle on a separate sheet.
- Cut it out.
- Tear the corners off.
- Paste the corners together on the right section of this page.
- Paste the remaining part of the triangle on the left section of this page.

Reaching the Finish Line



Honors Level

An altitude is drawn from the vertex angle to the base of an isosceles triangle. What four specific roles does this line segment simultaneously fulfill?

Can a triangle be both a right triangle and an equilateral triangle? Explain why or why not.

In an isosceles triangle, can the base ever be longer than the sum of the two congruent sides? Explain using a theorem.

How does the total length of the altitudes in an equilateral triangle compare to the total length of the medians?

Is it possible for an isosceles triangle to be an obtuse triangle? If so, where must the obtuse angle be located?

End-of-Course Prep

2.5 Angle, Segment, and Bisector

Warm-Up

Write all you know about the following terms.

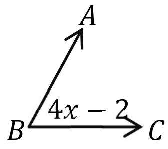
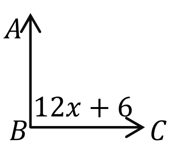
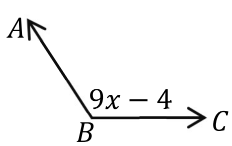
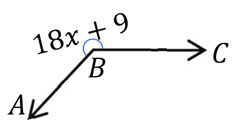
Angle	Segment	Bisector

Main Topic **Angle Construction**

TASK 1

- Construct an acute angle.
- Construct a right angle.
- Construct an obtuse angle.
- Construct a reflex angle.

Solve for x.

$m\angle ABC = 50°$	$\angle ABC$ is a right angle.	$m\angle ABC = 140°$	$m\angle ABC = 225°$
			

Main Topic **Segment Construction**

TASK 2

- Construct $\overline{AB}$.
- Locate the midpoint, M.
- Mark the segments that are congruent.

- Construct $\overline{CM}$.
- If M is the midpoint of $\overline{CD}$, locate endpoint D.
- Mark the segments that are congruent.

P is the midpoint of $\overline{XY}$ $\overline{XY} = 5x + 5$ and $\overline{XP} = x + 7$ What is $m\overline{PY}$?	T is the midpoint of $\overline{SU}$ $\overline{SU} = 3x - 3$ and $\overline{UT} = 15$ What is x?
R is the midpoint of $\overline{QS}$ $\overline{QR} = x + 2$ and $\overline{QS} = 12$ What is x?	M is the midpoint of $\overline{LN}$ $\overline{LN} = 4x - 4$ and $\overline{LM} = x + 5$ What is $m\overline{LM}$?

Need Help?

Main Topic | **Bisector Construction**

TASK 3

- Construct $\angle ABC$.
- Construct angle bisector $\overrightarrow{BD}$.
- Construct $\overline{ST}$.
- Construct perpendicular bisector $\overline{MU}$.

$\overrightarrow{KM}$ is an angle bisector of $\angle JKL$ $\angle JKL = 17x + 2$ and $\angle MKL = 9x - 1$ What is $m\angle JKM$?	$\overline{KI}$ is a perpendicular bisector $\overline{HJ}$ I is the midpoint of $\overline{HJ}$ $\llcorner KIJ = 4x + 1$ What is x?
$\overrightarrow{XZ}$ is an angle bisector of $\angle WXY$ $\angle WXZ = 4x + 6$ and $\angle ZXY = 2x + 28$ What is $m\angle WXY$?	$\overline{QR}$ is a perpendicular bisector $\overline{OP}$ R is the midpoint of $\overline{OP}$ $\overline{OR} = 4x + 4$ and $\overline{RP} = 6x$ What is $m\overline{OP}$?

Need Help?

The Corner Pavilion

A team is designing a **public park pavilion** with a unique, asymmetrical roof structure and a central access ramp. The drafting team must use construction principles to accurately lay out the key elements.

1. Segment and Midpoint (The Ramp Access)

The Problem: The design requires a ramp to provide central access to the pavilion deck. The drafters need to locate the exact midpoint of the main entry segment ($\overline{AB}$) to ensure the ramp is perfectly centered.

- **The Segment:** The entry segment $\overline{AB}$ is 30 feet long, spanning the front edge of the pavilion deck.
- **The Construction:** The drafter uses the compass tool to construct the perpendicular bisector of $\overline{AB}$.
- **The Result:** The point where the bisector intersects is the precise midpoint (M), establishing the center point for the ramp's start, ensuring it splits the entrance equally into two ____-foot segments ($\overline{AM}$ and $\overline{MB}$).

2. Angle and Angle Bisector (The Roof Truss)

The Problem: The architect wants a large structural **roof truss** ($\overline{AC}$) to provide uniform support and tension distribution across a critical corner angle ($\angle BAC$) of the pavilion's asymmetrical roof. The truss must perfectly bisect this angle.

- **The Angle:** The corner angle $\angle BAC$ is the space formed by two roof edge segments, $\overline{AB}$ and $\overline{AC}$, and measures $110°$ (an obtuse angle).
- **The Construction:** The drafter uses the compass tool to draw an arc intersecting both $\overline{AB}$ and $\overline{AC}$, then constructs the angle bisector ($\overrightarrow{AD}$).
- **The Result:** The resulting truss line ($\overline{AD}$) divides the angle into two equal, uniform angles of ____ each ($\angle BAD$ and $\angle CAD$). This geometric construction ensures the load is distributed evenly, fulfilling the structural requirement for uniform tension.

3. Perpendicular Bisector Theorem (Locating a Column)

The Problem: The project requires a central foundation column (K) to support the roof truss ($\overline{AD}$). To maintain aesthetic symmetry, the column must be placed such that it is exactly equidistant from the two main corner supports, A and B.

- **The Segments:** The drafter is dealing with the distance from column K to the two endpoints of the entry segment, A and B (the required distances KA and KB).
- **The Construction:** The drafter refers to the previously drawn perpendicular bisector of $\overline{AB}$.
- **The Result:** Applying the **Converse of the Perpendicular Bisector Theorem**, the drafter knows that any point on the perpendicular bisector is equidistant from A and B. The drafter selects a specific point (K) along that bisector that aligns with the structural grid. This placement guarantees $KA = KB$, satisfying the equidistant aesthetic requirement.

The Corner Pavilion

Materials: Ruler, Compass, Protractor, and Pencil

Fill in the blank

An ____________ is a geometric figure that consists of two rays that share a common endpoint. An __________ angle that measures less than $90°$ but more than $0°$ while an angle that measures more than $90°$ but less than $180°$ is called an ___________ angle. A ________ angle measures exactly $90°$ while a __________ angle measures more than $180°$ but less than $270°$. A ________ that cuts an angle into two equal parts is called a ______________.

A ____________ is a straight line with two endpoints. The point that is in the middle between two endpoints is a ______________. A __________________________ divides a segment into two equal parts passing its midpoint while forming angles that measure ________ degrees.

Construction Steps

List the construction steps for the following figures.

Angle Bisector	Perpendicular Bisector

Honors Level

What is the fundamental distinction between a geometric line and a line segment in terms of measurement and extent?

How does the Segment Addition Postulate conceptually relate the midpoint of a segment to the segment's total length?

Can a pair of vertical angles be complementary?
If so, what must be true about the measure of each angle?

What is the essential difference in the geometric result achieved by an angle bisector versus a segment bisector?

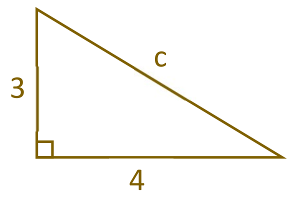
If the three angle bisectors of a triangle are constructed, what is the special name for their point of concurrency, and what does it represent geometrically?

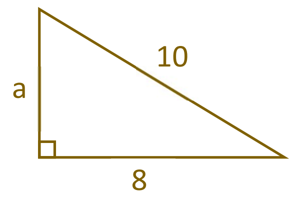
End-of-Course Prep

2.6 Triangle Construction

Warm-Up

Find the length of the missing side.





Main Topic | **Triangle Construction**

TASK 1

- Construct an isosceles triangle.

- List the steps.

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- Describe an isosceles triangle.

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Need Help?

TASK 2

- Construct a right triangle.

- List the steps.

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- Describe a right triangle.

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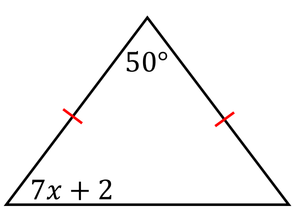
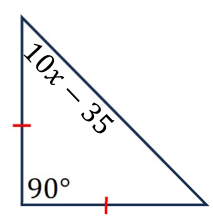
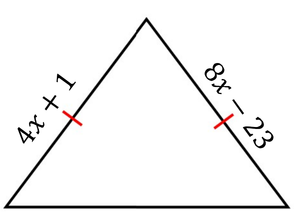
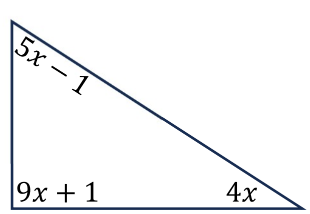
Solve for x.









Need Help?

TASK 3

- Construct an equilateral triangle.

- List the steps.

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- Describe an equilateral triangle.

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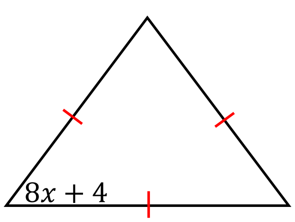
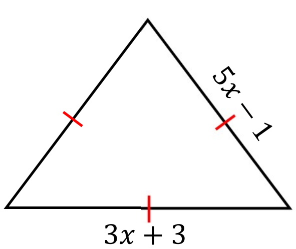
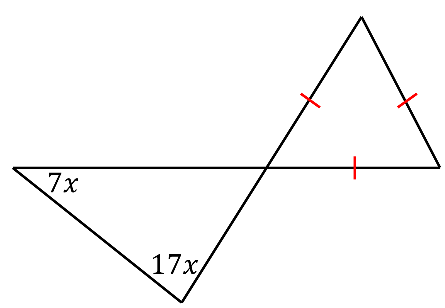
Solve for x.



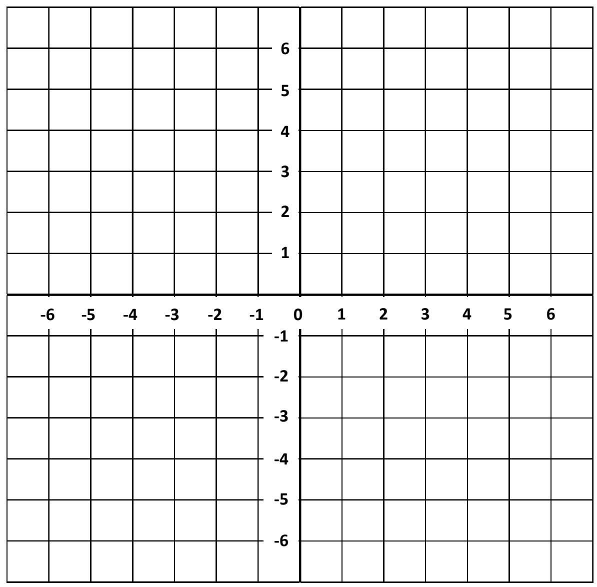




Need Help?

TASK 4

- Plot points A(-4, 3), B(-2, -5), C(4, 1).
- Connect and label the points.
- Determine if the triangle is isosceles, right, equilateral, or scalene. Prove algebraically.



The Green Community Center

A drafting team is working on the detailed plans for a new community center designed with various roof lines and structural elements to maximize natural light and rainwater collection.

<u>Right Triangle (Foundation and Stability)</u>

The Component: The corner foundation footing for the building's main support column (C) must be perfectly checked for a 90° angle.

- **The Construction:** The drafter uses a method based on the **Pythagorean Theorem** and the construction of a **right triangle**. They measure a segment 3 feet out from the corner along one foundation line ($\overline{CA}$) and 4 feet out along the perpendicular line ($\overline{CB}$).

- **The Result:** They then measure the hypotenuse ($\overline{AB}$). If the corner angle ($\angle ACB$) is a true a 90° right angle, the hypotenuse must measure exactly 5 feet ($3^2 + 4^2 = 9 + 16 = 25$, and $\sqrt{25} = 5$). This check is critical for establishing the squareness and stability of the entire foundation grid.

<u>Equilateral Triangle (Acoustic Ceiling Panels)</u>

The Component: The interior meeting room requires a pattern of decorative, sound-dampening panels that are all **equilateral triangles** to ensure uniform acoustic diffusion.

- **The Construction:** The architect specified that each side of the panel should measure 1.5 meters. The drafter uses the compass tool to construct the first side, $\overline{DE}$, at 1.5 m. They then set the compass to 1.5 m, place the point at D and draw an arc, then place the point at E and draw a second intersecting arc.

- **The Result:** The intersection point (F) is connected to D and E, creating ΔDEF, where all three sides ($\overline{DE}$, $\overline{EF}$, $\overline{DF}$) and all three interior angles are equal (60°). This ensures every panel in the schedule has the identical structural and acoustic properties.

Isosceles Triangle (Roof Gables)

The Component: The front entrance features a classic **isosceles triangular gable** (ΔGHI) whose apex (H) is centered over the entrance width ($\overline{GI}$).

- **The Construction:** The drafter first locates the **midpoint** (M) of the entrance segment ($\overline{GI}$) and constructs the perpendicular line upward from M to define the height. The height is set based on aesthetic rules (e.g., ¼ the width). By selecting any point H on this perpendicular line and connecting it to G and I, the drafter creates an isosceles triangle.

- **The Result:** By construction, the two roof segments ($\overline{GH}$ and $\overline{IH}$) are guaranteed to be equal in length. This is an application of the **Perpendicular Bisector Theorem**, which guarantees that any point on the bisector (M's line) is equidistant from the endpoints (G and I). This achieves a perfectly symmetrical, centered gable design.

Scalene Triangle (Rainwater Funnel)

The Component: A specialized roof section designed as a rainwater collector requires an unusually shaped, triangular funnel frame (ΔJKL) where no two sides or angles can be equal to properly direct water to a specific collection cistern.

- **The Construction:** The drafter plots three points (J, K, and L) based on the surrounding drainage pipes, ensuring that the distance $\overline{JK}$, $\overline{KL}$, and $\overline{JL}$ are all different lengths (e.g., 2m, 3m, and 4m).

- **The Result:** The resulting frame is a **scalene triangle**. This irregular shape ensures that the three internal angles ($\angle J, \angle K, \angle L$) are also all different, creating the specific angled slopes needed to guide the water path efficiently and uniquely toward the single target cistern.

The Green Community Center

Materials: Ruler, Compass, Protractor, and Pencil

Honors Level

What is the fundamental principle (or theorem) that governs whether or not three given side lengths can form a valid triangle?

Explain why Side-Side-Angle (SSA) is *not* a valid congruence criterion and often leads to the "ambiguous case" in construction.

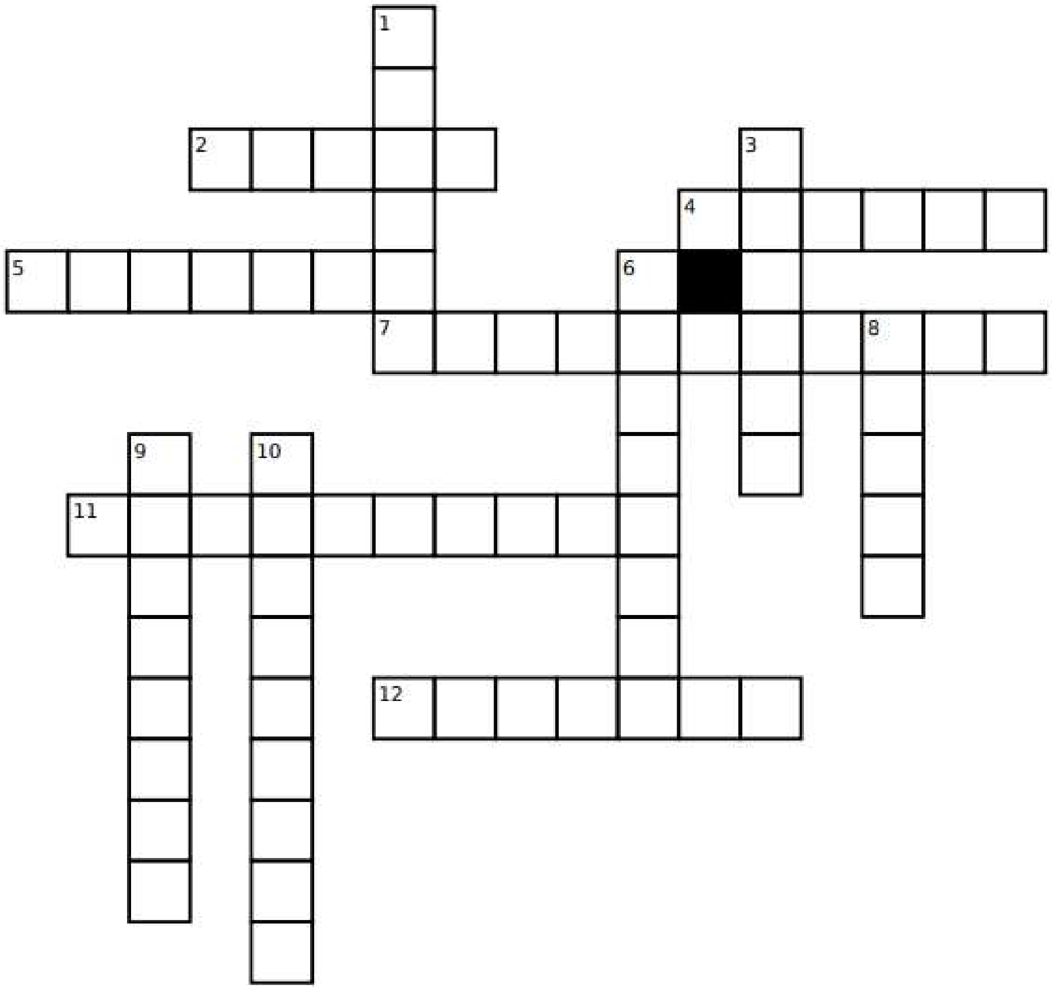
When constructing an angle bisector within a triangle, why is it essential that the compass setting remain constant for the second set of arcs?

When constructing a perpendicular bisector to find the circumcenter, why must the compass opening be set to *more than half* the length of the segment?

If an isosceles triangle is constructed, what relationship exists between the altitude drawn to the base and the angle bisector of the vertex angle?

End-of-Course Prep

Triangles and Construction



Across	Down
2. A triangle where all three angles are less than 90°. 4. A segment connecting a vertex to the midpoint of the opposite side. 5. A technical drawing instrument used for drawing circles and marking off distances. 7. A triangle where all three sides are equal. 11. A segment connecting the midpoints of two sides. 12. A triangle where all three sides have different lengths.	1. A triangle with one angle greater than 90°. 3. The point where the sides of a polygon or the edges of a polyhedron meet. 6. A perpendicular segment from a vertex to the line containing the opposite side (the "height"). 8. A triangle containing one 90° angle. 9. An angle ________ is a ray or segment that divides an angle into two equal parts. 10. A triangle with at two congruent sides.

Unit 3 | Parallel Lines, Transversal, and Triangular Proof

Architectural Drafting

High School Mathematics 2

North Carolina Math 2 Standards

Unit 3

Congruence *Prove geometric theorems.*	
NC.M2.G-CO.7	Use the properties of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
NC.M2.G-CO.8	• Use congruence in terms of rigid motion. • Justify the ASA, SAS, and SSS criteria for triangle congruence. Use criteria for triangle congruence (ASA,SAS,SSS,HL) to determine whether two triangles are congruent.
NC.M2.G-CO.9	Prove theorems about lines and angles and use them to prove relationships in geometric figures including: • Vertical angles are congruent. • When a transversal crosses parallel lines, alternate interior angles are congruent. • When a transversal crosses parallel lines, corresponding angles are congruent. • Points are on a perpendicular bisector of a line segment if and only if they are equidistant from the endpoints of the segment. • Use congruent triangles to justify why the bisector of an angle is equidistant from the sides of the angle.
NC.M2.G-CO.10	Prove theorems about triangles and use them to prove relationships in geometric figures including: • The sum of the measures of the interior angles of a triangle is 180º. • An exterior angle of a triangle is equal to the sum of its remote interior angles. • The base angles of an isosceles triangle are congruent. • The segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length.

3.1 Parallel Lines

Warm-Up

How do you determine if the lines are parallel?

Main Topic | **Parallel Lines Construction**

TASK 1

- Construct parallel lines.
- List the steps.

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- Define parallel lines.

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Need Help?

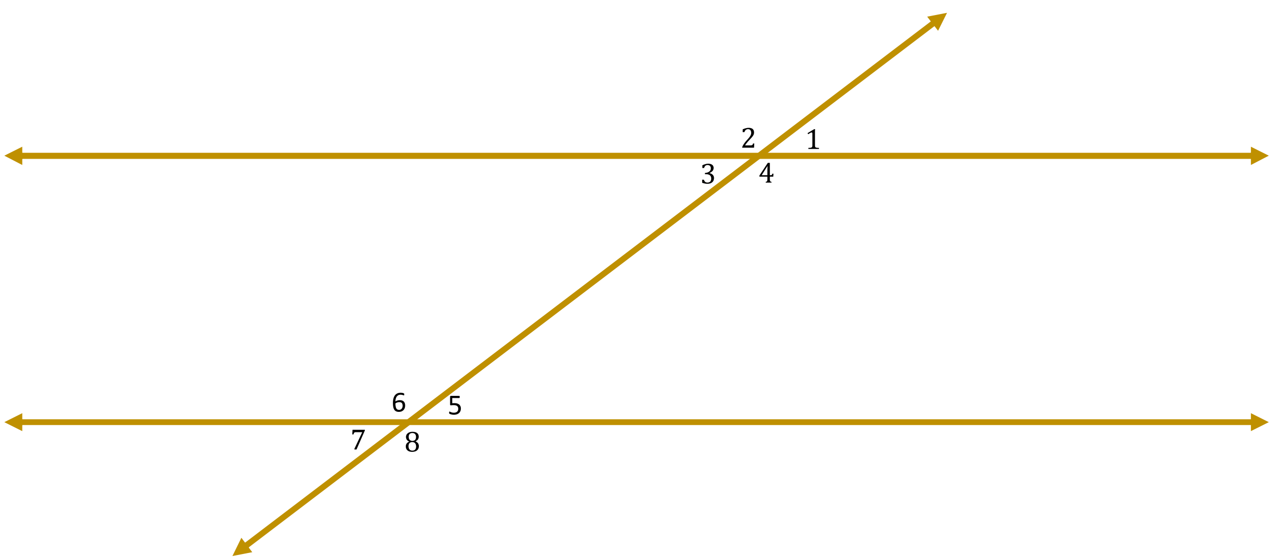
Parallel Lines Cutouts

Materials

Scissors Construction Paper Protractor Ruler

Directions

- Trace angle 1 using construction paper then cut it out.
- Trace angle 2 using construction paper then cut it out.
- Use the cutouts and start identifying congruent angles.



State if the pairs of angles are congruent or linear (supplementary).

Corresponding Angles	
$m\angle 1$ and $m\angle 5$	
$m\angle 2$ and $m\angle 6$	
$m\angle 4$ and $m\angle 8$	
$m\angle 3$ and $m\angle 7$	

Adjacent Angles	
$m\angle 1$ and $m\angle 2$	
$m\angle 1$ and $m\angle 4$	
$m\angle 3$ and $m\angle 4$	
$m\angle 2$ and $m\angle 3$	

Describe corresponding angles.

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Describe adjacent angles.

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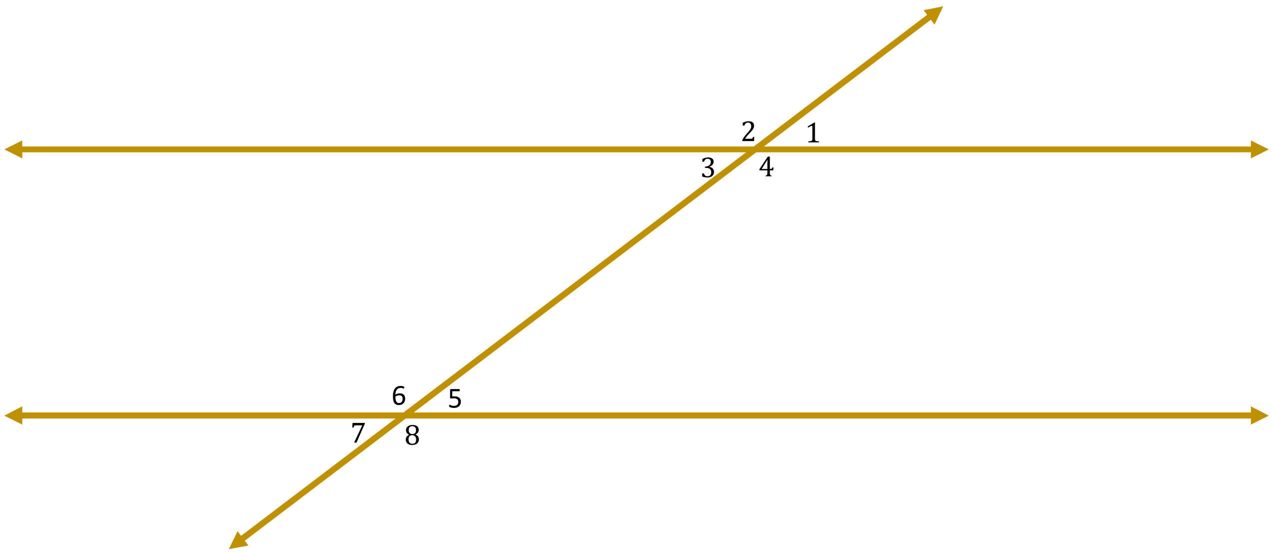

State if the pairs of angles are congruent or linear (supplementary).

Alternate Interior Angles	
$m\angle 3\ and\ m\angle 5$	
$m\angle 4\ and\ m\angle 6$	

Alternate Exterior Angles	
$m\angle 1\ and\ m\angle 7$	
$m\angle 2\ and\ m\angle 8$	

Describe alternate interior angles.

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Describe alternate exterior angles.

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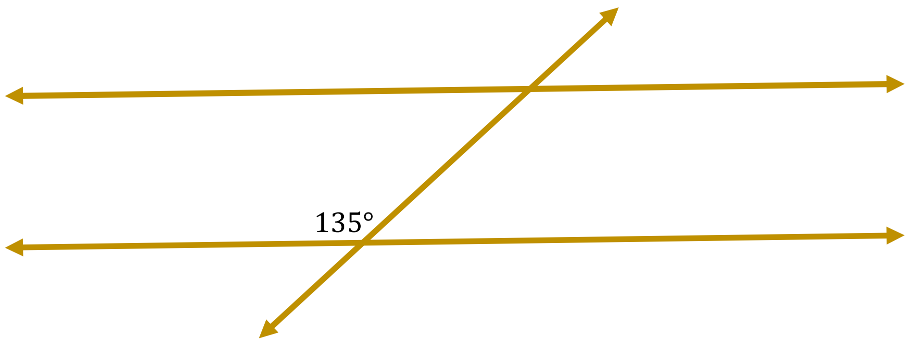
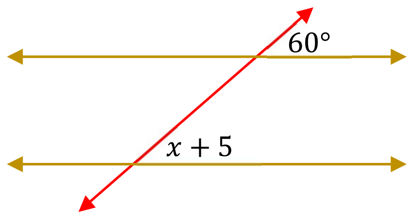
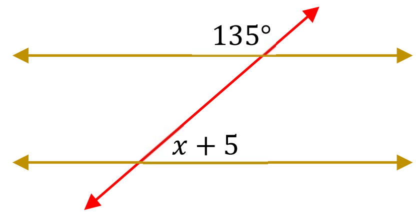
Find the measure of the remaining angles.

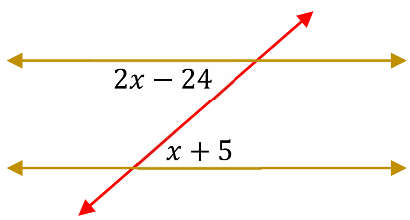


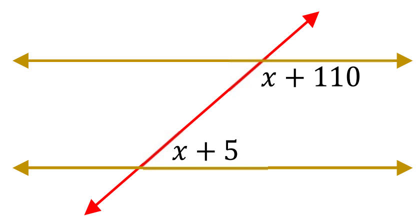
Main Topic **Solving Parallel Lines**

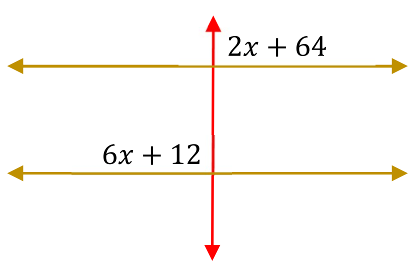
Solve for x.

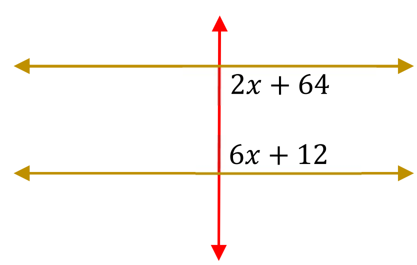










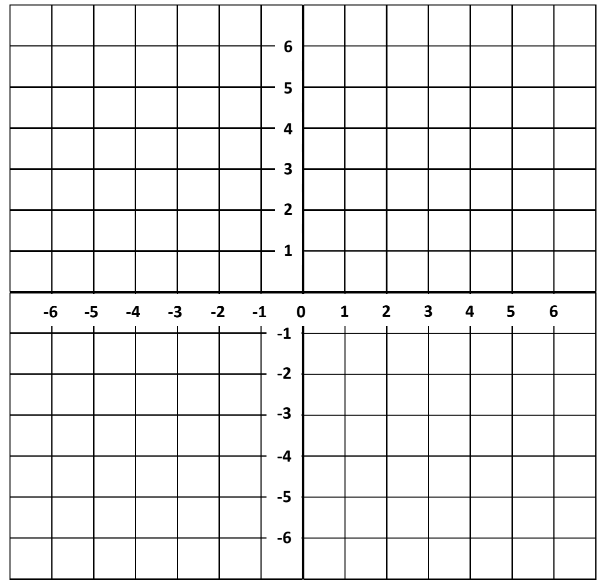


Need Help?

Main Topic | Parallel Lines on Cartesian Plane

TASK 2

- Draw a line that passes through A(-2, 2) and B(2, 4) and find its equation.



- Draw a line that passes through C(-3, -1) and D(1, 1) and find its equation.

- What do you notice about the lines?

- Draw the 3rd line, $2x + y = 3$.

- What do you notice about the 3rd line compared to the previous lines?

Fill in the blank

Two lines on Cartesian plane are ______________________ if they have the ________ slopes. If the slopes of the lines are ________________________, then they are perpendicular to each other.

Two lines on the same plane are considered ______________________ if they don't cross each other.

When a ______________________ line cuts parallel lines, it forms pairs of congruent angles such as __________________________ , __________________________, and __________________________.

The ___________________ are pairs of angles that are supplement to each other.

Supplementary angles are pairs of angles with a sum of __________ degrees.

Need Help?

Honors Level

What is the defining characteristic of two parallel lines on a plane, and why must they be equidistant?

Describe the logical relationship between *Alternate Interior Angles* and *Corresponding Angles* when two parallel lines are cut by a transversal.

In architectural drafting, why is the principle of parallel lines cut by a transversal critical for designing a staircase?

If you shift a parallel line 5 units up and 2 units right on the coordinate plane, how does the slope of the new line compare to the original line?

How can you use the concept of a transversal to prove that the opposite sides of a rectangle defined by coordinates are parallel?

End-of-Course Prep

3.2 Angle Bisector Theorem

Warm-Up

The measure of $\angle A\ is\ 80°$. If $\angle A$ is cut in half and the other half measures $5x + 5$, what is the value of x?

Main Topic **Angle Bisector Theorem**

TASK 1

- Use a ruler to construct a triangle on a separate sheet.
- Cut it out.
- Pick an angle and divide it equally by folding.
- Use a pencil to trace the crease.
- Paste the cutout on this page.
- Label the vertices with ABC, B has the crease.
- Label the other endpoint of a crease with D.
- Use the *cm* side of the ruler to determine the length of each segment.

$\overline{AB} =$ $\overline{BC} =$ $\overline{AD} =$ $\overline{CD} =$

- Find the ratios of the sides.

$\frac{\overline{AB}}{\overline{BC}} =$ $\frac{\overline{AD}}{\overline{CD}} =$

Write your observations between the ratios.

____

Compare your observations with other students.

____

Placing a Water Main Access Point

A team of civil engineers is designing a new residential subdivision. They need to install a central maintenance access point (manhole) for a new water main system that feeds two distinct service lines to different parts of the subdivision.

The main water line runs along a street intersection, forming a corner. The two service lines (AB and AC) branch out from this corner (A) at different angles and terminate at two existing connection points, B and C. The engineers have determined the lengths of the two service line segments are:

Service Line AB: 100 meters

Service Line AC: 150 meters

The maintenance crew requires the new access point, D, to be placed along the boundary line (BC) connecting the two existing points such that the main water pipe (AD) perfectly bisects the corner angle (∠BAC). This bisecting line provides the most uniform flow and pressure distribution into both service lines.

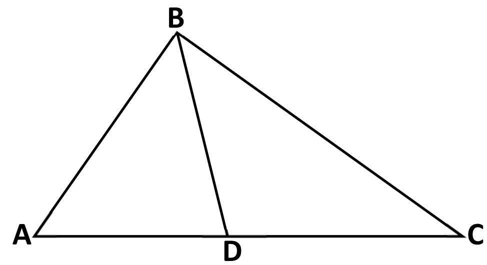
TASK 2

To determine the exact location of the access point D along the boundary line , the engineers apply the **Angle Bisector Theorem**. Use a ruler and a compass to locate the access point D.

The **Angle Bisector Theorem** is critical for placing the utility access point at the only location that ensures the new main pipe perfectly bisects the corner angle. This geometric principle is essential for hydraulic efficiency and for accurately staking out the required maintenance location on the ground.

TASK 3

- Given: $\overline{BD}$ is an angle bisector of $\angle ABC$.
- Prove: $\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{AD}}{\overline{DC}}$



Statement	Reason

Angle Bisector

A line or ray that divides an angle into two congruent parts.

Math 1 Review

Perform the operations.

$x(x+6)$ $(x-4)(3x+6)$

Two-Step Equation Review

Solve for x algebraically.

$2x - 1 = 5$	$2x - 2 = 5$
$2 - 3x = 7$	$3x + 1 = 13$

Speed Math

Answer as many questions as you can in 2 minutes.

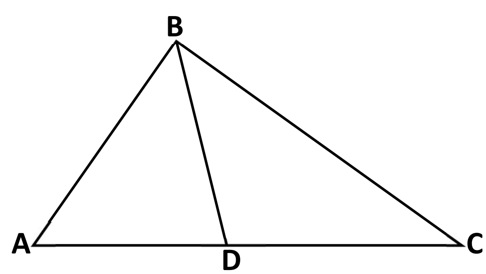
$2x - 1 = 9$	$2x + 1 = 9$	$2x - 1 = -9$	$-2x - 1 = 9$
$2x - x = 9$	$2x + x = 9$	$-2x + x = 9$	$-2x - x = 9$
$\frac{x}{2} = 9$	$\frac{x}{3} = 9$	$\frac{x}{2} = -9$	$-\frac{x}{2} = 9$

Given: $\overline{AB} = 20$, $\overline{BC} = 28$, $\overline{AD} = 2x + 3$, $\overline{DC} = 5x - 9$

Solve for x.

Given: $\overline{AB} = 2x + 1$, $\overline{BC} = 12$, $\overline{AD} = x + 2$, $\overline{DC} = 8$

Solve for x.



Given: $\overline{AB} = 15$, $\overline{BC} = 6x - 5$, $\overline{AD} = 12$, $\overline{DC} = 4x$

Solve for x.

Given: $\overline{AB} = 3x + 2$, $\overline{BC} = 4x + 8$, $\overline{AD} = 24$, $\overline{DC} = 36$

Solve for x.

Need Help?

TASK 4

- Given: $\overrightarrow{BD}$ is an angle bisector of $\angle ABC$.
- Point P is on $\overrightarrow{BD}$
- Construct a segment perpendicular to $\overrightarrow{BA}$ with endpoints P and F
- Construct a segment perpendicular to $\overrightarrow{BC}$ with endpoints P and G
- Prove: $\overline{PF} \cong \overline{PG}$

Statement	Reason

Angle Bisector Theorem

Write all you know about Angle Bisector Theorem

____

____

____

____

Honors Level

State the Angle Bisector Theorem in terms of segment and side ratios within a triangle.

The Converse of the Angle Bisector Theorem allows for a conclusion about an angle.
What is that conclusion?

What must be true about the distances from any point on an angle bisector to the two sides of the angle?

If an angle bisector is also the altitude of a triangle,
what can be concluded about the triangle's classification?

How does the Angle Bisector Theorem relate to the concept of similar triangles,
even though it doesn't involve them directly?

End-of-Course Prep

3.3 Perpendicular Bisector

Warm-Up

Point M is a midpoint of $\overline{AB}$. If $\overline{AM} = 2x + 2$ and $\overline{AB} = 80$, what is the value of x?

Main Topic | **Perpendicular Bisector**

A rural development project involves constructing a new community well to serve two existing isolated farmsteads, Farm A and Farm B, and a future development plot. The primary requirement from the community is that the well be located such that it is **equidistant from Farm A and Farm B**, ensuring fair and equal access for water distribution pipelines and maintenance.

- **Farm A** is located at coordinates .
- **Farm B** is located at coordinates .

TASK 1

The civil engineers need to identify all possible locations for the well that satisfy the "equidistant" requirement from these two points. Use a ruler and a compass to identify all possible locations.

The **Perpendicular Bisector Theorem** provides a precise and unambiguous method for defining all possible locations for the community well that are equidistant from Farm A and Farm B. This ensures fairness in access and simplifies the initial planning phase, allowing other practical considerations to narrow down the final choice along this geometrically defined line.

TASK 2

- Construct $\overline{AB}$.
- Locate the midpoint, M.
- Construct the perpendicular bisector $\overline{CD}$.
- $\overline{AM} = 2x - 2$ and $\overline{MB} = 10 - x$

- What is $m\overline{AM}$?
- What is $m\overline{MB}$?
- What is $m\overline{AB}$?

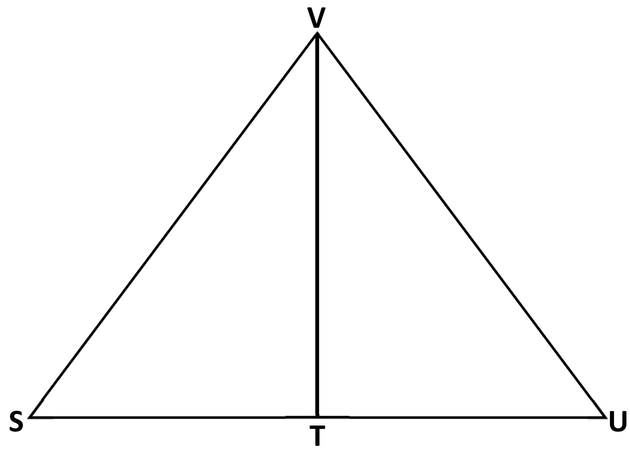
Perpendicular Bisector

A line intersecting a segment at a right angle and passing the midpoint of the segment.

Solve each problem and show all steps.

State the key property of any point that lies on the perpendicular bisector of a line segment AB.	Line $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$. If $AC = 5x - 3$ and $BC = 2x + 9$, what is the length of $\overline{AC}$?
Line l is the perpendicular bisector of segment $\overline{DE}$. If a point P on line l is such that the distance $PD = 5x - 8$ and $PE = 3x + 10$, what is the length of $\overline{PD}$.?	Segment $\overline{GH}$ is 16 cm long, and M is its midpoint. Line l is the perpendicular bisector of $\overline{GH}$. A point P on line l is 17 cm away from point G. How far is point P from the midpoint M?

Need Help?



Given: $\overline{VT}$ is a perpendicular bisector of $\overline{SU}$
Prove: $\Delta STV \cong \Delta UTV$

Statement	Reason

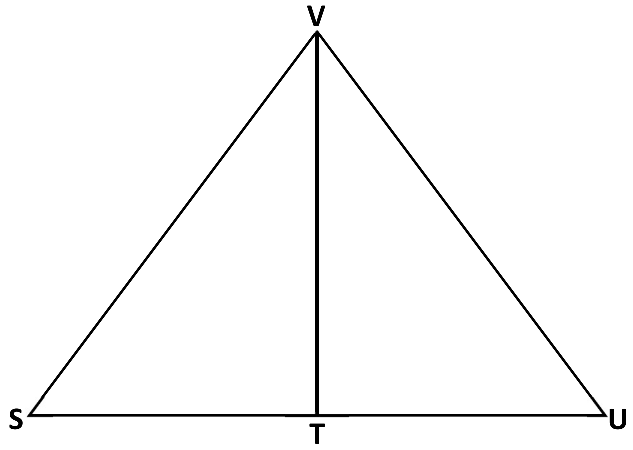
Given: $\overline{ST} = x + 1, \overline{UT} = 11 - 4x, \overline{VU} = 3x - 1$

Find $m\overline{ST}$.

Find $m\overline{SU}$.

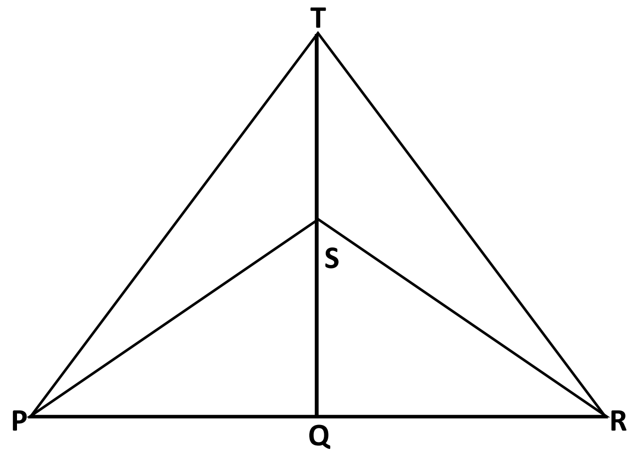
Find $m\overline{SV}$.

Find the perimeter of ΔSUV.



Given: $\overline{VT}$ is a perpendicular bisector of $\overline{SU}$
Prove: $\overline{SV} \cong \overline{UV}$

Statement	Reason



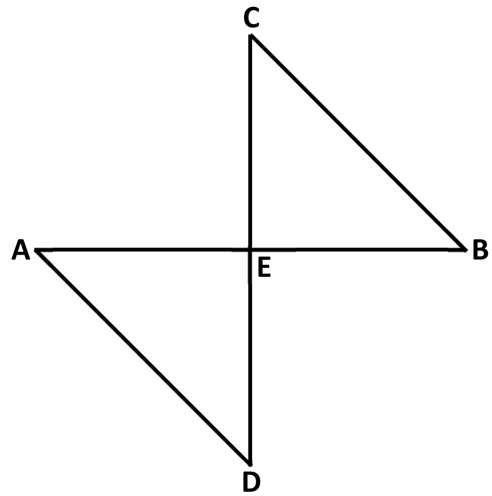
Given: $\overline{TQ}$ is a perpendicular bisector of $\overline{PR}$
Prove: $\Delta PST \cong \Delta RST$

Statement	Reason

Given: $\overline{PS} = 8$, $\overline{RS} = 5x - 7$, $\overline{RT} = 4x + 3$, $\overline{ST} = 6$

Find $m\overline{PT}$.

Find the perimeter of ΔRST.



Given: $\overline{AB}$ and $\overline{CD}$ are perpendicular bisector of each other
Prove: $\overline{AD} \cong \overline{BC}$

Statement	Reason

Given: $\angle A = 5x + 2$ and $\angle C = 48°$

Solve for x.

Honors Level

What is the defining geometric property of *all* points that lie on a perpendicular bisector, excluding the midpoint?

Can a perpendicular bisector be a ray or a segment?
Explain why or why not in terms of the theorem.

If a point P is closer to endpoint A than it is to endpoint B of segment AB, what can you definitively conclude about the location of point P relative to the perpendicular bisector of AB?

How does the Perpendicular Bisector Theorem differ conceptually from the Angle Bisector Theorem?

If a triangle is formed by connecting a point P on the perpendicular bisector of AB to the endpoints A and B, what geometric property must the resulting $\triangle PAB$ always possess?

End-of-Course Prep

3.4 Triangle Congruence

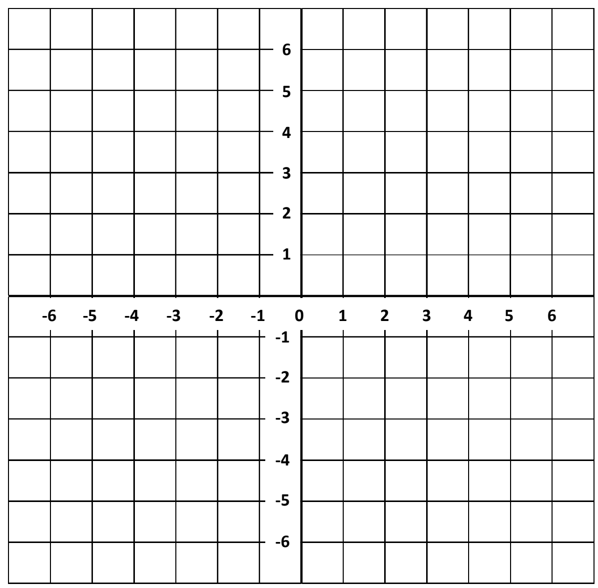
Warm-Up

Differentiate **similar** triangles and **congruent** triangles.

Main Topic | **Side-Side-Side Congruence (SSS)**

TASK 1

- Construct ΔABC with vertices at $A(-4, 1), B(-2, 4), C(0, 1)$
- Construct ΔDEF with vertices at $D(1, 0), E(4, -2), F(1, -4)$
- Are the triangles congruent? Prove it.

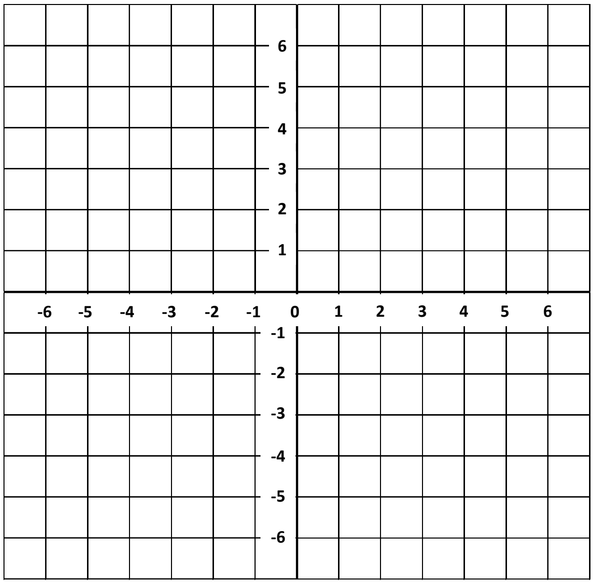


Need Help?

Main Topic | **Side-Angle-Side Congruence (SAS)**

TASK 2

- Construct a quadrilateral with vertices at $A(-2,3), B(1,4), C(4,3), D(1,-3)$
- Construct $\overline{BD}$.
- What do you call $\overline{BD}$ in relation to $\angle ABC$?
- Are the sides $\overline{AB}$ and $\overline{BC}$ congruent? Explain.
- Are the angles $\angle ABD$ and $\angle CBD$ congruent? Explain.
- Are the segments $\overline{BD}$ and $\overline{BD}$ congruent? Explain.

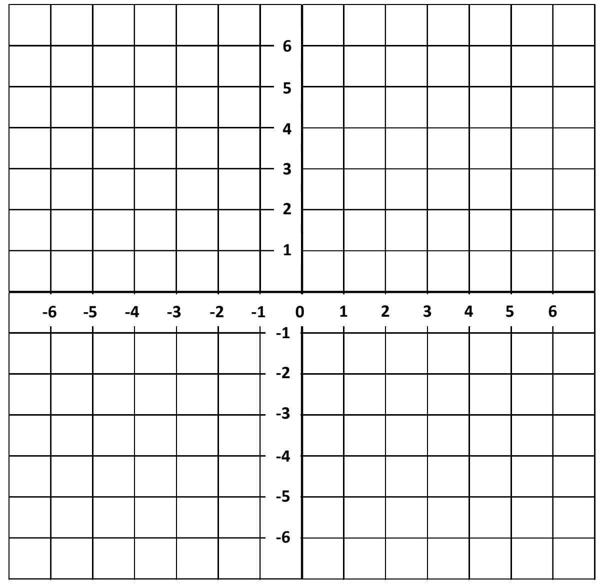


- Redraw ΔABD and ΔCBD.
- Label the corresponding parts of the triangles.
- Are the triangles ΔABD and ΔCBD congruent? Explain.

Main Topic | **Angle-Side-Angle Congruence (ASA)**

TASK 3

- Construct a quadrilateral with vertices at
  $$A(-2,-3), B(-2,4),$$
  $$C(3,2), D(3,-5)$$



- Construct $\overline{AC}$.

- Are the angles $\angle BAC$ and $\angle DCA$ congruent? Explain.

- Redraw ΔBAC and ΔDCA.
- Label the corresponding parts of the triangles.

- Are the segments $\overline{AC}$ and $\overline{AC}$ congruent? Explain.

- Are the angles $\angle BCA$ and $\angle DAC$ congruent? Explain.

- Are the triangles ΔBAC and ΔDCA congruent? Explain.

Standardizing Modular Truss Components

A team is designing a series of modular, prefabricated triangular steel trusses for the roof of a large civic gymnasium. To ensure every truss fits perfectly into the structural grid and minimizes waste during fabrication, the drafting team must prove that all trusses of a certain type are **congruent**.

The Congruence Challenge

The design calls for 15 identical **Side Truss Modules (Type A)** to span the length of the building. Each truss must be built to exacting standards to guarantee uniform load distribution. The drafters use different congruence theorems to verify the geometric specifications on the final blueprints.

<u>Side-Side-Side (SSS) for Fabrication Proof</u>

The fabrication shop needs the simplest method to ensure the finished product matches the drawing.

- **The Blueprint Specification:** The drafters specify the precise lengths of the three steel beams that form the exterior of the truss:
    - Top Chord ($\overline{AB}$): 24 feet
    - Bottom Chord ($\overline{BC}$): 16 feet
    - End Rafter ($\overline{AC}$): 18 feet

- **The Application:** The drawing explicitly lists these three dimensions. By relying on the **SSS Congruence Postulate**, the drafters guarantee that any truss fabricated using exactly these three lengths will be **congruent** to the design standard. The fabricators only need to measure the three sides to confirm the truss is correctly built.

Side-Angle-Side (SAS) for Installation Fixture Design

The installation crew needs a rigid fixture to temporarily hold the trusses in place during erection. The fixture must perfectly match the corner joint between the top chord and the bottom chord.

- **The Blueprint Specification:** The drafters detail the joint by specifying:
  - Length of Top Chord segment adjacent to the joint ($\overline{AD}$): 8 feet
  - Length of Bottom Chord segment adjacent to the joint ($\overline{BD}$): 6 feet
  - The **Included Angle** at the connection point ($\angle ADB$): $115°$

- **The Application:** By using the **SAS Congruence Postulate**, the drafters ensure the geometric profile of this specific joint area (ΔADB) is unique and can be replicated exactly for the installation fixture. This prevents rotational error when securing the truss to the main supports.

<u>Angle-Side-Angle (ASA) for Field Verification</u>

The site manager needs a quick way to verify that a truss fabricated off-site has the correct shape and angle, using only a protractor and a tape measure on one segment.

- **The Blueprint Specification:** The drafters include key angular dimensions and the length of a single common segment:
    - Angle at the Base ($\angle CAB$): 35°
    - Angle at the Apex ($\angle CBA$):$110°$
    - The **Included Side** (span of the truss, $\overline{AB}$): 20 feet

- **The Application:** Based on the **ASA Congruence Postulate**, if the site crew measures the 20-foot span and confirms the $35°$ and $110°$ angles at either end, they have mathematically proven that the entire truss is **congruent** to the design without having to measure all internal members or the third angle.

The Result: Uniformity and Efficiency

By applying these congruence theorems and clearly detailing the required measurements on the drawings, the architectural drafters achieve **complete standardization**. This ensures that all 15 trusses are dimensionally identical, simplifying fabrication, minimizing material waste, and guaranteeing structural uniformity across the roof.

Honors Level

What is the fundamental difference between proving two triangles are *congruent* versus proving they are *similar*?

What is the significance of the acronym CPCTC (Corresponding Parts of Congruent Triangles are Congruent) in a two-column proof?

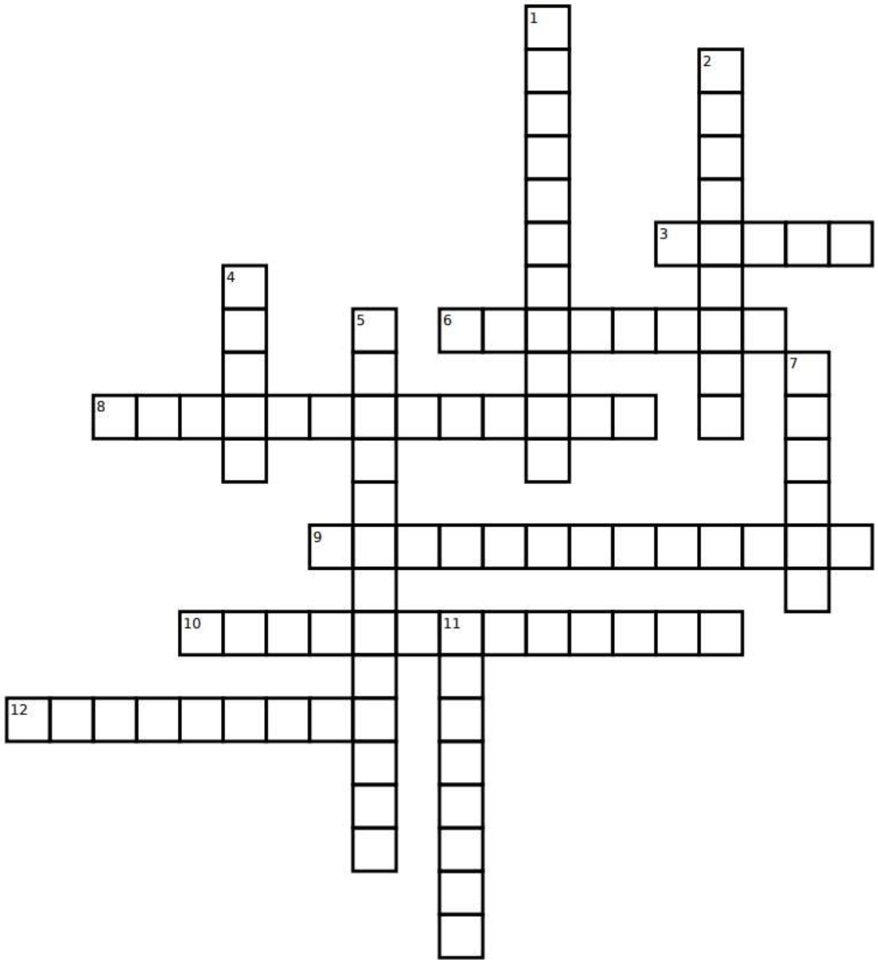
Why is Angle-Angle-Angle (AAA) not a valid congruence theorem?

When constructing the perpendicular bisector of a segment, which triangle congruence theorem justifies the step that proves the bisector is perpendicular?

How is the concept of congruence used to prove that a segment bisector drawn from the vertex of an isosceles triangle to the base is also an angle bisector?

End-of-Course Prep

Parallel Lines and Transversals



Across	Down
3. A logical argument that demonstrates the truth of a statement beyond any doubt. 6. Angles opposite each other when two lines cross. They share a vertex and are always congruent. 8. Two angles whose measures sum to exactly 180°. 9. Two lines that intersect to form a 90° (right) angle. 10. Angles that occupy the same relative position at each intersection where a straight line crosses two others. 12. Angles having the exact same measure or size.	1. A line that intersects two or more other lines (usually parallel lines) at distinct points. 2. __________ exterior angles are a pair of angles on opposite sides of the transversal and outside/between the two parallel lines. 4. A measure of the steepness and direction of a line. 5. Two angles whose measures sum to exactly 90°. 7. A pair of adjacent angles whose non-common sides are opposite rays (forming a straight line). 11. Two or more lines that lie in the same plane and never intersect, regardless of how far they are extended.

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